Advances in Adaptive Control Methods
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Constraint-Based Adaptive Control - Optimal Control Modification

Objective
- Introduce notion of constraint-based adaptive control that combines adaptive control with optimal control to achieve constrained error minimization.
- Develop robust optimal control modification adaptive law that enforces linear quadratic constraints.

Technical Challenges
- Persistent excitation (PE) can adversely affect robustness of adaptive control due to high frequency input signals.
- Nonlinear input-output mapping of adaptive control can result in unbounded performance.

Technical Approach
- Minimize $L_2$ cost function $J = \sum_{i=0}^{n} \int_0^T (x_i^2 + u_i^2) dt$
- Subject to control inputs $u_i = u_{opt}(\tilde{y}(i)) + \hat{v}(i)$
- Adaptation law $\dot{\hat{v}}(i) = \hat{v}(i) - \beta \hat{v}^2(i)$
- Linearization of adaptive law to ensure PE
- Optimal control modification: $u_{opt}(\tilde{y}(i)) = -\frac{1}{2} \frac{\partial J}{\partial \tilde{y}(i)}$

Example
- $\dot{x} = A_0 x + B_0 u + B_1 w$
- $y = C_0 x$
- $u_{opt}(\tilde{y}(i)) = -\frac{1}{2} \frac{\partial J}{\partial \tilde{y}(i)}$
- $\hat{v}(i) = \hat{v}(i) - \beta \hat{v}^2(i)$

Adaptive Control of Time-Delay Systems - Time Delay Margin of MRAC

Objective
- Derive stability analysis for time-delay adaptive system and analytical tool to compute time delay margin (TDM) based on Bounded Linear Stability Analysis.

Technical Challenges
- Cannot use analytical tool to define non-conservative and practical TDM estimate.

Technical Approach
- Input-delay adaptive system
- Bounded Linear Stability approximate adaptive system as a locally bounded LTI system using time-delay analysis.
- $\dot{x}(i) = A_1 x(i) + B_1 w(i)$
- $y(i) = C_1 x(i)$
- $\tau = \delta(k) - \delta(k-1)$
- $\Delta w(i) = w(i) - w(i-1)$
- $\Delta \tilde{y}(i) = \tilde{y}(i) - \tilde{y}(i-1)$
- TDM estimation by matrix measure approach - system is locally stable if time delay is less than TDM.

Summary
- New analytical method provides non-conservative TDM estimate.
- Method can easily be extended to eigenvalue modification and optimal control modification.

Matrix Measure Properties
- $\|\Delta w\|_2 \leq \sqrt{\lambda_{max}(C^T C)}$
- $\|\Delta \tilde{y}\|_2 \leq \sqrt{\lambda_{max}(C^T C)}$
- $\|x(i)\|_2 \leq \|x(i)\|_2$
- $\|y(i)\|_2 \leq \|y(i)\|_2$
- System is stable if $\tau = \delta(k) - \delta(k-1) < \gamma \lambda_{max}(C^T C)$
- System is stable independent of time delay if $\|x(i)\|_2 < \|x(i)\|_2$
- Method can easily be extended to eigenvalue modification and optimal control modification.