Simulation of non-Markovian dynamics on IBM QX

Filip A. Wudarski
filip.a.wudarski@nasa.gov

Introduction

Currently available quantum computers allow us to run proof of principle algorithms that are unitary in their nature. Therefore, this architecture is unoptimized for simulation of an open quantum system. Here we present a method that helps us to overcome unitarity. We show how to run a non-Markovian evolution of a qubit system. We discuss all the discrepancies from theoretical predictions.

non-Markovianity

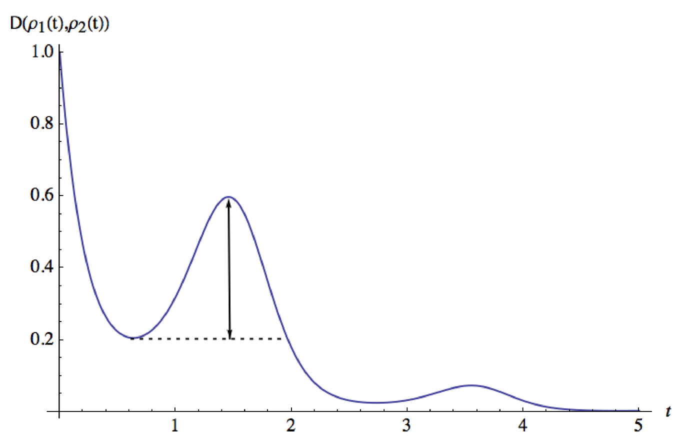
For open quantum systems dynamics described by a dynamical map Λ_t, we call it Markovian, if for all times $t \geq 0$ and for all pairs of state $\rho_1(t)$ and $\rho_2(t)$ undergoing the evolution, their distinguishability is monotonically decreasing in time, i.e.

$$\frac{d}{dt} D\Big(\Lambda_t(\rho_1(0)), \Lambda_t(\rho_2(0))\Big) \leq 0, \tag{1}$$

where

$$D\Big(\rho_1, \rho_2\Big) = \frac{1}{2}\|\rho_1 - \rho_2\|_1, \tag{2}$$

with trace norm $\|A\|_1 = \mathrm{Tr}(\sqrt{AA^\dagger})$.
If Eq. (1) is violated, we say that we witness non-Markovian evolution



Conclusion

We performed simulation of an open quantum systems dynamics on a quantum computer, when we were able to witness non-Markovianity. The take home message from this project is that:

1. leaving some qubits from the quantum register unmeasured, acts effectively as tracing them out, hence allows us to simulate non-unitary evolution,
2. one can associate parameters of the circuit (angles of the single qubit rotations) with time parameter,
3. imperfections of currently available devices strongly hinder the visibility of the results.

References

[1] H.-P. Breuer, E.-M. Laine, and J. Piilo, Phys. Rev. Lett. **103**, 210401 (2009).
[2.] H.-P. Breuer, G. Amato, and B. Vacchini, New J. Phys. **20**, 043007 (2018).

Collaboration

This work was done in collaboration with Giulio Amato, Heinz-Peter Breuer, Andreas Buchleitner form Albert-Ludwigs-Universität Freiburg, Panagiotis Barkoutsos from IBM Zurich Research Lab and Bassano Vacchini from Univeristy of Milan.

Dynamical model and Quantum Circuit

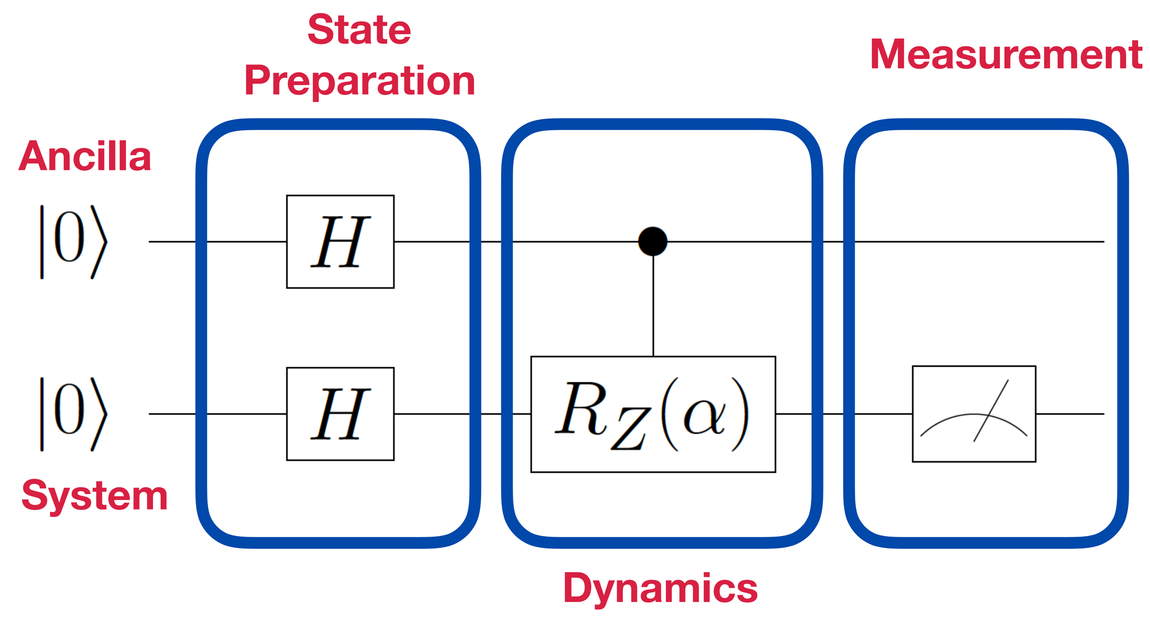
We focus on a qubit system that is a mixture of two unitaries (for thorough description of this model see [2])

$$\Lambda_t(\rho) = \frac{1}{2}\mathcal{U}_1(\rho) + \frac{1}{2}\mathcal{U}_2(\rho) = \frac{1}{2}U_t\rho U_t^\dagger + \frac{1}{2}\rho, \tag{3}$$

where

$$\mathcal{U}_1(\rho) = U_t\rho U_t^\dagger, \quad \mathcal{U}_2(\rho) = \rho, \qquad U_t = \exp(-it\sigma_z). \tag{4}$$

The above evolution is completely positive and trace preserving (CPTP), which cannot be directly implemented on a quantum computer. However, if we use additional qubits acting as an environment, and leave them unmeasured, one may achieve effectively non-unitary CPTP map. In our case, this is realized by the following quantum circuit



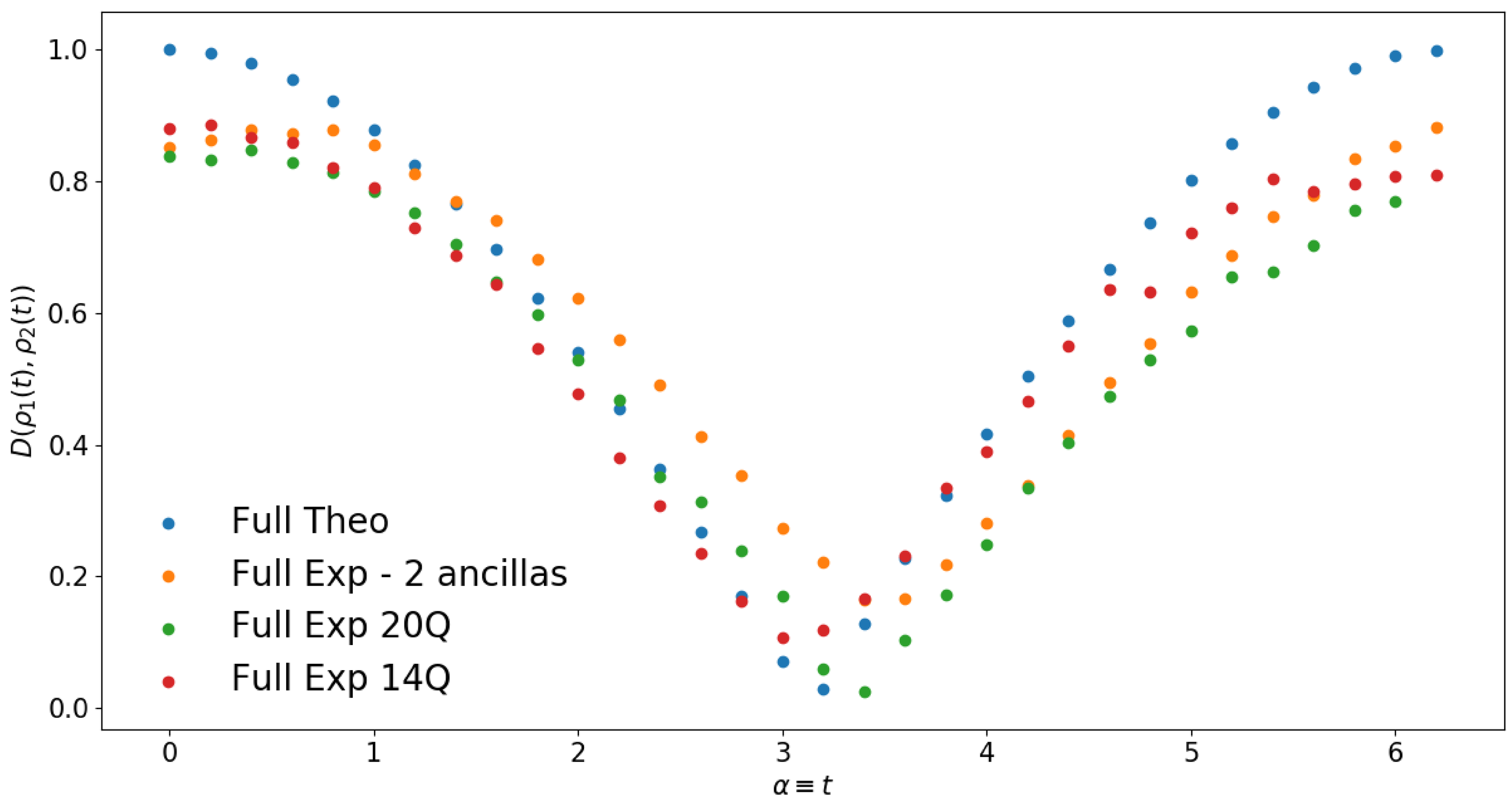
One associates rotational angle α with time t of the evolution.

Results

We tested this model on various quantum devices available through cloud services from IBM. In order to get trace distance, we mimicked the evolution of two orthogonal states $\|+\rangle$ and $\|-\rangle$ (eigenstates of σ_x). For each state we increased $\alpha \equiv t$ in steps of 0.2 radians and each evolved state was reconstructed with state tomography (in case of a qubit dynamics it suffices to measure Bloch vector components, i.e. measure in σ_x, σ_y and σ_z basis). Each measurement consisted of 8000 projections, from which we restored the expectation values of Pauli operators $\langle\sigma_k\rangle$, that is a Bloch vector component.
After having all states in Bloch vector representation, we computed trace distance between each pair of states for a given angle (time)

$$D(\rho_+, \rho_-) = \frac{1}{2}\sqrt{(x_+ - x_-)^2 + (y_+ - y_-)^2 + (z_+ - z_-)^2}, \tag{5}$$

where $x_\pm, y_\pm$ and $z_\pm$ are Bloch vector components for either $\|+\rangle$ or $\|-\rangle$ initial states.



We see substantial deviation of the results obtained from either machine and theoretical ones. The main reason for that lays in device's imperfections, that account for:

- Low gate fidelity (single qubit gates $\approx 99\%$, CNOT $\approx 95\%$),
- read-out errors (around 95%),
- inevitable interaction with the surroundings.

In one case we used two qubits as ancillary (imitating environment), however this case yielded even worse results, since, we had to use more gates.