Direct Adaptive Control of Non-Minimum Phase Linear Distributed Parameter Models of Large Flexible Structures

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ABSTRACT

Linear Distributed Parameter Systems are governed by partial differential equations. They are linear infinite dimensional systems described by a closed, densely defined linear operator that generates a continuous semigroup of bounded operators on a general Hilbert space of states and are controlled via a finite number of actuators and sensors. Many distributed applications are included in this formulation, such as large flexible aerospace structures, adaptive optics, diffusion reactions, smart electric power grids, and quantum information systems. Using a recently developed normal form for these systems, we have developed the following stability result: an infinite dimensional linear system is Almost Strictly Dissipative (ASD) if and only if its high frequency gain CB is symmetric and positive definite and the open loop system is minimum phase, i.e. its transmission zeros are all exponentially stable. In this paper, we focus on infinite dimensional linear systems that are non-minimum phase because a finite number of transmission zeros are unstable. Several methods to compensate for this issue modify the output of the infinite dimensional plant and then control this modified output rather than the original control output. Here we use a finite dimensional residual mode filter to modify the output to produce a fully minimum phase system. Then direct adaptive control for the infinite dimensional plant can use this modified output rather than the original output, to achieve ASD and produce asymptotically stability of the states on the Hilbert space. These results are illustrated by application to direct adaptive control of general linear systems on a Hilbert space that are described by operators with compact resolvent.

Keywords: Distributed Parameter Systems, Flexible Mechanical Structures, Adaptive Control Systems

1. INTRODUCTION

Linear Distributed Parameter Systems are governed by partial differential equations. They are linear infinite dimensional systems described by a closed, densely defined linear operator that generates a continuous semigroup of bounded operators on a general Hilbert space of states and are controlled via a finite number of actuators and sensors. Many distributed applications are included in this formulation, such as large flexible aerospace structures, adaptive optics, diffusion reactions, smart electric power grids, and quantum information systems. Using a recently developed normal form for these systems, we have developed the following stability result: an infinite dimensional linear system is Almost Strictly Dissipative (ASD) if and only if its high frequency gain CB is symmetric and positive definite and the open loop system is minimum phase, i.e. its transmission zeros are all exponentially stable.

In this paper, we focus on infinite dimensional linear systems that are non-minimum phase because a finite number of transmission zeros are unstable. Several methods to compensate for this issue modify the output of the infinite dimensional plant and then control this modified output rather than the original control output. Here we use a finite dimensional residual mode filter to modify the output to produce a fully minimum phase system. Then direct adaptive control for the infinite dimensional plant can use this modified output rather than the original output, to achieve ASD and produce asymptotically stability of the states on the Hilbert space. These results are illustrated by application to direct adaptive control of general linear systems on a Hilbert space that are described by operators with compact resolvent.

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2. ADAPTIVE DISTRIBUTED PARAMETER SYSTEM CONTROL

Let $\tilde{X}$ be an infinite dimensional *separable* Hilbert space with *inner product* (x, y) and corresponding norm $\|x\| \equiv \sqrt{(x,x)}$. Let $\tilde{A}$ be a closed linear operator with domain $D(\tilde{A})$ dense in $\tilde{X}$. Consider the Linear Infinite Dimensional System:

$$\begin{cases} \dfrac{\partial \tilde{x}}{\partial t}(t) = \tilde{A}\tilde{x}(t) + \tilde{B}u(t), \tilde{x}(0) \equiv \tilde{x}_0 \in D(\tilde{A}) \\ \tilde{B}u \equiv \sum_{i=1}^{m} b_i u_i \\ \tilde{y}(t) = \tilde{C}\tilde{x}(t), \tilde{y}_i \equiv (c_i, \tilde{x}(t)), i = 1 \ldots m \end{cases} \tag{1}$$

where $\tilde{x}(t) \in D(\tilde{A})$ is the plant state, $b_i \in D(\tilde{A})$ are linearly independent actuator influence functions, $c_i \in D(\tilde{A})$ are linearly independent sensor influence functions, *u(t)*, $\tilde{y}(t) \in \mathrm{R}^M$ are the control input and plant output M-vectors respectively. The closed linear operator $\tilde{A}$ with dense domain $D(\tilde{A})$ dense in $\tilde{X}$ generates a C_0 semigroup of bounded linear operators $\tilde{U}(t)$ on $\tilde{X}$. The dynamics stated above will be assumed to be *minimal* in the sense that they are fully realized in the corresponding infinite dimensional plant transfer function: $\tilde{P}(s) = \tilde{C}(sI - \tilde{A})^{-1}\tilde{B}$, and there are no hidden plant dynamics.

Now for this paper, we want to describe the plant in (1) as being decomposed into an infinite dimensional subsystem (A, B, C) and a finite dimensional stable subsystem (A_Q, B_Q, C_Q). *The subsystem* (A, B, C) *will be assumed to be Almost Strictly Dissipative as defined in later sections of this paper.* So we rewrite (1) as

$$\tilde{x} \equiv \begin{bmatrix} x \\ x_Q \end{bmatrix} \in \underbrace{X \times X_Q}_{\tilde{X}}$$

$$\begin{cases} \dfrac{\partial}{\partial t}\begin{bmatrix} x \\ x_Q \end{bmatrix} = \underbrace{\begin{bmatrix} A & 0 \\ 0 & A_Q \end{bmatrix}}_{\tilde{A}}\begin{bmatrix} x \\ x_Q \end{bmatrix} + \underbrace{\begin{bmatrix} B \\ B_Q \end{bmatrix}}_{\tilde{B}} u(t) \\ x(0) \equiv x_0 \in D(A) \text{ dense in } X \\ \& \dim X_Q \equiv N < \infty \\ \tilde{y}(t) = \underbrace{\begin{bmatrix} C & C_Q \end{bmatrix}}_{\tilde{C}}\begin{bmatrix} x \\ x_Q \end{bmatrix} = \underbrace{Cx}_{y} + \underbrace{C_Q x_Q}_{y_Q} \end{cases} \tag{1}$$

where *A* generates a C_0 semigroup of bounded linear operators *U(t)* on *X* and A_Q is a <u>stable</u> finite dimensional bounded linear operator on X_Q.

Furthermore (A, B, C) will be *assumed* to be <u>Almost Strictly Dissipative (ASD)</u> as defined in the next section.

What we will mean here by *asymptotic state regulation* is the following:

$$x(t)\xrightarrow[t\to\infty]{}0 \text{ for all } x_0 \in D(A) \tag{2}$$

This control objective will be accomplished using *only* the *modified output feedback* by a Direct Adaptive Control Law of the form:

$$u = Gy_c \tag{3a}$$

The direct adaptive controller will have adaptive gains given by:

$$\dot{G} = y_c y_c^{*} \gamma; \gamma > 0 \tag{3b}$$

where y_c will be the *new control output modified* by an RMF to be described later.

3. ALMOST STRICT DISSIPATIVITY

This section will refer only to the subsystem (A,B,C) above. Since there can be some ambiguity in the literature with the definition of strictly dissipative systems, we modify the suggestion of Wen in [8] for finite dimensional systems and expand it to include infinite dimensional systems.

Definition 1**:** The triple (A_c, B, C) is said to be *Strictly Dissipative (SD)* if A_c is a densely defined ,closed operator on $D(A_c) \subseteq X$ a separable complex Hilbert space with inner product (x,y) and corresponding norm $\|x\| \equiv \sqrt{(x,x)}$ and generates a C_0 semigroup of bounded operators $U(t)$,and (B,C) are (bounded) finite rank input/output operators with rank M where $B: R^m \to X$ and $C: X \to R^m$. In addition there exist Hermitian bounded operators P and Q on X such that $0 \le p_{\min} \|x\|^2 \le (Px,x) \le p_{\max} \|x\|^2$ and $0 \le q_{\min} \|x\|^2 \le (Qx,x) \le q_{\max} \|x\|^2$, i.e. P, Q are bounded and coercive, and the infinite dimensional Kalman-Yacubovic Conditions:

$$\begin{cases} \operatorname{Re}(PA_c x,x) \equiv \frac{1}{2}[(PA_c x,x) + \overline{(PA_c x,x)}] \\ \qquad = \frac{1}{2}[(PA_c x,x) + (x,PA_c x)] \\ \qquad = -(Qx,x) \le -q_{\min} \|x\|^2; \\ x \in D(A_c) \text{ and } PB = C^{*} \end{cases} \tag{4}$$

We also say that (A,B,C) is Almost Strictly Dissipative (ASD) when there exists G_* an *mxm* gain such that (A_c,B,C) is SD with $A_c \equiv A + BG_*C$. Similarly, (A,B,C) is Almost Dissipative (AD) when $Q=0$. Our definition of ASD is an extension of the concept of m-accretivity for infinite dimensional systems; see [9] pp278-280. Note that if P=I in (4), by the Lumer-Phillips Theorem [10], p405, we would have $\|U_c(t)\| \le e^{-\sigma t}; t \ge 0\ ; \sigma \equiv q_{\min} > 0$.

We will make the following set of assumptions:

Hypothesis 1: Assume the following:

i.) (A,B,C) is ASD, i.e. there exists a gain, G_* such that the triple $(A_C \equiv A + BG_*C, B, C)$ is SD

ii.) A is a densely defined ,closed operator on $D(A) \subseteq X$ and generates a C_0 semigroup of bounded operators $U(t)$,

iii.) $\operatorname{Re}(A_C x,x)$ is bounded on bounded subsets of $D(A) \subseteq X$.

From (3), we have $u \equiv Gy = G_* y + \underbrace{\Delta G y}_{\rho}$. And then, from (1), the Closed Loop System becomes

$$\begin{cases} \dfrac{\partial x}{\partial t} = \underbrace{(A + BG_* C)}_{A_c} x + B\Delta G y = A_c x + B\rho; \\ x \in D(A); \rho \equiv \Delta G y \\ y = Cx \\ \Delta \dot{G} = \dot{G} - \dot{G}_* = \dot{G} = -yy^* \gamma \ ; \quad \gamma > 0 \end{cases} \tag{5}$$

Since *B* and *C* are finite rank operators, so is $BG_* C$. Therefore $A_c \equiv A + BG_* C$ with $D(A_c) = D(A)$, generates a C_o semi-group $U_c(t)$ because *A* does; see [9] Theo 2.1 p 497.

From Appendix I in [30] using a Hilbert space version [16] of Lyapunov-Barbalat. We have the following *Adaptive Stabilization Theorem*

Theorem 1: Consider the coupled system of differential equations where $\begin{bmatrix} x \\ G \end{bmatrix} \in \bar{X} \equiv XxR^{mxm}$ Hilbert space with $\left(\begin{bmatrix} x_1 \\ G_1 \end{bmatrix}, \begin{bmatrix} x_2 \\ G_2 \end{bmatrix} \right) \equiv (x_1, x_2) + tr(G_1 \gamma^{-1} G_2)$ and

$$\left\| \begin{bmatrix} x \\ G \end{bmatrix} \right\| \equiv \left(\|x\|^2 + tr(G\gamma^{-1}G) \right)^{\frac{1}{2}}$$

$$\begin{cases} \dfrac{\partial x}{\partial t} = A_c x + B\underbrace{\left(G(t) - G_*\right)}_{\Delta G} y; x \in D(A_C), y \in R^m \\ y = Cx \\ \dot{G}(t) = -yy^* \gamma; \gamma > 0 \end{cases} \tag{6}$$

where $G(t)$ is the mxm adaptive gain matrix and γ is any positive definite constant matrix, each of appropriate dimension. *Under Hypothesis 1*, we have asymptotic state (and output) regulation, i.e. the system state $x(t)$ globally asymptotically converges to zero, and the adaptive gain matrix $G(t)$ is bounded.

On close examination, Theo.1 in various forms is the essence of direct adaptive control stability proofs in *finite dimensions*, when adaptive output feedback is used [20]-[21], [28].

4. NORMAL FORM FOR LINEAR INFINITE-DIMENSIONAL SYSTEMS

In this section we review the idea of a Normal Form for linear infinite dimensional systems (A, B, C).

The *High Frequency Gain* of the transfer function of (1) is defined as the *mxm* matrix: $CB \equiv [(c_i, b_j)]$.

In the *special case* that $(c_i, b_j) = (b_i, c_j)$ for all i, j then $(CB)^* = CB$ and when CB is nonsingular we have $CB > 0$. This is particularly true when $b_i = c_i \forall i$. In [30]-[31] we have shown that if CB is nonsingular then $P_1 \equiv B(CB)^{-1}C$ is a (non-orthogonal) bounded projection onto the finite dimensional range of *B*, $R(B)$, along the *null space of* $C, N(C)$ with $P_2 \equiv I - P_1$ the complementary bounded projection, and $X = R(B) \oplus N(C)$, as well as $D(A) = R(B) \oplus [N(C) \cap D(A)]$. We see that $R(B) = sp\{b_1, b_2, ..., b_m\}$ and $\dim R(B) = m$. But $N(C)$ will be infinite dimensional, and in general $N(C) \neq R(B)^{\perp}$.

Now for the above pair of projections $(P_1.P_2)$ we will have

$$\begin{cases} \dfrac{\partial P_1 x}{\partial t} = P_1 \dfrac{\partial x}{\partial t} \\ \quad = \underbrace{(P_1 A P_1)}_{A_{11}} P_1 x + \underbrace{(P_1 A P_2)}_{A_{12}} P_2 x + \underbrace{(P_1 B)}_{B} u \\ \quad = A_{11} P_1 x + A_{12} P_2 x + Bu \\ \dfrac{\partial P_2 x}{\partial t} = P_2 \dfrac{\partial x}{\partial t} \\ \quad = \underbrace{(P_2 A P_1)}_{A_{21}} P_1 x + \underbrace{(P_2 A P_2)}_{A_{22}} P_2 x + \underbrace{(P_2 B)}_{=0} u \\ \quad = A_{21} P_1 x + A_{22} P_2 x \\ y = \underbrace{(CP_1)}_{C} P_1 x + \underbrace{(CP_2)}_{=0} P_2 x = CP_1 x \end{cases}$$

because $y = Cx = C(B(CB)^{-1}C)x = CP_1 x$; $P_1 x = B(CB)^{-1}Cx = B(CB)^{-1}y$; $CP_2 = C - CB(CB)^{-1}C = 0$; and $P_2 B = B - B(CB)^{-1}CB = 0.$

From [30]-[31], we have the *Normal Form Theorem*:

Theorem 2: If CB is nonsingular, then with $l_2 \equiv \left\{ (x_k)_{k=1}^{\infty} / x_k \text{ complex}, \sum_{k=1}^{\infty} |x_k|^2 < \infty \right\}$, and $\left\langle (x_k)_{k=1}^{\infty}, (y_k)_{k=1}^{\infty} \right\rangle \equiv \sum_{k=1}^{\infty} x_k^* y_k$, there exists an invertible, bounded linear operator $W \equiv \begin{bmatrix} C \\ W_2 \end{bmatrix} : X \to \bar{X} \equiv \Re^m x l_2; \bar{B} \equiv WB = \begin{bmatrix} CB \\ 0 \end{bmatrix}$, $\bar{C} \equiv CW^{-1} = \begin{bmatrix} I_m & 0 \end{bmatrix}$, and $\bar{A} \equiv WAW^{-1}$ with $W^{-1} = \begin{bmatrix} Q_1 & Q_2 \end{bmatrix} = \begin{bmatrix} B(CB)^{-1} & W_2^* P_2 \end{bmatrix}$.

This coordinate transformation puts (1) into *normal form:*

$$\begin{cases} \dot{y} = \bar{A}_{11} y + \bar{A}_{12} z_2 + CBu \\ \dfrac{\partial z_2}{\partial t} = \bar{A}_{21} y + \bar{A}_{22} z_2 \end{cases} \tag{8}$$

where the subsystem: $(\bar{A}_{22}, \bar{A}_{12}, \bar{A}_{21})$ is called the *zero dynamics* of (1) and

$$\bar{A}_{11} \equiv CAB(CB)^{-1}; \bar{A}_{12} \equiv CAW_2^*;$$
$$\bar{A}_{21} \equiv W_2 AB(CB)^{-1}; \bar{A}_{22} \equiv W_2 A W_2^*$$

And $W_2 : X \to l_2$ by $W_2 x \equiv \begin{bmatrix} (\theta_1, P_2 x) \\ (\theta_2, P_2 x) \\ (\theta_3, P_2 x) \\ \cdots \\ \end{bmatrix}$ is an isometry from $N(C)$ onto $S_2 \equiv R(W_2) \subseteq l_2$.

Also we define the *Zero Dynamics Transfer Function* as $\overline{Z}(\lambda) \equiv \overline{A}_{12}(\lambda I - \overline{A}_{22})^{-1}\overline{A}_{21}$.

5. TRANSMISSION ZEROS AND THE ZERO DYNAMICS OF LINEAR INFINITE DIMENSIONAL SYSTEMS

Now it is possible to relate the point spectrum $\sigma_p(\overline{A}_{22}) \equiv \{\lambda / \lambda I - \overline{A}_{22} \text{ not } 1\text{-}1\}$ to the set $Z(A,B,C)$ of transmission (or transmission blocking) zeros of (A,B,C). Analogous to the finite-dimensional case [16], we can see that

$$Z(A,B,C) \equiv \left\{ \begin{array}{l} \lambda / H(\lambda) \equiv \begin{bmatrix} \lambda I - A & B \\ C & 0 \end{bmatrix}: \\ D(A)x\Re^m \to Xx\Re^m \text{ linear operator is not 1-1} \end{array} \right\}$$

From [29], we have:

Theorem 3:

$Z(A,B,C) = \sigma_p(\overline{A}_{22}) \equiv \{\lambda / \lambda I - \overline{A}_{22} \text{ is not } 1\text{-}1\}$ which is the point spectrum of $\overline{A}_{22}$. So the transmission zeros of the minimal infinite-dimensional open-loop plant (A,B,C) are the "eigenvalues (and limits of eigenvalues)" of its zero dynamics $(\overline{A}_{22}, \overline{A}_{12}, \overline{A}_{21})$.

Finite dimensional versions of this result appear in for example [14]-[15].

In this section we review the fact, from Appendix II of [30], that the ASD property is equivalent to two simple open-loop properties of the infinite dimensional system (A,B,C) involving the high frequency gain CB and the transmission zeros $Z(A,B,C)$.

Theorem 4: The Linear Infinite Dimensional System (A,B,C) is Almost Strictly Dissipative (ASD) if and only if the High Frequency Gain Matrix $CB \equiv [(c_i, b_j)]$ is a Hermitian and Sign Definite matrix (either positive definite or negative definite) and the Transmission Zeros, $Z(A,B,C)$, are (exponentially) stable, i.e., the zero dynamic subsystem $(\overline{A}_{22}, \overline{A}_{21}, \overline{A}_{12})$ is (uniformly) exponentially stable or equivalently: for all $z \in S_2 \subseteq l_2$, there exists a positive constant δ_z such that $\int_0^\infty \left\| \overline{U}_{22}(t) z \right\|^2 dt \le \delta_z < \infty$ where $\overline{U}_{22}(t)$ is the C_0 semigroup generated by $\overline{A}_{22}$.

This result gives necessary and sufficient conditions for ASD in terms of two *open-loop* system properties on the high frequency gain and the transmission zeros; no closed loop information is required. Theo.4 uses results from [11] and [22].

For some applications, the high frequency gain $CB \equiv [(c_i, b_j)]$ may be exactly known. If it is nonsingular, we can modify the input $u \equiv (CB)^{-1} v$ in (1) and treat v as the plant input or equivalently scale the gain in (3a): $u = (CB)^{-1} G y$. This will replace CB with the identity matrix I_m in the normal form (8).

In *finite dimensions*, a number of versions of Theo. 4 appear [13], [24]-[28], although they are often *not* both necessary and sufficient. Ref [13] does give necessary and sufficient conditions for ASD and also for the weaker condition: Almost Dissipative (AD). Note: In *finite_dimensional theory*, ASD is equivalent to Almost Strict Positive Real (ASPR) and AD is equivalent to Almost Positive Real.

We have shown that modifying the transmission zeros of a linear infinite dimensional system by static or dynamic output feedback is not possible. Clearly coordinate transformations do not change the transmission zeros.

Theorem 5 (Transmission Zero Invariance): The transmission zeros $Z(A,B,C)$ of the infinite dimensional plant (1) are invariant under *dynamic output feedback:* General Linear Infinite Dimensional Dynamic Output Feedback Controller

$$\begin{cases} u = L_{11}y + L_{12}\eta + L_{13}u_r \\ \dfrac{\partial \eta}{\partial t} = L_{21}y + L_{22}\eta + L_{23}u_r ; \eta(0) = \eta_0 \in D(L_{22}) \subseteq X \end{cases}$$

Proof: See [19].

6. ADAPTIVE CONTROL OF NONMINIMUM PHASE INFINITE DIMENSIONAL SYSTEMS USING A RESIDUAL MODE FILTER

This section contains the main result of this paper. In the above we have assumed that the full plant model $(\tilde{A},\tilde{B},\tilde{C})$ in (1) is nonminimum phase, i.e., there are a finite number of unstable transmission zeros. Consequently it is not ASD and hence the results in Theo. 4 do not apply to $(\tilde{A},\tilde{B},\tilde{C})$. However if $(\tilde{A},\tilde{B},\tilde{C})$ can be partitioned as in (1'), then we can modify the adaptive control law with a residual mode filter (RMF) so that the new system is ASD and then the results of Theo. 1 will apply. We must assume that the dynamics of the finite dimensional (A_Q,B_Q,C_Q) are known, even though we only know the infinite dimensional dynamics (A,B,C) has relative degree one (CB>0) and is minimum phase (all transmission zeroes are stable).

Consider the finite dimensional RMF:

$$\begin{cases} \hat{y}_Q = C_Q \hat{x}_Q \\ \dot{\hat{x}}_Q = A_Q \hat{x}_Q ; \hat{x}_Q(0) = 0 \end{cases} \tag{9}$$

And the modified control output:

$$y_c \equiv \tilde{y} - \hat{y}_Q \tag{10}$$

We note that the RMF is a finite dimensional state estimator for the (A_Q,B_Q,C_Q) subsystem with *estimator error*

$$\begin{aligned} & e_Q \equiv \hat{x}_Q - x_Q \\ & \Rightarrow \dot{e}_Q = A_Q e_Q \end{aligned}$$

Since A_Q is a bounded stable operator on the finite dimensional space X_Q, we have the following: $\exists P_Q > 0 \ni \operatorname{Re}(P_Q A_Q x_Q, x_Q) \le -\varepsilon_Q \left\| x_Q \right\|^2 \forall x_Q \in X_Q$.

The following is our Main Result:

Theorem 6: Under the Hypotheses on the plant $(\tilde{A},\tilde{B},\tilde{C})$ in the above development including that the subsystem (A,B,C) is ASD and the finite dimensional subsystem (A_Q,B_Q,C_Q) is stable, the direct adaptive controller with a residual mode filter RMF:

$$\begin{cases} u = Gy_c; \dot{G} = -y_c y_c^* \gamma; \gamma > 0 \\ y_c = \tilde{y} - \hat{y}_Q \\ \hat{y}_Q = C_Q \hat{x}_Q \\ \dot{\hat{x}}_Q = A_Q \hat{x}_Q + B_Q u \end{cases} \tag{11}$$

will produce globally asymptotic state regulation $x \xrightarrow[t\to\infty]{} 0$ with bounded adaptive gain G.

Proof: Consider

$$y_c \equiv \tilde{y} - \hat{y}_Q = [y + y_Q] - C_Q \hat{x}_Q = y - C_Q e_Q$$

$$\Rightarrow \begin{cases} \dfrac{\partial}{\partial t}\begin{bmatrix} x \\ e_Q \end{bmatrix} = \underbrace{\begin{bmatrix} A & 0 \\ 0 & A_Q \end{bmatrix}}_{\bar{\bar{A}}} \begin{bmatrix} x \\ e_Q \end{bmatrix} + \underbrace{\begin{bmatrix} B \\ 0 \end{bmatrix}}_{\bar{\bar{B}}} u \\ y_c = \underbrace{\begin{bmatrix} C & -C_Q \end{bmatrix}}_{\bar{\bar{C}}} \begin{bmatrix} x \\ e_Q \end{bmatrix} \end{cases}$$

From the definition of transmission zeros for infinite dimensional systems, form the operator $\bar{\bar{H}} \equiv \begin{bmatrix} \lambda I - \bar{\bar{A}} & \bar{\bar{B}} \\ \bar{\bar{C}} & 0 \end{bmatrix} = \begin{bmatrix} \lambda I - A & 0 & B \\ 0 & \lambda I - A_Q & 0 \\ C & -C_Q & 0 \end{bmatrix}$.

Then

$$\bar{\bar{H}}_e \equiv \begin{bmatrix} I & 0 & 0 \\ 0 & 0 & I \\ 0 & I & 0 \end{bmatrix} \bar{\bar{H}} \begin{bmatrix} I & 0 & 0 \\ 0 & 0 & I \\ 0 & I & 0 \end{bmatrix} =$$

$$= \begin{bmatrix} \lambda I - A & B & 0 \\ C & 0 & -C_Q \\ 0 & 0 & \lambda I - A_Q \end{bmatrix}$$

$$\therefore Z(\bar{\bar{A}}, \bar{\bar{B}}, \bar{\bar{C}}) = Z(A, B, C) \cup \sigma_p(A_Q)$$

Since (A, B, C) is minimum phase and A_Q is finite dimensional and stable, then $(\bar{\bar{A}}, \bar{\bar{B}}, \bar{\bar{C}})$ is minimum phase. Also $\bar{\bar{C}}\bar{\bar{B}} = \begin{bmatrix} C & -C_Q \end{bmatrix} \begin{bmatrix} B \\ 0 \end{bmatrix} = CB > 0$. Therefore $(\bar{\bar{A}}, \bar{\bar{B}}, \bar{\bar{C}})$ is ASD by Theo.4. Consequently by Theo.1 we have $\begin{bmatrix} x \\ e_Q \end{bmatrix} \xrightarrow[t\to\infty]{} 0$ and the adaptive gain G is bounded.

This completes the proof of Theo. 6.

7. ILLUSTRATIVE EXAMPLE: ADAPTIVE CONTROL OF LINEAR CAUCHY PROBLEMS

We will apply the above direct adaptive controller with RMF on the following single-input/single-output Cauchy problem:

$$\begin{cases} \dfrac{\partial \tilde{x}}{\partial t} = \tilde{A}\tilde{x} + \tilde{B}u, \tilde{x}(0) \equiv \tilde{x}_0 \in D(\tilde{A}) \subseteq \tilde{X} \text{ Hilbert Space} \\ \tilde{B}u = bu \\ \tilde{y} = \tilde{C}\tilde{x} = (c, \tilde{x}), \text{ with } b, c \in D(\tilde{A}) \end{cases} \tag{12}$$

We will assume that A is closed and densely defined *operator with compact resolvent*. This means A has discrete complex spectrum: $\{\lambda_k\}_{k=1}^{\infty}$ and

$\tilde{A}x = \sum_{k=1}^{\infty} \lambda_k (\varphi_k, x)\varphi_k$ where $\{\varphi_k\}_{k=1}^{\infty}$ is an orthonormal (Schauder) basis of eigenvectors for $\tilde{X}$; a special case for this occurs when $\tilde{A}$ is self-adjoint [9] Theo 6.29 p187.

Now we assume that (possibly with some rearrangement) that

$\tilde{A}\tilde{x} = \underbrace{\sum_{k=1}^{N} \lambda_k (\varphi_k, \tilde{x})\varphi_k}_{A_Q x_Q} + \underbrace{\sum_{k=N+1}^{\infty} \lambda_k (\varphi_k, \tilde{x})\varphi_k}_{Ax}$ where $x_Q \equiv \sum_{k=1}^{N} (\varphi_k, \tilde{x})\varphi_k$, $x \equiv \sum_{k=N+1}^{\infty} (\varphi_k, \tilde{x})\varphi_k$, and

$\mathrm{Re}\,\lambda_k < 0$ for $k = 1, 2, ..., N$ (*i.e.* A_Q is stable). Also, $B_Q u = \sum_{k=1}^{N} (\varphi_k, b)\varphi_k u$, and $Bu = \sum_{k=N+1}^{\infty} (\varphi_k, b)\varphi_k u$, $y_Q = (c, x_Q) = C_Q x_Q; y = (c, x) = Cx$. Furthermore, we assume that (A, B, C) is ASD.

Then the Modified Adaptive Controller

$$\begin{cases} u = G y_c \\ \dot{G} = -y_c y_c^{*} \gamma \\ y_c \equiv \tilde{y} - \hat{y}_Q \end{cases} \text{ with RMF } \begin{cases} \hat{y}_Q = C_Q \hat{x}_Q \\ \dot{\hat{x}}_Q = A_Q \hat{x}_Q + B_Q u \end{cases}$$

will produce globally asymptotic state regulation with bounded adaptive gain via Theo.6.

8. CONCLUSIONS

In this paper we have examined the theory of Direct Adaptive Model Reference Control for nonminimum phase linear systems on infinite-dimensional Hilbert spaces. Our main stability result is Theorem 6 which is based upon relating almost strict dissipativity to the equivalent conditions of minimum phase open-loop (all transmission zeros are exponentially stable) and relative degree one (CB>0); see Theo. 4. These results show that a simple direct adaptive controller modified by a finite dimensional residual mode filter (RMF) can produce asymptotic state regulation while the adaptive gains remain bounded.

The foundation for Theo 6 is the ability to partition the nonminimum phase infinite dimensional plant into the sum of a minimum phase infinite dimensional subsystem and a finite dimensional stable residual subsystem which is the basis for the RMF. At this time it is still an open question as to whether this can always be done for modal systems. What our results do show is that with a certain amount of trial and error, modal subsystems can be deleted one at a time from a distributed parameter model of a plant like a linear flexible structure to see if such a process will reveal a minimum phase subsystem. And from this a RMF modified adaptive controller can be created to provide globally asymptotic state regulation with bounded adaptive gains. This trial and error approach has worked for adaptive control of utility-scale wind turbines described by large scale structure approximations [34], which proves nothing but does suggest that finding such a partion of a modal system may not be as hard as it first appears.

We have illustrated our results by applying them to infinite dimensional system Cauchy problems governed by an operator with compact resolvent on an arbitrary Hilbert space. It is reasonable to expect that these results can be extended to handle model tracking and mitigation of persistent disturbances for nonminimum phase infinite dimensional systems.

Of course these results are valid for large-scale *finite dimensional* systems as well.

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