Multi-Objective Adaptive Control for Load Alleviation and Drag Minimization of Flexible Aircraft

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This paper describes a multi-objective flight control system design for flexible aircraft to take advantage of the availability of multi-functional distributed flight control surfaces to simultaneously gain aerodynamic efficiency, maneuver and gust load alleviation, and aeroservoelastic (ASE) mode suppression while maintaining traditional pilot command-tracking tasks. A multi-objective optimal control design is developed to establish a reference model of the closed-loop plant to achieve simultaneously the four objectives: pilot command tracking, drag optimization, maneuver load alleviation, and ASE mode suppression. An adaptive gust load alleviation control is designed to estimate the gust disturbance for the disturbance rejection using a modified least-squares gradient adaptive law method. Simulations of the gust load alleviation and drag optimization control on NASA Generic Transport Model equipped with flexible wings demonstrate the effectiveness of the multi-objective flight control method.

I. Introduction

Aircraft are typically designed to maintain efficient structural design for safe load-carrying capacity. Modern engineered materials such as composites have begun to appear in new airframe constructions for weight reduction purposes. A typical modern transport aircraft wing can provide less structural stiffness while maintaining the same load-carrying capacity as an older aluminum wing construction. There is a realization that next-generation aircraft concepts can be developed to take advantage of the structural flexibility afforded by modern engineered materials to improve the aerodynamic performance.

As aircraft wings become more flexible, adverse aerodynamics at off-design performance can result from changes in wing shapes due to aeroelastic deflections. Increased drag, hence increased fuel burn, is one such potential consequence. Without means for aeroelastic compensation, the benefit of weight reduction from the use of light-weight material could be offset by less optimal aerodynamic performance at off-design flight conditions. Performance Adaptive Aeroelastic Wing (PAAW) technology can potentially address these technical challenges for future flexible wing transports. PAAW technology leverages multi-disciplinary solutions to maximize the aerodynamic performance payoff of future adaptive wing design, while addressing simultaneously operational constraints that can prevent the optimal aerodynamic performance from being realized. These operational constraints include reduced flutter margins, increased airframe responses to gust and maneuver loads, and degraded pilot handling qualities as well as ride qualities. All of these constraints while seeking the optimal aerodynamic performance present themselves as a multi-objective flight control problem.

A multi-objective flight control framework has been developed to address these operational constraints and the efficiency goal simultaneously in order to arrive at optimal solutions that can provide good compromise between the efficiency goal and operational constraints. These optimal solutions take advantage of a multi-functional flight control system called the Variable Camber Continuous Trailing Edge Flap (VCCTEF) concept in this study. The VCCTEF is a possible candidate PAAW concept developed under the NASA Advanced Air Transport Technology (AATT) project.

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The VCCTEF concept was originally developed by a NASA Innovation Fund study entitled “Elastically Shaped Future Air Vehicle Concept” in 2010. This study examined new concepts that can enable active control of wing aeroelasticity to achieve drag reduction. The results showed that a highly flexible wing could be elastically shaped in-flight by active control of wing twist and vertical deflection in order to optimize the local angles of attack to improve the aerodynamic efficiency. The VCCTEF concept provides spanwise load tailoring via a continuous trailing edge formed by multiple spanwise flap sections which are joined together by elastomer transition sections as shown in Fig. 1. The spanwise load tailoring allows optimal lift distributions to be achieved throughout a given flight envelope, thereby enabling a mission-adaptive performance. A secondary benefit of the continuous trailing edge flap is the potential noise reduction during take-off and landing when the flap is configured for high-lift operations. In contrast, conventional flaps with gaps produce a significant noise source at high-lift conditions.

![Figure 1. GTM with with Variable Camber Continuous Trailing Edge Flap](image)

The VCCTEF also provides chordwise pressure shaping via a variable camber flap having three chordwise segments as shown in Fig. 2. These three chordwise flap segments can be individually commanded or actuated in unison when a flap deflection command is given. By varying the deflections of the individual chordwise flap segments, any camber surface can be created to achieve a desired aerodynamic performance. In general, a cambered flap is more efficient in producing lift than a straight conventional flap by achieving the same lift at a lower drag. The chordwise pressure shaping modifies the pressure distribution on a wing surface to achieve a drag reduction or to reduce the shock formation on the wing’s upper surface, thus allowing a higher cruise speed or reducing the transonic drag rise.

Initial study results indicate that the VCCTEF system can offer a potential pay-off in drag reduction that could translate into significant fuel savings. In order to realize the potential benefit of drag reduction by aeroelastic wing shaping control while meeting all other performance requirements and operational constraints, an integrated multidisciplinary approach is developed to incorporate a flight control system design into a modeling environment to maximize the system benefits of PAAW technology. Figure 1 illustrates the VCCTEF concept installed on a flexible wing GTM.

![Figure 2. Three-Segment Variable Camber Flap](image)

Subsequently, Boeing Research & Technology conducted a joint study with NASA in 2012 to develop a system design of the VCCTEF as shown in Fig. 3. This study was built upon the development of the original VCCTEF
system for the NASA Generic Transport Model (GTM) which is essentially based on the Boeing 757 airframe. The VCCTEF design includes 14 spanwise sections attached to the outer wing and 3 spanwise sections attached to the inner wing, as shown in Fig. 3. Each 24-inch section has three chordwise cambered flap segments that can be individually commanded. These cambered flaps are joined to the next section by a flexible and supported material (shown in blue) installed with the same shape as the camber and thus providing continuous flaps throughout the wing span with no drag producing gaps. The VCCTEF design results in the ability to control the wing twist shape along the wing span, thereby effectively producing a change to the wing twist to establish the best lift-to-drag ratio (L/D) at any aircraft gross weight or mission segment. The design wing twist on a traditional commercial transport is dictated by the aeroelastic deflection of a fixed “jig twist” shape applied during manufacturing. The design of this jig twist is set for only one cruise configuration, usually for a 50% fuel loading or mid-point on the gross weight schedule. The VCCTEF offers different wing twist settings, hence different spanwise loadings, for each gross weight condition and also different settings for climb, cruise and descent. This ability is a major factor in obtaining best L/D flight conditions.

Figure 3. GTM Wing Configured with the Variable Camber Continuous Trailing Edge Flap

Fight control design of conventional aircraft has a long heritage with the single-axis flight control philosophy for pitch control by the elevator, roll control by the aileron, and yaw control by the rudder. The VCCTEF represents a new class of multi-functional flight control for future PAAW technology. This potentially could open up a new flight control paradigm. In the presence of multi-functional flight control surfaces such as the VCCTEF, a conventional flight control task such as a pitch command could be designed in conjunction with other flight control requirements particularly for flexible wing aircraft. These additional requirements may include: 1) drag minimization, 2) aeroelastic mode suppression, 3) maneuver load alleviation, and 4) gust load alleviation. In addressing these requirements, a flight control system could provide improvements in pilot handling qualities and passenger ride comfort.

To address all of these flight control objectives simultaneously can be a challenge in a flight control design. A multi-objective optimization framework can be developed to address the needs for satisfying multiple, competing flight control requirements. This paper will present a multi-objective adaptive control approach to address some of these multi-disciplinary interactions in a flexible wing aircraft employing PAAW technology. A multi-objective flight control system has been previously developed to simultaneously gain aerodynamic efficiency, maneuver load alleviation, and aeroservoelastic (ASE) mode suppression while maintaining traditional pilot command-tracking tasks for guidance and navigation. This study continues to extend the previously developed multi-objective optimal control design to include adaptive control for gust load alleviation.
II. Multi-Objective Flight Control for Drag Minimization and Load Alleviation

Multi-objective flight control design is an enabling feature of PAAW technology. A typical flight control design usually takes into account different sets of requirements for performance and stability that must be considered during a design process. Performance in the context of flight control usually implies the ability for a flight control system to follow a pilot command. However, in this study, a new notion of aerodynamic performance is introduced into the flight control framework. The goal of the new vehicle is to achieve low drag through adaptive aeroelastic wing shaping control actuation. Thus, drag penalty due to the VCCTEF should be considered in a flight control design. Hence, a new concept of multi-objective flight control has been proposed to not only achieve a pilot command-following objective but also a drag reduction objective\textsuperscript{1,5–9} during maneuvers.

Stability is of paramount importance for any flight vehicle. Structural flexibility of airframes can cause significant aeroelastic interactions that can degrade vehicle stability margins, potentially leading to loss of control. There exists a trade-off between structural flexibility and flutter margins. With increased wing flexibility, a flutter margin could possibly occur below the FAA flutter clearance requirement. Thus, a flight control system must be able to stabilize any potential flutter modes. For obvious reasons, it is generally not acceptable to design an unstable transport aircraft that relies on feedback control for flutter suppression. Thus, in practice, an aeroelastically stable airframe will always be required, but the increase in structural flexibility may manifest itself as an increase in the airframe response to flight loads. Then, the role of a flight control system would be relegated to stability augmentation for aeroelastic mode suppression as opposed to a more demanding task of aeroelastic stabilization. This is generally considered to be more acceptable in the certification of flexible aircraft flight control since existing aircraft flight control systems already include many stability augmentation features such as the yaw and pitch dampers to provide desired damping characteristics to meet pilot handling quality requirements.

Gust and maneuver load alleviation control is also an important part of the overall flight control strategy for flexible aircraft. As structural flexibility increases, the vehicle aeroelastic response to a wind gust disturbance or during a maneuver can result in handling and ride quality issues. Gust load alleviation control can reduce the aeroelastic response by reactive feedback control or predictive feedforward control using early detection turbulence sensors. Similarly, maneuver loads can be kept to within a required load envelope by means of maneuver load alleviation control.

In terms of control actuation, the VCCTEF is designed with dual purposes. The two inner chordwise flap segments are driven by shaped memory alloy (SMA) actuators which are slow actuators suitable only for changing the VCCTEF settings for cruise drag optimization either by scheduling or real-time drag optimization\textsuperscript{4}. This is considered as a guidance feature that could be used in an auto-pilot cruise control. For fast-acting flight control functions, the outermost chordwise flap segment is designed to be a fast acting control surface driven by electro-mechanical actuators (EMA).\textsuperscript{4} This flap segment is spanned the entire wing and is assumed to have the required bandwidth and control power for roll control and aeroelastic mode suppression control.

The multi-objective flight control framework is envisioned to comprise of the following objectives all acting in a synergistic manner: 1) traditional pilot command-following flight control, 2) drag minimization, 3) aeroelastic mode suppression, and 4) gust and maneuver load alleviation. Each of these objectives can be a major control system design in its own right. Thus, a multi-objective flight control system can be a complex flight control design that takes into account multiple competing requirements to achieve optimal flight control solutions that have the best compromise for these requirements. Figure 4 illustrates an architecture of a multi-objective flight control system.\textsuperscript{5} In addition, a real-time drag minimization control strategy is included in the guidance loop.\textsuperscript{10} This feature utilizes system identification methods to estimate aerodynamic parameters for the on-line optimization. The aerodynamic parameters are also used in the multi-objective flight control for drag minimization and load alleviation control.

The ASE state space model can be written as

\[
\frac{\dot{x}_r}{\dot{x}_e} = \begin{bmatrix} A_{rr} & A_{re} \\ A_{er} & A_{ee} \end{bmatrix} \begin{bmatrix} x_r \\ x_e \end{bmatrix} + \begin{bmatrix} B_{rr} & B_{re} \\ B_{er} & B_{ee} \end{bmatrix} \begin{bmatrix} u_r \\ u_e \end{bmatrix} + \begin{bmatrix} w_r \\ w_e \end{bmatrix}
\]

(1)

where \(x_r(t)\) is rigid-body state vector which is available for feedback, \(x_e(t)\) is the elastic state vector including the actuator state vector of the VCCTEF dynamics which is not measured, \(u_r(t)\) is the rigid aircraft control input vector, \(u_e(t)\) is the VCCTEF control input vector, \(w_r(t)\) is the disturbance to the rigid aircraft state, and \(w_e(t)\) is the disturbance to the elastic state. For longitudinal dynamics, \(u_r(t) = \delta_e(t)\) is the elevator control surface deflection and \(u_e(t)\) are the symmetric VCCTEF control surface deflections. For lateral-directional dynamics, \(u_r(t) = \begin{bmatrix} \delta_a(t) \\ \delta_r(t) \end{bmatrix}^\top\) are the aileron and rudder control surface deflections and \(u_e(t)\) are the anti-symmetric VCCTEF control surface deflections.
The wing sensors which could be accelerometers or strain gauges are represented by

\[
a_w = a_w x + a_w u + a_w w = \begin{bmatrix} a_{wx} & a_{wu} \\ a_{wv} & a_{ww} \end{bmatrix} \begin{bmatrix} x_r \\ x_e \end{bmatrix} + \begin{bmatrix} a_{wur} & a_{wue} \\ a_{wwr} & a_{wwr} \end{bmatrix} \begin{bmatrix} u_r \\ u_e \end{bmatrix} + \begin{bmatrix} w_r \\ w_e \end{bmatrix} \tag{2}
\]

where \( x(t) = \begin{bmatrix} x_r^T(t) \\ x_e^T(t) \end{bmatrix} \), \( u(t) = \begin{bmatrix} u_r^T(t) \\ u_e^T(t) \end{bmatrix} \), and \( w(t) = \begin{bmatrix} w_r^T(t) \\ w_e^T(t) \end{bmatrix} \).

Collectively, the aircraft rigid aircraft states and wing acceleration measurements form the output of the system denoted by

\[
y = Cx + Du + Ew = \begin{bmatrix} I & 0 \\ a_{wxe} & a_{wwxe} \end{bmatrix} \begin{bmatrix} x_r \\ x_e \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ a_{wur} & a_{wwur} \end{bmatrix} \begin{bmatrix} u_r \\ u_e \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ a_{wwe} & a_{wwwe} \end{bmatrix} \begin{bmatrix} w_r \\ w_e \end{bmatrix} \tag{3}
\]

where \( y(t) = \begin{bmatrix} x_r^T(t) \\ a_{wxe}^T(t) \end{bmatrix} \).

The aircraft drag coefficient can be expressed as

\[
\Delta C_D = \left( \varepsilon_L^T + x^T C_{Ld} + u^T C_{Ld} \right) K (\varepsilon_L + C_{Ld} x + C_{Lu} u) - \varepsilon_L^T K \varepsilon_L = C_{Dx} x + C_{Du} u + x^T C_{Dx} x + x^T C_{Du} u + u^T C_{Du} u \tag{4}
\]

where \( K > 0 \) is a drag polar parameter, \( C_{Dx} = 2 \varepsilon_L^T K C_{Ld} \), \( C_{Du} = 2 \varepsilon_L^T K C_{Lu} \), \( C_{Dx} = C_{Ld}^T K C_{Ld} \), \( C_{Du} = 2 C_{Ld}^T K C_{Lu} \), and \( C_{Du} = C_{Ld}^T K C_{Lu} \). We assume that the drag derivatives are known or can be estimated in real time and \( \Delta C_D > 0 \).

The wing root bending moment is assumed to be available by measurements. It is given by

\[
M = M_w \tag{5}
\]

In addition, we assume that the normal acceleration measurements of the aircraft are available at the aircraft center of gravity (CG) and at a location near the tail. The aircraft acceleration vector is given by

\[
a_z = a_{zg} x + a_{zu} u + a_{zw} w \tag{6}
\]
We formulate a multi-objective cost function as follows:

\[
J = \lim_{T \to \infty} \frac{1}{2} \int_0^T \left[ (G_x x - G_r r)^T Q (G_x x - G_r r) + u^T Ru + q_D \Delta C_D + q_M M^T M \right] dt \tag{7}
\]

where \(G_x = \text{diag}(F,0), \ G_r = \text{diag}(I,0), \ Q = \text{diag}(Q_x, Q_r), \) and \(R = \text{diag}(R_r, R_v).\)

The rigid aircraft controller \(u_r(t)\) can be designed to enable a rigid aircraft state \(z(t) = F x_r(t)\) to track a constant command signal \(r(t)\) and load alleviation. The flexible wing controller \(u_w(t)\) is designed to provide drag minimization, aeroelastic mode suppression, and load alleviation.

The Hamiltonian for the whole aircraft is established as

\[
H = \frac{1}{2} (G_x x - G_r r)^T Q (G_x x - G_r r) + \frac{1}{2} u^T Ru + \frac{1}{2} q_D \left( C_{D_2} x + x^T C_{D_2} x + x^T C_{D_2 a} u + u^T C_{D_2 a}^T u \right) + \frac{1}{2} q_M M_x^T M_x + \lambda^T (Ax + Bu + w) \tag{8}
\]

The necessary conditions for optimality are given by

\[
\dot{\lambda} = -\frac{\partial H}{\partial x} = -G_x^T Q (G_x x - G_r r) - \frac{1}{2} q_D \left( C_{D_2} x + 2C_{D_2 a} x + C_{D_2 a} u \right) - q_M M_x^T M_x - A^T \lambda \tag{9}
\]

\[
\frac{\partial H}{\partial u} = Ru + \frac{1}{2} q_D \left( C_{D_2} x + C_{D_2 a} x + 2C_{D_2 a} u \right) + B^T \lambda = 0 \tag{10}
\]

The optimal control is then obtained as

\[
u = - \left( R + \frac{1}{2} q_D C_{D_2 a} \right)^{-1} \left( B^T \lambda + \frac{1}{2} q_D C_{D_2} x + \frac{1}{2} q_D C_{D_2 a}^T x \right) \tag{11}
\]

Let \(\dot{\lambda}(t) = W(t) x(t) + V(t).\) Then, differentiating \(\dot{\lambda}(t)\) yields

\[
W x + W \dot{A} x + \dot{\Lambda} W x - WBR^{-1} B^T W x + \dot{Q} x + \dot{\Lambda} V - WBR^{-1} B^T V - WBR^{-1} \frac{1}{2} q_D C_{D_2}^T + W w = G_x^T Q G_r r - \frac{1}{2} q_D C_{D_2}^T + \frac{1}{2} q_D C_{D_2 a} \tilde{R}^{-1} \frac{1}{2} q_D C_{D_2 a}^T \tag{12}
\]

where

\[
\tilde{R} = R + \frac{1}{2} q_D C_{D_2 a} \tag{13}
\]

\[
\tilde{A} = A - \frac{1}{2} q_D BR^{-1} C_{D_2 a} \tag{14}
\]

\[
\tilde{Q} = G_x^T Q G_x + q_D C_{D_2 a}^T - \frac{1}{4} q_D C_{D_2 a} \tilde{R}^{-1} C_{D_2 a}^T + q_M M_x^T M_x \tag{15}
\]

We choose \(q_D\) and \(q_M\) appropriately to ensure \(\tilde{Q} > 0.\)

For the infinite time-horizon optimal control, \(W = 0\) and \(V = 0.\) Therefore, we obtain

\[
W \dot{A} + \dot{\Lambda} W - WBR^{-1} B^T W + \tilde{Q} = 0 \tag{16}
\]

\[
V = V_0 + V_r r + V_w w \tag{17}
\]

\[
V_0 = \frac{1}{2} q_D \left( \dot{\Lambda} - WBR^{-1} B^T \right)^{-1} \left[ \left( WB + \frac{1}{2} q_D C_{D_2 a} \right) \tilde{R}^{-1} C_{D_2 a}^T - C_{D_2 a}^T \right] \tag{18}
\]

\[
V_r = \left( \dot{\Lambda} - WBR^{-1} B^T \right)^{-1} G_x^T Q G_r \tag{19}
\]

\[
V_w = - \left( \dot{\Lambda} - WBR^{-1} B^T \right)^{-1} W \tag{20}
\]

The multi-objective optimal controller is expressed as

\[
u = K_x \dot{x} + K_r r + K_w \dot{w} + \Lambda_0 \tag{21}
\]
where
\[ K_s = -\tilde{R}^{-1} \left( B^T W + \frac{1}{2} q_d C_{D_m}^T \right) \]  \hspace{1cm} (22)
\[ K_r = -\tilde{R}^{-1} B^T V_r \]  \hspace{1cm} (23)
\[ K_w = -\tilde{R}^{-1} B^T V_w \]  \hspace{1cm} (24)
\[ A_0 = -\tilde{R}^{-1} \left( B^T V_0 + \frac{1}{2} q_d C_{D_m}^T \right) \]  \hspace{1cm} (25)

Note that \( x_e (t) \) is not available and therefore has to be estimated by a Kalman filter observer design as
\[ \dot{x} = A \dot{x} + L (y - \dot{y}) + Bu + \dot{w} \]  \hspace{1cm} (26)
where \( L \) is the Kalman filter observer gain.

Then, the nominal closed-loop plant without the gust disturbance is expressed as
\[ \dot{x} = (A + BK_e) \dot{x} + L (y - \dot{y}) + BK_r r + B \Lambda_0 \]  \hspace{1cm} (27)

The nominal closed-loop plant is then used to establish the following reference model:
\[ \dot{x}_m = A_m x_m + B_m r + B \Lambda_0 \]  \hspace{1cm} (28)
where \( A_m = A + BK_e \) is Hurwitz and \( B_m = BK_r \).

### III. Adaptive Gust Load Alleviation

If the gust load only exists in the longitudinal axis, then the elevator can be used to cancel out the pitching moment component of the gust load which is normally the greatest. The lift and drag components can be effectively canceled out using the VCCTEF on the wings. Since there are multiple flight control surfaces that form the VCCTEF, only one or two flap elements are needed to cancel out the lift and drag components. For the gust load acting on the wings, the most active ASE modes are identified and ranked based on their damping values. Generally, only the first few lowest ASE modes are actively participating in the flexible aircraft dynamics. Sufficient control surface allocation can be designed to suppress the gust load components for these ASE modes.

Suppose \( x_e (t) \) is ordered from most active ASE modes to the least active ASE modes based on the pole locations and the damping values. It is assumed that the gust loads associated with the first \( n \) ASE modes are to be suppressed. The ASE state space model can be reduced in the order by the singular perturbation method\(^{11}\) to retain only those active ASE modes.

Consider a symmetric vertical gust case, the rigid aircraft state vector is described by \( x_r (t) = \left[ \begin{array}{c} \alpha (t) \\ q(t) \\ \theta (t) \end{array} \right] \).

The gust load on the rigid aircraft is denoted as \( w_r (t) = \left[ \begin{array}{c} w_x \alpha (t) \\ w_q (t) \\ 0 \end{array} \right] \). The aircraft acceleration measurements can be used to estimate the gust components due to the rigid aircraft. Since there are two acceleration measurements, the gust load on the rigid aircraft can be estimated.

The estimated acceleration measurements can be expressed as
\[ \dot{a}_z = a_{z_i} I_{aq} \dot{\hat{a}}_r = a_{z_i} I_{aq} \left( A_{rr} x_r + A_{re} \dot{x}_e + B_{re} u + \dot{w}_r \right) \]  \hspace{1cm} (29)
with
\[ a_{z_i} = \left[ \begin{array}{cc} V & 0 \\ V & l_r \end{array} \right] \]  \hspace{1cm} (30)
and
\[ I_{aq} = \left[ \begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \end{array} \right] \]
where \( V \) is the speed of the aircraft and \( l_r \) is the distance between the aircraft CG and the location of the aft accelerometer near the tail. Note that Eq. (30) neglects the normal acceleration component due to \( V \alpha \) which is assumed to be small when an aircraft is in trim.

The estimation error of the acceleration measurements is computed as
\[ \varepsilon_r = \dot{a}_z - a_z = a_{z_i} I_{aq} \left( A_{re} \dot{x}_e + \dot{w}_r \right) \]  \hspace{1cm} (31)
where \( \dot{x}_e (t) = \dot{x}_e (t) - x_e (t) \) and \( \dot{w}_r (t) = \dot{w}_r (t) - w_r (t) \).
The gust load on the rigid aircraft can then be estimated using a modified least-squares gradient adaptive law as follows:

$$I_{aq} \dot{w}_r = \begin{bmatrix} \dot{\hat{w}}_r \\ \dot{\hat{w}}_q \end{bmatrix} = -\Gamma_{wr} \left( a_{\hat{x}_r}^{-1} \varepsilon_r + v_{wr} I_{aq} \hat{w}_r \right)$$  \hspace{1cm} (32)

where $\Gamma_{wr} > 0$ is an adaptation rate matrix and $v_{wr} > 0$ is a tuning parameter designed to improve stability of the gust estimation.

To estimate the gust load on the flexible wing, we use the acceleration measurements on the wing which can be estimated as

$$\hat{a}_w = a_w \hat{x} + a_w u + a_{w_{\hat{w}}} \hat{w}$$  \hspace{1cm} (33)

We form the estimation error as

$$\varepsilon_e = \hat{a}_w - a_w = a_w \hat{x} + a_{w_{\hat{w}}} \hat{w}_r + a_{w_{\hat{w}}} \hat{w}_e$$  \hspace{1cm} (34)

where $\hat{x}(t) = \hat{x}(t) - x(t)$ and $\hat{w}_e(t) = \hat{w}_e(t) - w_e(t)$.

Since the number of gust load components is greater than the number of acceleration measurements on the wing, the estimation of the gust load on the flexible wing is provided by a modified least-squares gradient adaptive law as follows:

$$\dot{\hat{w}}_e = -\Gamma_{we} \left[ \begin{array}{cccc} a_{w_{\hat{w}}} & a_{w_{\hat{w}}} & \vdots & a_{w_{\hat{w}}} \end{array} \right]^{-1} \varepsilon_e + v_{we} \hat{w}_e$$  \hspace{1cm} (35)

where $\Gamma_{we} > 0$ is an adaptation rate matrix and $v_{we} > 0$ is a tuning parameter designed to improve stability of the gust estimation. Since there are fewer outputs than the number of elements in the gust load on the flexible wing, the estimation of the gust load on the flexible wing is in a least-squares sense.

### IV. Simulations

A coupled ASE longitudinal dynamic model of the flexible wing GTM is constructed.\textsuperscript{12} The wing stiffness is reduced by 50\% from the baseline stiffness. The model has 5 rigid aircraft states in the longitudinal direction but the airspeed and altitude states are removed, 10 aeroelastic modes with 2 elastic states and 4 aerodynamic lag states per mode, two rigid aircraft flight control inputs; namely, the elevator and engine throttle but the engine throttle is removed, and 16 VCCTEF inputs to the outermost chordwise flap segments. Thus, the model has a total of 63 states and 17 control variables.

![Figure 5. Sensors for Gust Load Alleviation Control](image-url)
Due to the stiffness imposed by the elastomer transition section, a constraint on the relative deflection of any two adjacent flaps is imposed on the control input command. In order to address this relative deflection constraint, a virtual control concept is used whereby the VCCTEF deflection is mapped into a mathematically smooth shape function whose coefficients are the virtual control variables. A cubic Chebyshev polynomial is selected as a candidate shape function. Then, the flap deflection of the $i$-th flap is expressed as

$$\delta_i = c_0 + c_1 k + c_2 (2k^2 - 1) + c_3 (4k^3 - 3k)$$

where $k = \frac{i-1}{n-1}$, $i = 1, 2, \ldots, 16$, $n = 16$ with $\delta_1$ being the inboard flap, and $c_j(t)$, $j = 0, 1, 2, 3$ are the virtual control variables. Thus, the flexible wing control vector is represented by $u_e(t) = \left[ c_0(t) \ c_1(t) \ c_2(t) \ c_3(t) \right]^T$.

Two accelerometers are placed in pairs at the wing tips. One accelerometer is placed forward of the elastic axis and one is placed aft of the elastic axis. The accelerometer locations on the left wing are indicated by the two red dots in Fig. 5. By placing the accelerometers in pairs forward and aft of the elastic axis, it is possible to not only measure the vertical acceleration of the wing section, but also the angular acceleration, i.e., twist acceleration. At a minimum, the two wing tip accelerometers suffice for a flight control design. Care must be exercised in the accelerometer signal conditioning to ensure synchronous sampling to prevent a phase shift among the accelerometer signals which could affect the control objective. Two normal acceleration measurements are available from two accelerometers placed on the fuselage center line at the aircraft CG and at a location 70 ft behind the aircraft CG near the quarter chord of the mean aerodynamic chord of the horizontal tail, as shown in Fig. 5. Additionally, a strain gauge sensor is placed in the proximity of the wing root on a load carrying member of the wing box. The strain gauge can be calibrated as a function of known static wing root bending moment to give a calibration constant for estimating the wing root bending moment in flight.

Figure 6. Gust Estimation Due to Full State Feedback without Load Alleviation Control but with Feedforward Control of Gust Estimates
An initial simulation with a simulated light continuous von Karman gust disturbance is conducted. A flight path angle controller is designed with \( Q = 1 \) for the flight path angle and \( Q = 0 \) for all other states. The control weight matrix is selected to be \( R = I \). The controller is designed without the drag optimization term \( (q_D = 0) \) and load alleviation control term \( (q_M = 0) \) in the LQR design. However, the feedforward control of the gust estimates using the feedforward gain \( K_w \) is active. The flight condition is Mach 0.8 at an altitude of 35,000 ft. A full state feedback design is first implemented for the estimation of the gust load in order to determine the effectiveness of the gust estimation as a benchmark for the observer design. Adaptation rate matrices \( \Gamma_{w_r} = 100I \) and \( \Gamma_{w_e} = 100I \) are used in the simulation. Figure 6 show the estimates of the gust disturbances for the angle of attack, pitch rate, and the first two elastic modes as compared to their corresponding actual gust disturbances. It can be seen that the estimates of the gust disturbances in the angle of attack and pitch rate dynamics with \( v_{w_r} = 0 \) are highly oscillatory. Using \( v_{w_r} = 1 \), the estimates of the gust disturbances in the angle of attack dynamics and especially the pitch rate dynamics are much improved. Similarly, the estimates of the gust disturbances for the elastic modes are also improved with \( v_{w_e} = 1 \), particularly for the second mode as shown in Fig. 6.

The load alleviation control is turned on next by setting \( q_M = \frac{1}{||M_x||^2} \). The gust estimation is shown in Fig. 7. It is noticed that the gust disturbance in the angle of attack has improved significantly while the other estimates remain about the same.

![Figure 7. Gust Estimation Due to Full State Feedback with Load Alleviation Control and Feedforward Control of Gust Estimates](image-url)

The gust load alleviation control effectiveness is due to both the load alleviation control term \( q_M \) and the feedforward control term \( K_w \) using the estimates of the gust disturbance. The aircraft response and the wing root bending
moment due to the baseline controller without the gust load alleviation control for the full-state feedback design by setting \( q_M = 0 \) and \( K_w = 0 \) are shown in Fig. 8. The maximum wing root bending moment reaches to a value of about 1.25 million lb-ft. It can be seen that the load alleviation control is quite effective in reducing the wing root bending moment to the gust load. On the other hand, the feedforward control of the gust estimates is more effective in reducing the aircraft response. When this term is not present, the flight path angle significantly deviates from zero, as shown by the red line in Fig. 8. When both control actions are present, both the aircraft response and wing root bending moment are substantially reduced, as shown by the green line in Fig. 8. A reduction in the maximum wing root bending moment by a factor of five is achieved with both the load alleviation control and the feedforward control of the gust estimates.

Figure 8. Aircraft Response and Wing Root Bending Moment Due to Full State Feedback with and without Load Alleviation Control and Feedforward Control of Gust Estimates

Figure 9 shows the control surface deflections for the four controller designs with and without the load alleviation control and the feedforward control of the gust estimates. All the control surface deflections are within the flap position limits. The control surface deflections are much more active with the gust load alleviation control than without when \( q_M = 0 \) and \( K_w = 0 \). The load alleviation control with \( q_M = \frac{1}{||M_x||^2} \) increases the effectiveness of the VCCTEF deflections as compared to that with \( q_M = 0 \), which can be seen in the two plots on the right of 9. On the other hand, the feedforward control of the gust estimates increases the activity of the elevator deflection, as can be seen in the two upper plots of 9.
Figure 9. Control Surface Deflections Due to Full State Feedback with and without Load Alleviation Control and Feedforward Control of Gust Estimates

Figure 10. VCCTEF Deflections Due to Full State Feedback with and without Load Alleviation Control and Feedforward Control of Gust Estimates
Figure 10 shows the VCCTEF deflections for the baseline controller design without load alleviation control ($q_M = 0$) and feedforward control of the gust estimates ($K_w = 0$) and for the gust load alleviation control. It can be seen that the VCCTEF is responsible for the effectiveness of the gust load alleviation control by acting as a direct lift control device. The inboard flap at the wing root (flap number 1) has the largest flap deflection of about 2°. The VCCTEF deflections then decrease monotonically toward the outboard flap at the wing tip (flap number 16) with a negative flap deflection of $-0.1^\circ$. The smoothly varying VCCTEF deflections are enabled by the use of the virtual control shape function. With the baseline controller, the VCCTEF deflections are essentially at zero.

The sensitivity of the gust load alleviation control to the load alleviation control weighting factor $q_M$ and the control weighting matrix $R$ is investigated next. The control weighting matrix is denoted by $R = \text{diag}(R_e, R_c)$ where $R_e$ is the control weighting factor for the elevator and $R_c$ is the control weighting matrix for the VCCTEF. Figure 11 shows the aircraft response and wing root bending moment for two values of $q_M = \frac{1}{||M_e||^2}$ and two values of $R_e = 0.1I / I$. Increasing $q_M$ from $\frac{1}{||M_e||^2}$ to $\frac{10}{||M_e||^2}$ while keeping $R_e = I$ causes the maximum wing root bending moment to decrease from $4.4 \times 10^5$ lb-ft to $1.6 \times 10^5$ lb-ft, a factor of almost 3. This is on top of a factor of 2 reduction in the maximum wing root bending moment with $q_M = \frac{1}{||M_e||^2}$ and $R_e = I$ at the same time instance, as shown in Fig. 8. Decreasing $R_e$ from $I$ to $0.1I$ while keeping $q_M = \frac{1}{||M_e||^2}$ has the about the same effect as increasing $q_M$ alone by a factor of 10. When $q_M$ increases to $\frac{10}{||M_e||^2}$ and $R_e$ decreases to $0.1I$, a huge reduction in the maximum wing root bending moment is achieved with the largest amplitude of about $0.2 \times 10^5$, or a factor of more than 20. This large reduction in the wing root bending moment comes from the reduction in the angle of attack as well as the pitch rate. Due to the presence of the VCCTEF on the wing as a distributed lift control system, this gust load alleviation control is possible. The VCCTEF effectively generates an opposing lift to cancel out the gust load on the wing.

![Figure 10: VCCTEF deflections for different load alleviation control settings.](image1)

![Figure 11: Aircraft Response and Wing Root Bending Moment Due to Full State Feedback with $q_M = \frac{1}{||M_e||^2}$, $R_e = I / I$.](image2)

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The control surface deflections for two values of $q_M = \frac{1}{\|M_x\|^2}$, $\frac{10}{\|M_x\|^2}$ and two values of $R_e = 0.1/1$ are plotted in Fig. 12. As can be seen, increasing $q_M$ to $\frac{10}{\|M_x\|^2}$ alone causes the maximum VCCTEF deflection to increase from 2.04° to 5.61° while the elevator only slightly decreases from $-1.62°$ to $-1.93°$. Thus, the reduction in the wing root bending moment can be attributed largely to the VCCTEF. Decreasing $R_e$ to 0.1/1 alone also causes the maximum VCCTEF deflection to increase to about 4.16° while the elevator stays the same. Finally, with $q_M = \frac{10}{\|M_x\|^2}$ and $R_e = 0.1/1$, the maximum VCCTEF deflection increases to about 8.58°.

Figure 12. Control Surface Deflections Due to Full State Feedback with $q_M = \frac{1}{\|M_x\|^2}$, $\frac{10}{\|M_x\|^2}$ and $R_e = 0.1/1$. 
An observer design is implemented next using $q_M = \frac{1}{|M_x|}$ and $R = I$. For the state estimation, the rigid aircraft states are used in conjunction with the accelerometers on the wing. The estimates of the elastic states are then used in conjunction with the rigid aircraft states to construct the partial state estimation of the whole aircraft. Alternatively, the full state estimation can be used. The state estimation is computed with the Kalman filter design with $Q = 1 \times 10^3 I$ and $R = \text{diag}(I, 1 \times 10^5 I)$. The controller includes both the load alleviation control with $q_M = \frac{1}{|M_x|}$ and feedforward control of the gust estimates. While the estimates of the gust disturbances in the angle of attack and pitch rate dynamics with the observer design are seen to be in a reasonable agreement with their actual gust disturbances as shown in Fig. 13, the estimation of the gust disturbances in the elastic modes is much less accurate and is not surprising. This inaccurate estimation is due to the state estimation for the elastic modes which in turn is dependent on the accurate gust estimation.

Figure 14 shows the state estimation of the angle of attack, pitch rate, and the generalized coordinates of the first two modes. As can be seen, while the estimates of the angle of attack and pitch rate are quite accurate, the estimates of the generalized coordinates of the elastic modes show poor agreement with the actual generalized coordinates.
Figure 14. State Estimation Due to Observer Design with Load Alleviation Control \( q_M = \frac{1}{\|M_x\|} \) and Feedforward Control of Gust Estimates

The aircraft response and wing root bending moment with the observer design of the gust load alleviation control are plotted in Fig. 15. The effectiveness of the gust load alleviation control on the wing root bending moment reduction is still retained with the observer design. With \( q_M = \frac{1}{\|M_x\|} \) and \( R = I \), the maximum wing root bending moment is seen to be reduced by a factor 5 with the observer design, which is the about the same as that with the full-state feedback design. The aircraft response is significantly reduced in amplitude with the observer design.

Figure 16 shows the control surface deflections for the observer design of the gust load alleviation control. The VCCTEF deflections exhibit a similar distribution as those for the full-state feedback design with the maximum VCCTEF deflection of \( 2.1^\circ \) occurring at the inboard flap number 1 and minimum VCCTEF deflection of \( -0.3^\circ \) at the outboard flap number 16.
Figure 15. Aircraft Response and Wing Root Bending Moment Due to Observer Design with and without Load Alleviation Control and Feedforward Control of Gust Estimates

Figure 16. Control Surface Deflections Due to Observer Design with Load Alleviation Control and Feedforward Control of Gust Estimates
The drag optimization control is now examined in the context of gust load alleviation based on the multi-objective flight control approach. A drag optimization control weighting factor $q_D = 1$ is used in the observer design. Figure 17 shows the plot of the drag coefficient of the GTM with gust load alleviation control alone ($q_D = 0$ and $q_M = 1/||M_x||^2$), combined gust load alleviation and drag optimization control ($q_D = 1$ and $q_M = 1/||M_x||^2$), and drag optimization control alone ($q_D = 1$ and $q_M = 0$). A control weighting matrix $R = I$ is used. The feedforward control of the gust estimates is also used in the control design. The GTM at the design cruise condition has a lift-to-drag ratio of about 18 corresponding to a trim lift coefficient of 0.51. Thus, the trim drag coefficient is estimated about 283.3 counts ($1$ drag count = 0.0001). The total drag coefficient is plotted in Fig. 17. The gust load alleviation control creates higher drag than the drag optimization control with and without gust load alleviation control. The drag coefficient for the gust load alleviation control is 310.7 counts versus 303.8 drag counts for the combined gust load alleviation and drag optimization control, a 2.3% drag reduction, and 293.2 drag counts with the drag optimization control alone, a 6.0% drag reduction. The maximum wing root bending moment is reduced with the drag optimization control turned on, but the drag optimization control actually causes an increase in the peak-to-peak amplitude of the wing root bending moment. With the combined gust load alleviation and drag optimization control, the flight path angle is seen to decrease by about 0.1° while the angle of attack increases by about the same amount. The bias term $\Lambda_0$ in the drag optimization control effectively seeks a new trim point for the aircraft to minimize the drag coefficient.

Figure 18 shows the control surface deflections for the combined gust load alleviation and drag optimization control. The VCCTEF deflections now exhibit a different pattern with a nearly linear distribution from flap number 1.
with a deflection of $1.2^\circ$ to flap number 16 with a deflection of $-0.7^\circ$.

**Figure 18. Control Surface Deflections Due to Observer Design with Combined Gust Load Alleviation and Drag Optimization Control**

Figure 19 shows the control surface deflections for the drag optimization control alone. The VCCTEF deflections are entirely negative and varies from the inboard flap number 1 at $-2.2^\circ$ to the outboard flap number 16 at $-0.2^\circ$. The negative VCCTEF deflections imply the VCCTEF causes the wing lift to decrease, but this lift decrease is offset by an increase in the angle of attack which causes the wing lift to increase. The lift distribution on the wing is also influenced by the aeroelastic deflections due to wing bending and torsion which create an effective wash-out twist (twist angle decreasing toward wing tip).

**Figure 19. Control Surface Deflections with Observer Design with Combined Gust Load Alleviation and Drag Optimization Control**
Figure 20 shows the incremental wing bending deflection and torsional twist about the elastic axis for the three controllers. With drag optimization control active, the wing bending deflection increases, while the torsional twist causes the wing leading edge to pitch up more. This implies that the wing actually sees higher lift distribution than when the drag optimization control is not present. Thus, drag optimization control can be viewed as a wing shaping control device which changes the wing shape for improved aerodynamic performance of the wing.

A simulation for a moderate one-minus cosine gust is also conducted for the observer design of the gust load alleviation control. Figure 21 show the estimates of the gust disturbances using the same setting as the design for the continuous von Karman gust. While the estimates of the gust disturbances for the angle of attack and pitch rate dynamics are reasonable and follow the right trend, the estimates for the elastic modes are not effective.
Figure 21. Gust Estimation Due to Observer Design with Load Alleviation Control \( q_M = \frac{1}{|M_x|} \) and Feedforward Control of Gust Estimates for Discrete Gust

Figure 22 shows the aircraft response and the wing root bending moment with gust load alleviation and drag optimization control. The effectiveness of the gust load alleviation control alone is demonstrated in Fig. 22. The maximum wing root bending moment greatly decreases by more than an order of magnitude. The aircraft response also decreases substantially to about zero. With the combined gust load and drag optimization control, the wing root bending moment shifts slightly negative due to the effective re-trim of the aircraft. The drag coefficient is reduced by the combined gust load alleviation and drag optimization control by about 2.7% relative to the drag coefficient produced by the gust load alleviation control alone. The flight path angle decreases by about 0.1° due to the effective re-trim of the aircraft.

Figure 23 shows the control surface deflections for the gust load alleviation control alone. The VCCTEF deflections are mostly negative with the inboard flap number 1 deflected to \(-2.2^\circ\) and the outboard flap number 16 deflected to \(0.4^\circ\). The negative flap deflections reduce the lift force generated by the gust.

Figure 24 shows the control surface deflections for the combined gust load alleviation and drag optimization control. The VCCTEF deflections are now entirely negative with the inboard flap number 1 deflected to \(-3^\circ\) and the outboard flap number 16 deflected to \(-0.1^\circ\). The incremental negative VCCTEF deflections are accompanied by an increase in the angle of attack to change the lift distribution on the wing for improved aerodynamic performance.
Figure 22. Aircraft Response and Wing Root Bending Moment Due to Observer Design with and without Load Alleviation Control and Drag Optimization Control

Figure 23. Control Surface Deflections Due to Observer Design with Gust Load Alleviation Alone
V. Conclusions

This paper presents a gust load alleviation control design for a flexible wing aircraft. A multi-objective flight control approach is developed using the optimal control method with the drag minimization and load alleviation objectives. A least-squares parameter estimation of gust disturbances is presented. The use of a pair of accelerometers to measure the acceleration of a rigid aircraft at two locations is seen to be effective in providing accurate gust estimation for the rigid aircraft dynamics. On the other hand, accelerometers on the wing can be used to estimate the gust disturbances for the elastic modes. The gust load alleviation control is designed with a load alleviation component obtained from the optimal control method and a feedforward control of the gust estimates obtained from modified least-squares gradient adaptive laws. Simulations of the gust load alleviation and drag optimization control on NASA Generic Transport Model equipped with flexible wings demonstrate the effectiveness of the multi-objective flight control method.

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References

