Advances in Adaptive Control Methods

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Integrated Resilient Aircraft Control

Constraint-Based Adaptive Control - Optimal Control Modification

Objective

- Introduces notion of **constraint-based adaptive control** that combines adaptive control with optimal control to achieve constrained error minimization.
- Develops robust **optimal control modification** adaptive law that **enforces linear quadratic constraints**.

Technical Challenges

- **Persistent excitation** (PE) can adversely affect robustness of adaptive control due to high-frequency input signals.
- **Nonlinear input-output mapping** of adaptive control can result in unpredictable performance.

Technical Approach

- Minimize LQ cost function $J=\lim_{t_f\to\infty}\frac{1}{2}\int_0^{t_f}\left[e(t)-\Delta(t)\right]^\top Q\left[e(t)-\Delta(t)\right]dt$

  subject to error dynamics $\dot{e}(t)=A_m e(t)+B\left[\tilde{\Theta}^\top(t)\Phi(x(t))-\epsilon(x(t))\right]$
- Approach based on application of Pontryagin's Minimum Principle
- **Optimal Control Modification Adaptive Law**

  $$\dot{\Theta}(t)=-\Gamma\Phi(x(t))\left[e^\top(t)P-\nu\Phi^\top(x(t))\Theta(t)B^\top PA_m^{-1}\right]B$$

  Lyapunov stability proof shows that the adaptive law is stable and tracking error is UUB.
- **Modification term proportional to persistent excitation** (PE) to counteract adverse effects of PE

Asymptotic Linearity for Linear Uncertainty

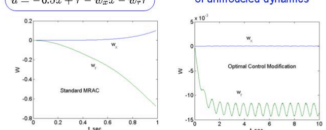
- Fast adaptation condition $\Phi^\top(x(t))\Gamma\Phi(x(t))\gg\|A_m\|\gg1$
- Asymptotic behavior $B\Theta^\top(t)\Phi(x(t))\to\frac{1}{\nu}P^{-1}A_m^\top Pe(t)$
- Linear tracking error dynamics for linear uncertainty

  $$\dot{e}(t)=-P^{-1}\left[\left(\frac{1+\nu}{2\nu}\right)Q-\left(\frac{1-\nu}{2\nu}\right)S\right]e(t)-B\Theta^{*\top}x(t)$$
- **Adaptive law can be designed to guarantee stability** for given bound on Θ^* using projection operator such that $A_c=-P^{-1}\left[\left(\frac{1+\nu}{2\nu}\right)Q-\left(\frac{1-\nu}{2\nu}\right)S\right]+B\Theta^{*\top}$ is Hurwitz and satisfies linear stability margin requirements everywhere inside projection bound $\Rightarrow$ **certifiable adaptive control**
- Note: e-modification or sigma-modification results in nonlinear error dynamics even for linear uncertainty

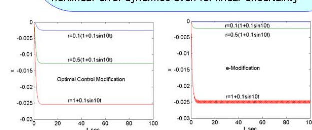
Example

$$\dot{x}=-x+2u-0.1y$$
$$\ddot{y}+100\dot{y}+100y=7x$$
$$r=1+0.1\sin 10t$$
$$u=-0.5x+r-w_x x-w_r r$$

1st-order plant with 2nd-order unmodeled dynamics and input at the same frequency as that of unmodeled dynamics



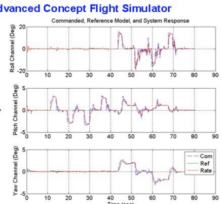
Robustness to Unmodeled Dynamics



Asymptotic Input-Output Linear Mapping

Pilot-in-the-Loop GTM Simulations

Pilot-in-the-Loop Advanced Concept Flight Simulator



10K ft, 250 Kn IAS
A = 0, B scaled
Doublet to capture flight director task

Summary

- Optimal Control Modification can provide stable fast adaptation to improve tracking
- Asymptotic linearity with fast adaptation can guarantee linear stability for linear structured uncertainty
- Pilot-in-the-loop simulations demonstrate effectiveness of the method

Adaptive Control of Time-Delay Systems - Time Delay Margin of MRAC

Objective

- Develops **stability analysis** for **time-delay adaptive system** and analytical tool to compute **time delay margin** (TDM) based on **Bounded Linear Stability Analysis**

Technical Challenges

- Currently no analytical tool exists to provide non-conservative and practical TDM estimate.

Technical Approach

- **Input-delay adaptive system**

  $$\dot{x}(t)=Ax(t)+B\left[u(t-t_d)+\Theta^{*\top}\Phi(x(t))\right]\qquad u_{ad}(t)=\Theta^\top(t)\Phi(x)$$
  $$u(t)=K_x x(t)+K_r r(t)-u_{ad}(t)\qquad \dot{\Theta}(t)=-\Gamma\Phi(x(t))e^\top(t)PB$$
- **Bounded Linearity Stability** approximates adaptive system as a locally bounded LTI system using time-window analysis

  $$\dot{\Theta}^\top(t)\Phi(x(t))=-B^\top Pe(t)\Phi^\top(x(t))\Gamma\Phi(x(t))\approx-\gamma B^\top Pe(t)$$
  $$\gamma=\frac{1}{T_0}\int_{t_i}^{t_i+T_0}\Phi^\top(x(\tau))\Gamma\Phi(x(\tau))d\tau>0\in\mathbb{R}$$
  $$t\in[t_i,t_i+T_0)\qquad i=0,1,\ldots,n$$

  $\Rightarrow$ $\dot{u}_{ad}(t)=\approx-\gamma B^\top Pe(t)+\Theta^\top(t)\dot{\Phi}(x(t))$

  γ can be viewed as integral product of adaptive gain and PE value
- Locally LTI approximation of tracking error dynamics

  $$\begin{bmatrix}\ddot{e}(t)\\ \dot{e}(t)\end{bmatrix}=C_i\begin{bmatrix}\dot{e}(t)\\ e(t)\end{bmatrix}-D_i\begin{bmatrix}\dot{e}(t-t_d)\\ e(t-t_d)\end{bmatrix}+\begin{bmatrix}d_1(t)+d_2(t-t_d)+d_3\\ 0\end{bmatrix}$$
  $$C_i=\begin{bmatrix}A+B\Theta^{*\top}\Phi_i' & 0\\ I & 0\end{bmatrix}\qquad D_i=\begin{bmatrix}A-A_m+B\Theta_i^\top\Phi_i' & \gamma BB^\top P\\ 0 & 0\end{bmatrix}$$
- **TDM estimation by matrix measure approach** - system is locally stable if time delay is less than TDM

  $$\omega_i<\overline{\mu}(-jC_i)+\|D_i\|$$
  $$t_{d_i}<\frac{1}{\omega_i}\cos^{-1}\frac{\overline{\mu}(C_i)+\overline{\mu}(jD_i)}{\|D_i\|}$$

Matrix Measure Properties

$$\overline{\mu}(C)=\max_{1\le i\le n}\lambda_i\left(\frac{C+C^*}{2}\right)=\lim_{\epsilon\to0}\frac{\|I+\epsilon C\|-1}{\epsilon}$$

$$\underline{\mu}(C)\le\mathrm{Re}\lambda_i(C)\le\overline{\mu}(C)\qquad \mathrm{Im}\lambda(C)\le\overline{\mu}(-jC)$$

$$\overline{\mu}(C)\le\|C\|$$

Given $\dot{x}(t)=Ax(t)-BKx(t-t_d),\ \lambda(A-BK)\in\mathbb{C}^-$

System is stable if $t_d<\frac{1}{\omega}\cos^{-1}\frac{\overline{\mu}(A)+\overline{\mu}(jBK)}{\|BK\|}$

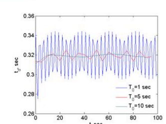
$$\omega<\overline{\mu}(-jA)+\|BK\|$$

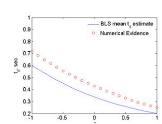
System is stable independent of time delay if

$$\overline{\mu}(A)<\|BK\|<-\underline{\mu}(A)$$

Example

$$\dot{x}=x+u+\theta^*x$$
$$\dot{x}_m=-x_m+\sin t$$
$$u=-2x+\sin t-\theta x$$





TDM estimate agrees well with simulation results

Summary

- New analytical method provides non-conservative TDM estimate
- Method can easily be extended to sigma-modification and optimal control modification