

PROBABILISTIC MATCHING OF LINES FOR THEIR HOMOGRAPHY

Taemin Kim^{*}, Jihwan Woo^{**}, and In So Kweon^{**}

^{*} NASA Ames Research Center, Moffett Field, CA, USA

^{**} Korea Advanced Institute of Science and Technology, Daejeon, Korea
taemin.kim@nasa.gov, {jihwoo, iskweon}@rcv.kaist.ac.kr

ABSTRACT

A probabilistic matching of lines, which form a homography in two images, is formulated in the framework of the forward stepwise regression. A membership matrix represents the likelihood of line correspondences to the homography. The correspondence measure is borrowed from the forward stepwise regression so that the squared error of the homography and the number of correspondences are balanced simultaneously. An alternating scheme for optimizing the membership and homography is provided. The experimental results on synthetic and real images validate the proposed method.

Index Terms—Line correspondences, Membership and homography, Forward stepwise regression

1. INTRODUCTION

In many computer vision applications, image registration is crucial for image analysis in which the information is obtained from the combination of various image sources. Image registration is to overlays two or more images of the same scene taken by different configurations such as times, viewpoints, and sensors. Typically, the registration methods consist of two main steps: feature matching and transform model estimation [1]. Estimating the transformation requires a set of reliable matches of features between the images while exact transformation helps to find the matches. Finding correspondence and transformation between two images brings us a chicken and egg problem.

Many invariant feature descriptors are developed to summarize their appearance and compared to find the correspondences. Appearance information includes color, intensity and edge orientation histogram at image features such as edges, corners and blobs [2][3]. The geometric invariance should be considered for robust feature matching. [5]. In the case that two images have largely different camera viewpoint and pose, appearance of features does not match well. For example of satellite and aerial images in Figure 1, only roofs of buildings are observed in the satellite image while façade parts dominate the appearance of buildings in the aerial image.

The graph matching problem has been approached in many different ways in the computer vision. A relaxation method for pair-wise attribute relations was proposed [7][8]. A dual-step expectation maximization algorithm was introduced to match geometric structure in 2D point-sets [9]. Spectral graph theory was applied to characterize the global structural properties [11][12][13]. Exact graph matching is proposed for rich structures such as road networks [6]. A probabilistic matching of line segments is founded in our previous paper [14]. However, it needs to describe some theoretical refinements. In addition, detection of line segments is vulnerable to noise more than lines.

A probabilistic matching of lines, which form a homography in two images, is formulated in the framework of the forward stepwise regression. The common features of satellite and aerial images (Figure 1) are line segments in the ground plane. The homography of two images induced by the ground plane is considered. A membership matrix represents the likelihood of line correspondences to the homography. The correspondence measure is borrowed from the forward stepwise regression so that the squared error of the homography and the number of correspondences are balanced simultaneously. An alternating scheme for optimizing the membership and homography is provided. The experimental results on synthetic and real images validate the proposed method.

2. PROBABILISTIC MATCHING OF LINES

Suppose a homography \mathbf{H} between two images π_1 and π_2



(a) Satellite image (b) Aerial image
Figure 1. Satellite and aerial images.

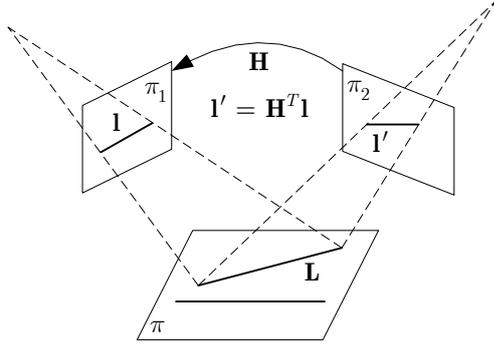


Figure 2. Line Homography

induced by a ground plane π (Figure 2). A line l' on π_2 is covariantly transformed to l on π_1 [5]:

$$l' = \mathbf{H}^T l. \quad (1)$$

Four line correspondences determine the homography \mathbf{H} .

The correspondence of two sets of lines is represented by a Boolean matrix which embeds a bijective one-to-one correspondence. Let L and L' be the line sets on π_1 and π_2 , respectively:

$$\begin{aligned} L &= \{l_1, l_2, \dots, l_n\} \text{ and} \\ L' &= \{l'_1, l'_2, \dots, l'_{n'}\}, \end{aligned} \quad (2)$$

where n and n' are the numbers of L and L' , respectively. The correspondence between L and L' is written by an indicator matrix $\mathbf{M} := [m_{ij}] \in \{0,1\}^{n \times n'}$:

$$m_{ij} = \begin{cases} 1 & \text{if } (i, j) \text{ is matched} \\ 0 & \text{otherwise.} \end{cases} \quad (3)$$

For all lines in L and L' , at most one correspondence is allowed to be embedded in \mathbf{M} . Let m_i and m'_j be the respective individual membership of $l_i \in L$ and $l'_j \in L'$:

$$\begin{aligned} m_i &:= \sum_{j=1}^{n'} m_{ij} \text{ and} \\ m'_j &:= \sum_{i=1}^n m_{ij}. \end{aligned} \quad (4)$$

Then they are restricted to be Boolean:

$$m_i, m'_j \in \{0,1\}. \quad (5)$$

The correspondence measure supporting the homography \mathbf{H} is defined to minimize its square error and to maximize the number of correspondences. The squared error is written by

$$SSE(\mathbf{M}, \mathbf{H}) = \sum_{i,j} m_{ij} SSE(i, j; \mathbf{H}), \quad (6)$$

where $SSE(i, j; \mathbf{H}) = \|\mathbf{H}^T l_i - l'_j\|^2$. The number of correspondence is written by

$$m := \sum_{i,j} m_{ij} \in (\frac{1}{2} \text{dof } \mathbf{H}, \min\{n, n'\}), \quad (7)$$

where dof denotes the degree of freedom. An objective

function which minimizes $SSE(\mathbf{M}, \mathbf{H})$ and maximizes m simultaneously is hard to define. Even if it is defined, computing the optimum \mathbf{M} is NP-hard.

The correspondence matrix is relaxed to be real value in $[0,1]$ to make the combinatorial optimization problem into a tractable nonlinear optimization problem. The membership matrix, the relaxed version of correspondence matrix, reflects the likelihood of the correspondences. Let us redefine $\mathbf{M} := [m_{ij}] \in [0,1]^{n \times n'}$, where m_{ij} is the membership of line correspondence (l_i, l'_j) . Each line in L

and L' cannot abuse its resources so that its individual memberships, the sum of all memberships of a line in one set to all lines in the other set, does not exceeds a unit. Accordingly, the relaxed constraints of individual memberships are

$$m_i, m'_j \in [0,1]. \quad (8)$$

The correspondence measure to minimize $SSE(\mathbf{M}, \mathbf{H})$ and maximize m simultaneously is borrowed from the forward stepwise regression. The forward stepwise regression is to select significant variables in the regression model [4]. It determines variables by the following significance test of a reduced model to full model:

$$F = \frac{MSR(\Delta \mathbf{M} | \mathbf{M}, \mathbf{H})}{MSE(\mathbf{M} + \Delta \mathbf{M}, \mathbf{H})}, \quad (9)$$

where $MSE(\bullet)$ and $MSR(\bullet | \bullet)$ means mean squared error of full and reduced models, respectively, and $\Delta \mathbf{M}$ is the indicator matrix which variables are added or removed. Forward (or backward) stepwise regression determines discrete $\Delta \mathbf{M}$ which maximizes (9) in each step. The relaxed

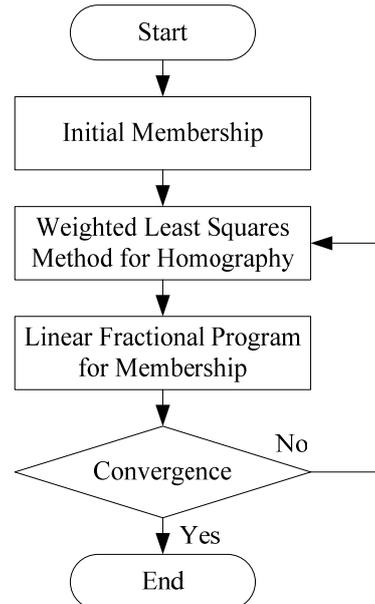


Figure 3. Flow chart of proposed algorithm

version of (9) induced by small $\Delta\mathbf{M}$ is derived:

$$\begin{aligned} F(\mathbf{M}, \mathbf{H}) &= \lim_{\Delta\mathbf{M} \rightarrow 0} \frac{MSR(\Delta\mathbf{M} | \mathbf{M}, \mathbf{H})}{MSE(\mathbf{M} + \Delta\mathbf{M}, \mathbf{H})} \\ &= \frac{m}{SSE(\mathbf{M}, \mathbf{H})} \sum_{i,j} SSE(i, j; \mathbf{H}). \end{aligned} \quad (10)$$

Therefore, membership \mathbf{M} and homography \mathbf{H} is obtained to maximize (10) subject to $\|\mathbf{H}\| = 1$.

The objective function $F(\mathbf{M}, \mathbf{H})$ is maximized in an alternating scheme is proposed because it is highly nonlinear with respect to \mathbf{M} and \mathbf{H} (Figure 3). On one hand, F of (10) is a quadratic function with respect to \mathbf{H} for a fixed \mathbf{M} . On the other hand, F is a linear fraction with respect to \mathbf{M} for a fixed \mathbf{H} . Given \mathbf{M} , \mathbf{H} is obtained by weighted least squares method. Conversely, \mathbf{M} is obtained by linear fractional program. Initial membership is crucial to find a correct correspondence and homography because F is not convex.

3. INVARIANTS OF FIVE COPLANAR LINES

The initial correspondences of line homography are restricted by the invariants of five coplanar lines in the case to match a small line set to a large reference one. Given five coplanar lines, labeled $\mathbf{l}_1, \mathbf{l}_2, \dots, \mathbf{l}_5$, two independent projective invariants are

$$\begin{aligned} I_1 &= \frac{\begin{vmatrix} L_{431} & L_{521} \\ L_{421} & L_{531} \end{vmatrix}}{\begin{vmatrix} L_{421} & L_{531} \\ L_{432} & L_{521} \end{vmatrix}} \text{ and} \\ I_2 &= \frac{\begin{vmatrix} L_{421} & L_{532} \\ L_{432} & L_{521} \end{vmatrix}}{\begin{vmatrix} L_{431} & L_{521} \\ L_{421} & L_{531} \end{vmatrix}}, \end{aligned} \quad (11)$$

where $L_{ijk} = [\mathbf{l}_i, \mathbf{l}_j, \mathbf{l}_k]$, and $|L_{ijk}|$ is the determinant of L_{ijk} . Notice that each L_{ijk} is viewed as the area of the triangle the vertices of which are the intersections of the lines $\mathbf{l}_i, \mathbf{l}_j$, and \mathbf{l}_k . Invariant vectors (I_1, I_2) of the reference set stored in a database are restricted by those of the small set, which drastically reduces the number of initial guesses.

The trigonometric invariants are defined to take into account degenerate configurations, uniform distribution of noise and cyclic continuity. For certain configurations, the labeling of lines in each L_{ijk} may make some determinants in the denominators vanish. Trigonometric invariants are defined as

$$\begin{aligned} \theta_1 &= \text{atan2}\left(\left|L_{431}\right| \left|L_{521}\right|, \left|L_{421}\right| \left|L_{531}\right|\right) \text{ and} \\ \theta_2 &= \text{atan2}\left(\left|L_{421}\right| \left|L_{532}\right|, \left|L_{432}\right| \left|L_{521}\right|\right), \end{aligned} \quad (12)$$

where atan2 is the two-argument variation of the arctangent function. In practice, the noise variance of cross-ratio is not uniform and is proportional to its values. Trigonometric invariants whiten the noise in somehow. Lastly, the configurations having cross-ratios of negative

and positive infinity are very close. The periodicity of trigonometric invariants accommodates this property. Figure 4 shows an example database of trigonometric invariants obtained from a satellite image.

4. EXPERIMENTAL RESULT

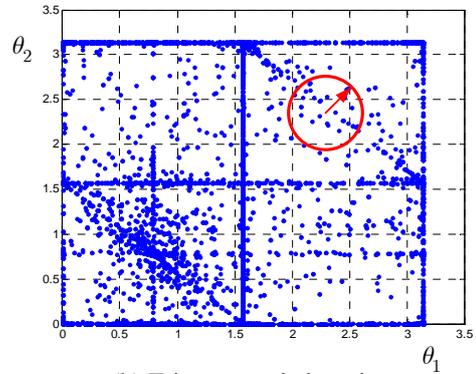
Simulation result of a toy example with three lines is shown in Figure 5. The line correspondence is set to be identity matrix and the homography is defined to enlarge the one image by the scaling factor 2. The initial membership is chosen from trigonometric invariants with small noise. The true and initial homographies are

$$\mathbf{H} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } \hat{\mathbf{H}}_0 = \begin{bmatrix} 0.2 & -2.1 & 4.7 \\ 0.2 & -2.0 & 4.3 \\ 0.0 & -0.4 & 1.0 \end{bmatrix}. \quad (13)$$

The initial lines transformed by \mathbf{H}_0 are distorted very much from the corresponding lines. The first four lines of image 1 (Figure 5a) are displayed in Figure 5b to show how much the initial homography distorts them. However, the proposed method recovers the perfect correspondence in a few iterations. The probabilistic matching is converged to



(a) Roadmap from a satellite image



(b) Trigonometric invariant

Figure 4. Trigonometric invariants of five coplanar lines

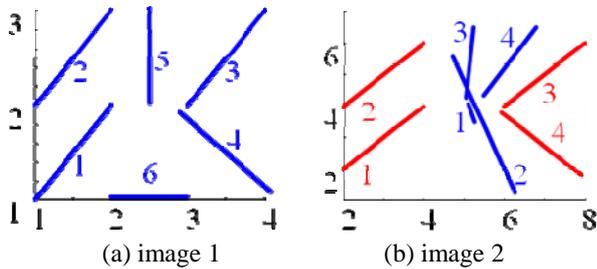


Figure 5. Toy example

the true correspondence from a reasonable initial guess. All lines are randomly rotated by the direction angle. The proposed method is tolerable to about 6° noise variance in direction angle of lines.

The experimental result for a real image is also convincing (Figure 6). Seven and eighteen lines are extracted manually from aerial and satellite images, respectively. The initial membership is chosen by the invariants of five coplanar lines. The homography is estimated by the proposed method:

$$\hat{\mathbf{H}} = \begin{bmatrix} 1.6 & 4.8 & 149 \\ -0.4 & 6.4 & 557 \\ 0.0 & 0.0 & 1.0 \end{bmatrix}. \quad (14)$$

The squared error of $\hat{\mathbf{H}}$ is 0.03471 and the relaxed number of correspondence is 7.21.

5. CONCLUSION

A probabilistic approach to match line between two images, taken from largely different camera configuration, is proposed to calculate their homography. A combinatorial problem of line matching is converted from the relaxed membership into nonlinear optimization. The forward stepwise regression speeds up the combinatorial optimization even though it is a suboptimal method. The relaxed version of the forward and backward stepwise regression is derived and maximized with respect to the

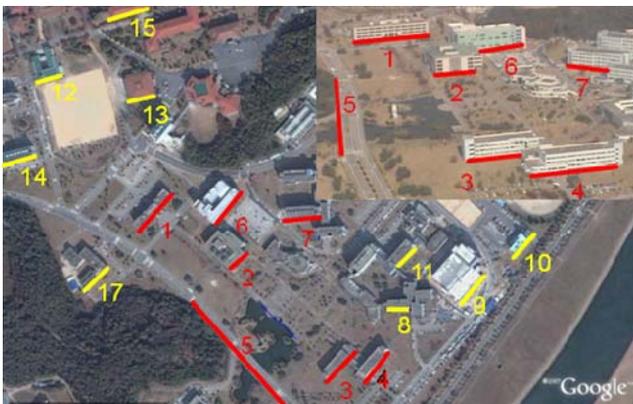


Figure 6. Aerial and satellite images

membership and homography. An alternating algorithm of the weighted least square method and linear fractional program provides the optimal solution. The probabilistic matching is converged to the optimal correspondence from a reasonable initial guess. Invariants of five coplanar lines restrict effectively the initial guess of the correspondence. The extensive error analysis is needed to verify the performance. The location of aerial vehicles is estimated from the homography.

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6. REFERENCES

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