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EVOLVING SYSTEMS: NONLINEAR ADAPTIVE KEY COMPONENT CONTROL WITH PERSISTENT DISTURBANCE REJECTION

Mark J. Balas* **and Susan A. Frost†**

This paper presents an introduction to Evolving Systems, which are autonomously controlled subsystems that self-assemble into a new Evolved System with a higher purpose. Evolving Systems of aerospace structures often require additional control when assembling to maintain stability during the entire evolution process. This is the concept of Adaptive Key Component Control which operates through one specific component to maintain stability during the evolution. In addition this control must overcome persistent disturbances that occur while the evolution is in progress. We present theoretical results for the successful operation of Nonlinear Adaptive Key Component control in the presence of such disturbances and an illustrative example.

INTRODUCTION

Evolving Systems are autonomously controlled subsystems which self-assemble into a new Evolved System with a higher purpose[1-2]. Evolving Systems of aerospace structures often require additional control when assembling to maintain stability during the entire evolution process[3-5]. An adaptive key component controller has been shown to restore stability in Evolving Systems that would otherwise lose stability during evolution[6]. The adaptive key component controller uses a direct adaptation control law to restore stability to the Evolving System through a subset of the input and output ports on one key component of the Evolving System. Much of the detail of Evolving Systems appears in the chapter (ref. 8). In this paper, we will deal with the situation where persistent disturbances can appear in some components and must be mitigated by the adaptive key component controller. Such disturbances will often be attendant in actively controlled rendezvous and docking.

The control laws used by the adaptive key component controller to restore stability in an Evolving System are guaranteed to have bounded gains and asymptotic tracking if the Evolved System is almost strictly dissipative. Hence, it is desirable to know when the dissipativity traits of the subsystem components, including the key component, are inherited in an Evolving System. We present results describing when an Evolving System will inherit the almost strict dissipativity traits of its subsystem components. Then we will present an adaptive key component controller that restores asymptotic stability with bounded adaptive gains and mitigates the effect of persistent disturbances during evolution.

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* Electrical and Computer Engineering Department, University of Wyoming, 1000 E. University Ave., Laramie, Wyoming 82071, U.S.A. E-mail: mbalas@uwyo.edu.

† Intelligent Systems Division, NASA Ames Research Center, M/S 269-1, Moffett Field, California 94035, U.S.A. E-mail: susan.a.frost@nasa.gov.

MATHEMATICAL FORMULATION OF EVOLVING SYSTEMS

A mathematical formulation of a nonlinear time-invariant Evolving System is given here. Consider a system of L components of individually, actively controlled subsystems which can be described by the following equations for the ith component:

$$\begin{cases} \dot{x}_i = f_i(x_i, u_i) \\ y_i = g_i(x_i, u_i) \end{cases} \tag{1}$$

where $i = 1, 2, \ldots L$. The ith component has a performance cost function J_i and a Lyapunov function V_i. These are the building blocks of the Evolving System. When these individual components are joined to form an Evolved System, the new entity becomes:

$$\begin{cases} \dot{x} = f(x, u) \\ y = g(x, u) \end{cases} \tag{2}$$

with $x = [x_1 \ldots x_L]^T$, $y = [y_1 \ldots y_L]^T$, performance cost function J, and Lyapunov function V. The ith component in the above Evolved System is given by:

$$\dot{x}_i = f_i(x_i, u_i) + \sum_{j=1}^{L} \varepsilon_{ij} f_{ij}(x_i, x_j, u_j); \quad 0 \le \varepsilon_{ij} \le 1 \tag{3}$$

where $f_{ij}(x_i, x_j, u_j)$ represents the interconnections between the ith and j^{th} components. Note that when $\varepsilon_{ij} = 0$, the system is in component form and when $\varepsilon_{ij} = 1$, the components are connected and the system is fully evolved. As the system evolves, or joins together, the ε_{ij}'s continuously change from 0 to 1.

The components of the Evolving System are actively controlled by means of local control. Local control means dependence only on local state or local output information of the component, i.e., $u_i = h_i(x_i)$ or $u_i = h_i(y_i)$. In general, the local controller on the ith component would have the form:

$$\begin{cases} u_i = h_i(y_i, z_i) \\ \dot{z}_i = l_i(y_i, z_i) \end{cases} \tag{4}$$

where z_i is the dynamical part of the control law. Local control will be used to keep the components stable and meet the individual component performance requirements, J_i.

Once the system is fully evolved (i.e., $\varepsilon_{ij} = 1$), the ith component becomes:

$$\dot{x}_i = f_i(x_i, u_i) + \sum_{j=1}^{L} f_{ij}(x_i, x_j, u_j) \tag{5}$$

A state space version of the ith individual component of an Evolving System where the components are connected through the states can be represented as:

$$\begin{cases} \dot{x}_i = A_i(x_i) + B_i(x_i)u_i + \sum_{j=1}^{L} \varepsilon_{ij} A_{ij}(x_i, x_j);\ x_i(0) \equiv x_{0_i} \\ y_i = C_i(x_i) \end{cases} \tag{6}$$

where $i = 1, 2, \ldots L$, $x_i \equiv [x_1^i \ldots x_{n_i}^i]^{\mathrm{T}}$ is the component state vector, $u_i \equiv [u_1^i \ldots u_{m_i}^i]^{\mathrm{T}}$ is the control input vector, $y_i \equiv [y_1^i \ldots y_{p_i}^i]^{\mathrm{T}}$ is the sensor output vector, $(A_i(x_i), B_i(x_i), C_i(x_i))$ are vector fields of dimension n_i x n_i, n_i x m_i, and p_i x n_i, respectively, and the connection forces between components are represented in the n_i x n_j connection matrix, $A_{ij}(x_i, x_j)$ with $\varepsilon_{ji} = \varepsilon_{ij}$. The state space representation of the Evolved System then becomes:

$$\begin{cases} \dot{x} = A(x) + B(x)u \\ y = C(x) \end{cases} \tag{7}$$

which can also be written as $(A(x), B(x), C(x))$.

INHERITANCE OF SUBSYSTEM TRAITS IN EVOLVING SYSTEMS

We say a subsystem trait, such as stability, is inherited when the Evolved System retains the characteristic of the trait from the subsystem. Previous papers have examined the inheritance of stability and shown that stability is not a generally inherited trait[3-5]. Inheritance of almost strict passivity of subsystems is desirable in Evolving Systems that use an adaptive key component controller to restore stability.

In previous papers[5-6], a key component controller has been proposed to restore stability to Evolving Systems which would otherwise lose stability during evolution. The design approach used by the key component controller is for the control and sensing of the components to remain local and unaltered except in the case of one key component which has additional local control added to stabilize the system during evolution. The key component controller operates solely through a single set of input-output ports on the key component, see Fig. 1.

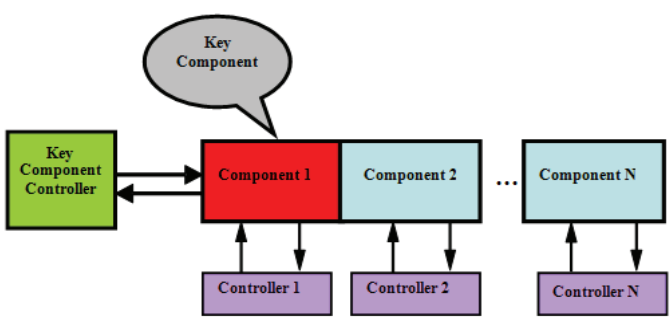

Figure 1. Key component controller.

Only the key component of the Evolving System needs modification to restore the inheritance of stability. A clear advantage of the key component design is that components can be reused in many different configurations of Evolving Systems without the need for component redesign. The reuse of components which are space-qualified, or at least previously designed and unit tested, could reduce the overall system development and testing time and should result in a higher quality system with potentially significant cost savings and risk mitigation.

In many aerospace environments and applications, the parameters of a system are poorly known and difficult to obtain. Adaptive key component controllers, which make use of a direct adaptation control law, are a good design choice for restoring stability in Evolving Systems where access to precisely known parametric values is limited. The sufficient condition for an Evolving System with an adaptive key component controller to be guaranteed to have bounded gains and to have asymptotic output tracking is that the system be almost strictly dissipative. So, we are interested in the conditions under which the inheritance of almost strict dissipativity can be guaranteed in Evolving Systems.

INHERITANCE OF ALMOST STRICT DISSIPATIVITY IN EVOLVING SYSTEMS

Inheritance of almost strict dissipativity of subsystems is desirable in Evolving Systems that use an adaptive key component controller to restore stability. Consider a nonlinear system of the form given by Eq. (7). We say this system is *Strictly Dissipative* when there exists a function $V(x) > 0 \, \forall x \neq 0$ such that the Lie derivatives satisfy:

$$\begin{cases} L_A V \equiv \nabla V A(x) \leq -S(x) \; \forall x \\ L_B V \equiv \nabla V B(x) = C^T(x); \nabla V \equiv \text{gradient } V \end{cases} \tag{8}$$

The function $V(x(t))$ is called the *Storage Function* for Eq. (7), and the above says that the storage rate is always less than the external power. This can be seen from

$$\begin{aligned} \dot{V} &\equiv \nabla V[A(x) + B(x)u] \\ &\leq -S(x) + C^T(x)u \\ &= -S(x) + \langle y, u \rangle \end{aligned} \tag{9}$$

Taking $u \equiv 0$, it is easy to see that Eq. (9) implies Eq. (8a) but not necessarily Eq. (8b); so Eq. (8) implies Eq. (9) but not conversely. They are only equivalent if Eq. (8a) is an equality. (When equality holds in Eq. (8) and Eq. (9), the property is known as *Strict Passivity*.)

We will say a system (u, y) is *Almost Strictly Dissipative* (ASD) when there is some output feedback, $u = G_* y + u_r$, so that the following is strictly dissipative:

$$\begin{cases} \dot{x} = A_C(x) + B(x)u_r; A_C(x) \equiv A(x) + B(x)G_* C(x) \\ y = C(x) \end{cases} \tag{10}$$

Now if each component of an Evolving System is ASD, then we have

$$\begin{cases} \nabla V_i[A_i(x_i)+B_i(x_i)G_iC_i(x_i)] \le -S_i(x_i)+\sum_{j=1}^{L}\varepsilon_{ij}\nabla V_i A_{ij}(x_j,u_j) \\ \nabla V_i B_i(x_i) = C_i^T(x_i) \end{cases} \tag{11}$$

where $\nabla V_i \equiv gradient V_i$. Due to the interconnection terms, Eq. (11) is not necessarily strictly dissipative. However, in some circumstances, the interconnection terms have a special form and ASD is inherited when the system evolves.

Suppose we have a pair of subsystems of the form:

$$\begin{cases} \dot{x}_i = A_i(x_i)+\varepsilon B_i(x_i)u_i + B_i^A(x_i)u_i^A \\ y_i = C_i(x_i) \\ y_i^A = C_i^A(x_i) \end{cases} \tag{12}$$

where $i=1,2$ and both subsystems $\left(\begin{bmatrix} u_1 \\ u_1^A \end{bmatrix},\begin{bmatrix} y_1 \\ y_1^A \end{bmatrix}\right)$ and $\left(\begin{bmatrix} u_2 \\ u_2^A \end{bmatrix},\begin{bmatrix} y_2 \\ y_2^A \end{bmatrix}\right)$ have storage functions V_i. We have the following result:

Theorem 1: If the subsystems (u_1^A, y_1^A) and (u_2^A, y_2^A) are ASD and

$$\nabla V_i B_i(x_i) = C_i^T(x_i);\ i=1,2 \tag{13}$$

then the resulting feedback connection, $y_1 = u_2$ and $\mathrm{u}_1 = -y_2$, will leave the composite system $\left(u_A \equiv \begin{bmatrix} u_1^A \\ u_2^A \end{bmatrix}, y_A \equiv \begin{bmatrix} y_1^A \\ y_2^A \end{bmatrix}\right)$ almost strictly dissipative.

Proof: See Appendix.

It was previously shown that the physical connection of components is equivalent to the feedback connection of the admittance of one to the impedance of the other[3-4]. When two components join to form an Evolved System, at their point of contact, their velocities are equal and the forces exerted are equal and opposite. If the two components are given by (f_1, v_1) and (f_2, v_2), then the contact dynamics of the Evolved System can be represented by:

$$\begin{cases} f_1 = -f_2 \\ v_1 \equiv \dot{q}_1 = v_2 \equiv \dot{q}_2 \end{cases} \tag{14}$$

This connection can be modeled as the admittance of one component connected in feedback with the impedance of the other component. When we use this idea of the joining of two components of an Evolving System as the feedback connection of their admittance and impedance, we can apply Theo. 1 from above to determine whether almost strict dissipativity is inherited by the Evolved System. Consequently, if the two subsystems (u_1, y_1) and (u_2, y_2) are in admit-

tance/impedance form, then Theo. 1 shows that ASD is an inherited property for nonlinear Evolving Systems.

MATHEMATICAL FORMULATION OF ADAPTIVE KEY COMPONENT CONTROLLER WITH PERSISTENT DISTURBANCE MITIGATION

In this section, we describe the mathematical formulation of an adaptive key component controller for restoring stability to an evolving system. The key component is chosen to be component #1 of the evolving system and will be modeled by the following nonlinear system with an external persistent disturbance:

$$\begin{cases} \dot{x}_1 = A_1(x_1) + \varepsilon B_1(x_1)u_1 + B_1^A(x_1)u_1^A + \Gamma_1(x_1)u_D \\ y_1 = C_1(x_1) \\ y_1^A = C_1^A(x_1) \end{cases} \tag{15}$$

All vector fields in Eq. (15) will have the appropriate compatible dimensions and be smooth in their arguments with a single equilibrium point at 0 in a neighborhood U.

The persistent disturbance input vector $u_D(t)$ is ND-dimensional and will be thought to come from the following disturbance generator:

$$\begin{cases} u_D = \Theta z_D \\ \dot{z}_D = F z_D ; z_D(0) = z_0 \end{cases}$$

where the disturbance state $z_D(t)$ is ND-dimensional. Such descriptions of persistent disturbances were first used to describe signals of known form but unknown amplitude[9]. For example, step disturbances yield $\Theta = 1$ and F=0 while sinusoidal disturbances can be described by

$$\begin{cases} \Theta = \begin{bmatrix} 1 & 0 \end{bmatrix} \\ F = \begin{bmatrix} 0 & 1 \\ -\omega_D^2 & 0 \end{bmatrix} \end{cases}$$

where the frequency ω_D is known but the amplitudes are unknown.

Assume that the disturbance generator parameter F is known. In many cases this is not a severe restriction as when the disturbance has the form of a sinusoidal or a step function. To improve understanding, the above can be written in the following equivalent form:

$$\begin{cases} u_D = \Theta z_D \\ z_D = L\varphi_D \end{cases} \tag{16}$$

where φ_D is a vector composed of the known basis functions for the solutions of $z_D(t)$ and (L, Θ) need not be known. The solution of $z_D(t)$ can be written as:

$$z_D(t) = e^{Ft} z_D(0) = \left[\varphi_1(t), \varphi_2(t), ..., \varphi_{N_D}(t)\right] z_D(0) = \sum_{i=1}^{N_D} z_D^i \varphi_i(t) = L\varphi_D$$

Note that L is directly related to F via its columns but not to θ. Some rearrangement of the entries in the columns of F is needed to create φ_D. A simple example of the above is given by the following:

$$\begin{cases} \dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u_D \\ u_D = a\sin(\omega_D t + b) = a_1 \sin(\omega_D t) + a_2 \cos(\omega_D t) \end{cases} \tag{17}$$

Assume $\left(\begin{bmatrix} u_1 \\ u_1^A \end{bmatrix}, \begin{bmatrix} y_1 \\ y_1^A \end{bmatrix}\right)$ is ASD. Also assume the following matching condition is true:

$$R(\Gamma_1(x_1)) \subseteq R(B_1^A(x_1)) \tag{18}$$

which says there exists H_* such that $B_1^A(x_1)H_* = \Gamma_1(x_1)$.

Component #2 will represent the rest of the evolving system and will be assumed to be strictly dissipative by choice of local controllers:

$$\begin{cases} \dot{x}_2 = A_2(x_2) + \varepsilon B_2(x_2) u_2 \\ y_2 = C_2(x_2) \end{cases} \tag{19}$$

The components are in admittance-impedance form so when they are joined $u_1 = -y_2$ and $u_2 = y_1$. The adaptive key component controller with disturbance mitigation works through the control input-output ports (u_1^A, y_1^A) of component #1 and is defined by:

$$\begin{cases} u_1^A = G_e y_1^A + G_D \phi_D \\ \dot{G}_e = -y_1^A (y_1^A)^T \gamma_e ; \gamma_e > 0 \\ \dot{G}_D = -y_1^A (\phi_D)^T \gamma_D ; \gamma_D > 0 \end{cases} \tag{20}$$

This produces $x \equiv \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \xrightarrow[t\to\infty]{} 0$ with bounded adaptive gains $\left(G_e, G_D\right)$ as the following convergence theorem shows:

Theorem 2: Assume that V_1 and V_2 are positive for any $x \neq 0$ and radially unbounded, and $\left(A(x), B(x), C(x)\right)$ are continuous functions of x and $S(x)$, above, is positive for any $x \neq 0$ and has continuous partial derivatives in x. Furthermore, assume:

1) The conditions of Theo.1 are satisfied; so that $\left(\begin{bmatrix} u_1 \\ u_1^A \end{bmatrix}, \begin{bmatrix} y_1 \\ y_1^A \end{bmatrix}\right)$ is almost strictly dissipative (ASD)

2) The matching condition: $R(\Gamma_1(x_1)) \subseteq R(B_1^A(x_1))$

3) ϕ_D is bounded (or F has only simple imaginary poles and no right half-plane poles)

Then the adaptive key component controller given by Eq. (20) produces $x \equiv \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \xrightarrow[t\to\infty]{} 0$ with bounded adaptive gains (G_e, G_D) when component 1 is joined with component 2 into an Evolved System and the outputs $y_i = C_i(x_i) \xrightarrow[t\to\infty]{} 0$.

Proof: See Appendix.

It should be noted that the above results might only hold on a neighborhood $N_i(0, r_i) \equiv \{x_i / \|x_i\| < r_i\}$. Then the stability in Theo. 2 is only locally asymptotic to the origin.

ILLUSTRATIVE EXAMPLE

Example 1, which follows, is a two component linear flexible structure Evolving System. The components of Ex. 1 are stable when they are unconnected components, but the Evolving System fails to inherit the stability of the components. This example will be used to demonstrate the inheritance and lack of inheritance of almost strict dissipativity in Evolving Systems.

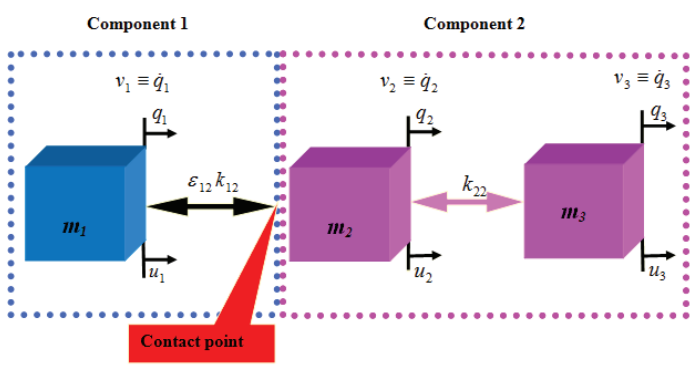

Figure 2. Example 1: A two component flexible structure Evolving System.

The dynamical equations for the components of Ex. 1 are:

$$\text{component 1: } \begin{cases} m_1\ddot{q}_1 = u_1 - \varepsilon_{12}k_{12}(q_1 - q_2) \\ y_1 = [q_1, \dot{q}_1]^{\mathrm{T}} \end{cases} \tag{21a}$$

$$\text{component 2:} \begin{cases} m_2\ddot{q}_2 = u_2 - \varepsilon_{12}k_{12}(q_2 - q_1) - k_{22}(q_2 - q_3) \\ m_3\ddot{q}_3 = u_3 - k_{22}(q_3 - q_2) \\ y_2 = [q_2, \dot{q}_2]^{\mathrm{T}} \\ y_3 = [q_3, \dot{q}_3]^{\mathrm{T}} \end{cases} \tag{21b}$$

with $m_1 = 30$, $m_2 = 1$, $m_3 = 1$, $k_{12} = 4$, and $k_{22} = 1$. Example 1 has controllers:

$$\begin{cases} u_1 = -(0.9s + 0.1)q_1 \\ u_2 = -\left(\dfrac{0.1}{s} + 0.2s + 0.5\right)q_2 \\ u_3 = -(0.6s + 1)q_3 \end{cases} \tag{22}$$

The subsystem components from Ex. 1 are stable in closed-loop form when they are unconnected, i.e., $\varepsilon_{12} = 0$. When $\varepsilon_{12} = 1$, the system is fully evolved and it has a closed-loop eigenvalue at 0.17, resulting in an unstable Evolved System.

A Simulink model was created to implement an adaptive key component controller for Example 1 as described in the previous section. Simulations were run in which the connection parameter, ε_{12}, ranged from 0 to 1, allowing the system to go from unconnected components to a fully Evolved System. The key component controller was able to maintain system stability during the entire evolution process when it used the input-output ports on mass 1 of component 1, see Fig. 3. When component 1 was the key component, the fully evolved system is almost strict positive real (ASPR), which is equivalent to almost strictly dissipativity for LTI systems[11].

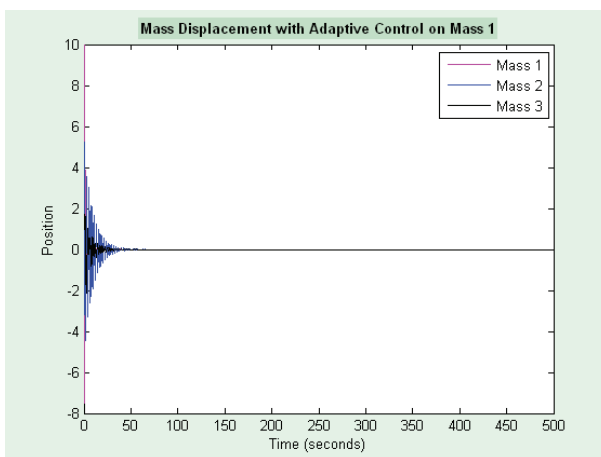

Figure 3. Adaptive key component controller on mass 1.

When the key component controller was located on component 2 and used the input-output ports on mass 3, stability was not maintained, see Fig. 4. The adaptive key component controller was not able to restore stability on mass 3 because that system was not ASPR, i.e., it had non-minimum phase zeros at 0.00515±0.2009i. Hence it did not satisfy the almost strictly dissipative condition of Theo. 2.

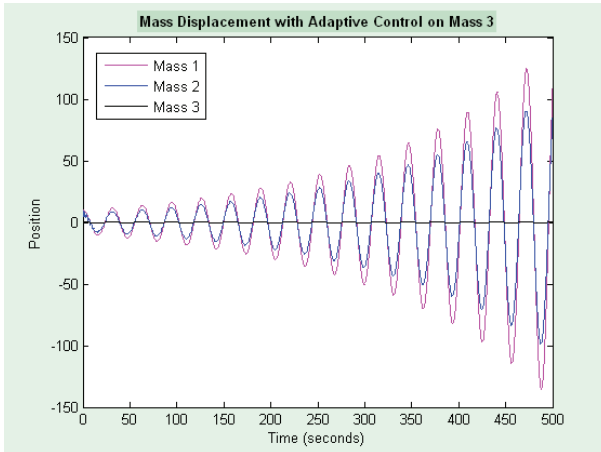

Figure 4. Adaptive key component controller on mass 3.

CONCLUSION

We have presented a result (Theo. 1) describing when an Evolving System will inherit the almost strict dissipativity traits of its subsystem components. An example was given of successful inheritance of almost strict dissipativity and failed inheritance of almost strict dissipativity. This result allows a control system designer to determine a sufficient condition for an Evolving System to use an adaptive key component controller to restore stability. We also presented a convergence result (Theo. 2) for an adaptive key component controller to restore stability during evolution and mitigate persistent disturbances.

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APPENDIX

Proof of Theorem 1:

Let (u_i^A, y_i^A) be ASD. From (9) and (11), there exists G_i^* such that

$$\begin{aligned} &\nabla V_i A_i^C(x_i) \equiv \nabla V_i \left[A_i(x_i) + B_i^A(x_i) G_i^* C_i^A(x_i) \right] \le -S_i(x_i) + \varepsilon_{ij} \nabla V_i A_{ij}\left(x_i, u_i, u_i^A\right) \\ &\nabla V_i B_i^A(x_i) = C_i^A(x_i)^T \end{aligned} \tag{A1}$$

If we connect (u_1, y_1) in feedback with (u_2, y_2), then $y_1 = u_2$ and $\mathrm{u}_1 = -y_2$ and, use (12) and (13), then we have $\nabla V_1 A_{12}(x_1, u_1, u_1^A) = \nabla V_1 B_1(x_1) u_1 = C_1^T(x_1)[-y_2] = -y_1^T y_2$ and similarly, $\nabla V_2 A_{21}(x_2, u_2, u_2^A) = y_2^T y_1$.

Let $x \equiv \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ and, from Eq. (12),

$$\begin{cases} \dot{x} = A(x) + B(x)u = \begin{bmatrix} A_1^C(x_1) + \varepsilon_{12} A_{12}(x_2) \\ A_2^C(x_2) + \varepsilon_{21} A_{21}(x_1) \end{bmatrix} + \begin{bmatrix} B_1^A(x_1) & 0 \\ 0 & B_2^A(x_2) \end{bmatrix} \begin{bmatrix} u_1^A \\ u_2^A \end{bmatrix} \\ y = \begin{bmatrix} y_1^A \\ y_2^A \end{bmatrix} = C(x) = \begin{bmatrix} C_1^A(x_1) \\ C_2^A(x_2) \end{bmatrix} \end{cases} \tag{A.2}$$

with $V = V_1 + V_2$, using Eq. (13) and $\varepsilon_{ji} = \varepsilon_{ij} \equiv \varepsilon$ from Eq. (3),

$$\begin{aligned} \nabla V A(x) &= \left[\nabla V_1 \ \nabla V_2\right] \begin{bmatrix} A_1^C(x_1) + \varepsilon A_{12}(x_2) \\ A_2^C(x_2) + \varepsilon A_{21}(x_1) \end{bmatrix} \\ &= \nabla V_1 A_1(x_1) + \varepsilon\left(-y_1^T y_2\right) + \nabla V_2 A_2(x_2) + \varepsilon\left(y_2^T y_1\right) \\ &\le -\left[S_1(x_1) + S_2(x_2)\right] + \varepsilon\left(-y_1^T y_2\right) + \varepsilon\left(y_2^T y_1\right) \\ &= -S(x) \end{aligned}$$

and

$$\nabla V B(x) = \left[\nabla V_1 \quad \nabla V_2\right] \begin{bmatrix} B_1^A(x_1) & 0 \\ 0 & B_2^A(x_2) \end{bmatrix} = \begin{bmatrix} C_1^A(x_1) \\ C_2^A(x_2) \end{bmatrix}^T = C^T(x)$$

Therefore $\left(u_A \equiv \begin{bmatrix} u_1^A \\ u_2^A \end{bmatrix}, y_A \equiv \begin{bmatrix} y_1^A \\ y_2^A \end{bmatrix} \right)$ is ASD with output feedback $\begin{bmatrix} u_1^A \\ u_2^A \end{bmatrix} \equiv \begin{bmatrix} G_1^* & 0 \\ 0 & G_2^* \end{bmatrix} \begin{bmatrix} y_1^A \\ y_2^A \end{bmatrix} + \begin{bmatrix} u_1^{Ar} \\ u_2^{Ar} \end{bmatrix}$ as desired. End of proof.

Proof of Theorem 2:

Since the physical connection of component 1 to component 2 is equivalent to the feedback connection $u_1 = -y_2$ and $u_2 = y_2$,

By Theo.1 we have that the closed-loop system (u_1^A, y_1^A) below is ASD:

$$\begin{cases} \dot{x}_1 = A_1(x_1) - \varepsilon B_1(x_1)C_2(x_2) + B_1^A(x_1)u_1^A \\ \dot{x}_2 = A_2(x_2) + \varepsilon B_2(x_2)C_1(x_1); 0 \le \varepsilon \le 1 \\ y_1^A = C_1^A(x_1) \end{cases} \tag{A3}$$

Rewrite Eq. (20) to obtain,

$$\begin{cases} u_1^A = G_e y_1^A + G_D \varphi_D = G_e^* y_1^A + G_D^* \varphi_D + \underbrace{\Delta G \eta}_{w} \\ \Delta G \equiv G - G^* = \begin{bmatrix} \Delta G_e & \Delta G_D \end{bmatrix}; \eta \equiv \begin{bmatrix} y_1^A \\ \varphi_D \end{bmatrix} \\ \Delta \dot{G} = \dot{G} = -y_1^A (y_1^A)^T \gamma; \gamma \equiv \begin{bmatrix} \gamma_e & 0 \\ 0 & \gamma_D \end{bmatrix} > 0 \end{cases} \tag{A4}$$

Combining (A.3) and (A.4) yields:

$$\begin{cases} \dot{x}_1 = A_1^C(x_1) - \varepsilon B_1(x_1)C_2(x_2) + \left[B_1^A(x_1)G_D^* + \Gamma_1(x_1)\theta L \right]\varphi_D + B_1^A(x_1)w \\ \dot{x}_2 = A_2(x_2) + \varepsilon B_2(x_2)C_1(x_1); 0 \le \varepsilon \le 1 \\ y_1^A = C_1^A(x_1) \end{cases} \tag{A5}$$

with $w \equiv \Delta G \eta$ and $G_D^* \equiv -H_* \theta L$ from Eq. (20) and $A_1^C(x_1) \equiv A_i(x_i) + B_i^A(x_i)G_i^* C_i^A(x_i)$.

Let $V = V_1 + V_2$ and we have:

$$\dot{V} = -S(x) + \left\langle y_1^A, w \right\rangle \tag{A6}$$

Now form $V_G \equiv \frac{1}{2} tr(\Delta G \gamma^{-1} \Delta G^T)$ and obtain from Eq. (A.3):

$$\begin{aligned} \dot{V}_G &\equiv tr(\Delta \dot{G} \gamma^{-1} \Delta G^T) = -tr(y_1^A (y_1^A)^T \Delta G^T) = -tr(y_1^A (w)^T) \\ &= -\left\langle y_1^A, w \right\rangle \end{aligned} \tag{A7}$$

Define: $V(x,\Delta G) \equiv V(x) + V_G(\Delta G)$ and, from Eq. (A.5) and Eq. (A7), we have:

$$\begin{aligned}\dot{V}(x,\Delta G) &\equiv \dot{V}(x) + \dot{V}_G(\Delta G) = -S(x) + \left\langle y_1^A, w \right\rangle - \left\langle y_1^A, w \right\rangle \\ &= -S(x) \le 0\end{aligned} \quad \text{(A8)}$$

This guarantees that all trajectories $(x,\Delta G)$ are bounded. If $\dot{V}(x,\Delta G)$ is uniformly continuous or $\ddot{V}(x,\Delta G)$ is bounded, then Barbalat's Lemma[10] yields:

$$S(x) \xrightarrow[t\to\infty]{} 0$$

and the positivity and continuity of $S(x)$ imply that $x \equiv \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \xrightarrow[t\to\infty]{} 0$. Now consider

$$\begin{aligned}\ddot{V}(x,\Delta G) = -\dot{S}(x) \le \left|\dot{S}(x)\right| &= \left|\frac{\partial S(x)}{\partial x}\dot{x}\right| \le \left\|\frac{\partial S(x)}{\partial x}\right\| \|\dot{x}\| \le \left\|\frac{\partial S(x)}{\partial x}\right\| \left[\|A(x)\| + \|B(x)\| \left\|w_1^A\right\|\right] \\ &\le \left\|\frac{\partial S(x)}{\partial x}\right\| \left[\|A(x)\| + \|B(x)\| \|\Delta G\| \left\|C_1^A(x_1)\right\|\right]\end{aligned}$$

which is bounded because $(x,\Delta G)$ is bounded, $S(x)$ has continuous partial derivatives and $\left(A(x), B(x), C(x)\right)$ are continuous, and a continuous function of bounded $x(t)$ is also bounded in *t*. So, $y_i = C_i(x_i) \xrightarrow[t\to\infty]{} 0$ because $C_i(x_i)$ is continuous. End of proof.