Adaptive Maneuver Load Alleviation for Flexible Wing Aircraft with Nonminimum Phase Zeros

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A two-part control system is designed for flight path angle command tracking that includes an adaptive control component to reduce maneuver load. The primary controller is nonadaptive and facilitates the tracking goal. The secondary controller is adaptive and is tasked only with reducing the maneuver load resulting from the tracking objective. It is designed to function in addition to the primary controller and is allocated control surfaces that are separate from those used by the primary. The secondary controller utilizes an output feedback model reference adaptive control framework that can accommodate systems with nonminimum phase zeros so long as an estimate of these zeros is available. Performance of the control design is demonstrated in simulation of a reduced stiffness, transport-type aircraft.

Nomenclature

G	=	transfer function matrix
W	=	closed-loop or reference transfer function matrix
r	=	command signal
l	=	number of inputs, outputs
n, n_m, n_u	=	polynomial degrees
d	=	plant relative degree
n_c	=	user-choice filter polynomial degree
α, α_m	=	plant, reference model denominator polynomial
β, β_m	=	plant, reference model numerator polynomial matrix
β_u	=	polynomial matrix for plant nonminimum phase zeros
$\beta_{d,i}$	=	i^{th} column of plant gain matrix β_d
β_{du}	=	nonminimum phase zero structure
β_{dm}	=	gain matrix for reference model
β_r	=	polynomial matrix for additional reference model zeros
e_i	=	i^{th} column of $l \times l$ identity matrix
L, M, N	=	control law unknown parameters
Θ, θ	=	vector, matrix of control law unknown parameters
ϕ, Ψ	=	vector, matrix of control law regressor
Φ	=	filtered regressor
$\otimes$	=	Kronecker product
z	=	output tracking error

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z_s	=	surrogate tracking error
P	=	covariance matrix used in parameter update law
a_f, b_f	=	user-selected polynomials for filter design
C_f	=	user-selected polynomial matrix for filter design
γ	=	flight path angle
M_y	=	wing root bending moment
Subscript		
n	=	nominal system quantity
Δ	=	delta system quantity
r	=	reference quantity
i	=	index counter

I. Introduction

Adaptive control is a well-established branch of modern control design. An adaptive control law can be used to compensate for environmental or plant model uncertainty by updating its parameters during operation. However, adaptive control techniques are often avoided in aerospace systems due to reasons ranging from restrictive plant structural requirements to validation difficulty. Here we seek to remove some barriers to implementation of adaptive laws on flight vehicles by applying them to a non-essential, performance-enhancing task.

This paper specifically considers the popular model reference adaptive control (MRAC) technique as a solution for load alleviation of an aircraft during a flight path angle tracking maneuver. A two-part control structure is proposed such that MRAC can be implemented for maneuver load alleviation (MLA) in conjunction with a nonadaptive control law that facilitates the tracking task. The two-part structure utilizes separate control surfaces for the adaptive and nonadaptive portions. The nonadaptive, or nominal, control law provides acceptable tracking performance for the aircraft when implemented on its own. The adaptive, or delta, control law is intended to reduce the load on the aircraft wing during a maneuver when compared to the load experienced using the nominal law alone. The delta law is not intended to be operated on its own.

Use of an adaptive law for the MLA task is well motivated when considering how the load is assessed. For example, here wing root bending moment is used as the load performance metric. This quantity can be determined from an appropriately placed strain gauge measurement. However, the sensitivity matrices that map state and control values to the measured output are likely poorly known by the user. The adaptive feature could help compensate for such uncertainties. Further, the structure of the proposed control design also allows some of the risk associated with implementing an adaptive law to be mitigated by having the essential flight task managed by a nonadaptive law. Careful design of the delta system's reference model can be used to adjust the aggressiveness of the adaptive delta law's task and the subsequent size of the calculated ideal control input.

To accommodate use of the control surface designated for MLA, the proposed design uses a variant of MRAC that can handle nonminimum phase systems. This variant is a continuous time, output feedback MRAC design based on a quantity known as the surrogate tracking error. It can accommodate both single-input, single-output (SISO) and multi-input, multi-output (MIMO) systems. The design was originally proposed in Refs. 1,2 and fully proved in Ref. 3. It facilitates nonminimum phase systems so long as, among other requirements, the nonminimum phase zeros are known. However, it has been observed that an estimate of the nonminimum phase zeros is sufficient for lesser performance.

The proposed MLA MRAC design is demonstrated in a flight path angle tracking simulation for a flexible version of the Generic Transport Model (GTM). The aircraft model is equipped with a multi-segment trailing edge flap that is actuated here as a single unit for the purposes of demonstrating the design. The two-part control system is shown to reduce the wing root bending moment during commanded changes in flight path angle when compared to control by the nominal controller only, but moderately degrades tracking performance as well.

The paper is organized as follows: A discussion of other relevant investigations in the literature is provided in Section II. Section III presents the partitioned MRAC design and describes the general form of the nominal and delta control laws that will be used. A general description of the GTM implementation is given in Section IV. Simulation results are presented in Section V, and some concluding remarks are made in Section VI.

II. Related Work

Restriction to minimum phase systems is an often seen limitation in adaptive control literature, though a few techniques have been developed that can accommodate nonminimum phase behavior. Periodic control laws, for example, can be used with discrete time implementations for nonminimum phase systems but produce intermittent control.[4] Some indirect adaptive control techniques such as adaptive pole placement will also work.[5] Some $\mathcal{L}_1$ adaptive control formulations can be used with nonminimum phase systems.[6] If lesser performance is acceptable, a minimum phase version of the plant can be assumed, despite the true nonminimum phase nature, and the difference treated as a large uncertainty.[7] In this case perfect tracking is no longer attainable.

Extension of MRAC to nonminimum phase systems has been slower to develop. One possibility is to use a feedforward block in parallel with the plant to make the system appear minimum phase to the adaptation mechanism.[8] Here the output magnitude of the extra block must be small so as to not significantly disturb the tracking performance of the actual plant. An observer-based adaptive control design offers a potentially robust approach that can be used on "squared-up" systems with nonminimum phase transmission zeros.[9] Retrospective Cost Adaptive Control can also be used in the MRAC framework for discrete-time systems.[10] Initial attempts to create a similar control law for continuous-time systems using a surrogate tracking error quantity have been proposed as well.[1,2] Further stability results for the continuous-time version are provided in Ref. 3.

Implementation of adaptive control laws on aircraft is further hindered due to difficulty in characterizing stability margins. Regardless, significant effort has been made to construct adaptive designs with features that might enable them to still serve as practical aircraft control systems. For example, a variant of Retrospective Cost Adaptive Control has been applied to uncertain, nonminimum phase aircraft models.[11] MRAC robustness modifications, such as the optimal control modification,[12] have enabled adaptive control systems to be flown on an F/A-18 aircraft.[13,14] Recently, an $\mathcal{L}_1$ adaptive controller was implemented on a Learjet and used for manned flight.[15] A survey of other adaptive control designs and implementations for aircraft is provided in Ref. 16.

III. Partitioned Model Reference Adaptive Control

The structure of the two-part MRAC scheme is presented first. The design assumes a linear, multi-input plant description and combines the use of two control laws: a nonadaptive nominal control law and an adaptive delta control law. The two control laws utilize separate control surfaces, but measurements can be fed back for use by both control systems. A general block diagram of the proposed design is provided in Fig. 1.

As an example of the partitioned structure consider a linear plant with four inputs and two outputs described by the transfer function relationship

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} G_{11}(s) & G_{12}(s) & G_{13}(s) & G_{14}(s) \\ G_{21}(s) & G_{22}(s) & G_{23}(s) & G_{24}(s) \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}. \tag{1}$$

Inputs u_1 and u_2 are assigned to the nominal control law and u_3 and u_4 to the delta control law. Both outputs y_1 and y_2 are fed back to both control systems. Each output thus consists of two independent portions, one resulting from each control law:

$$\begin{aligned} y_1 &= G_{11}(s)u_1 + G_{12}(s)u_2 + G_{13}(s)u_3 + G_{14}(s)u_4 \\ &= G_{1,n}(s)u_n + G_{1,\Delta}(s)u_\Delta \\ &= y_{1,n} + y_{1,\Delta} \end{aligned} \tag{2}$$

where $G_{1,n} = [G_{11} \;\; G_{12}]$, $G_{1,\Delta} = [G_{13} \;\; G_{14}]$, $u_n = [u_1 \;\; u_2]^T$, and $u_\Delta = [u_3 \;\; u_4]^T$. A similar result holds for y_2.

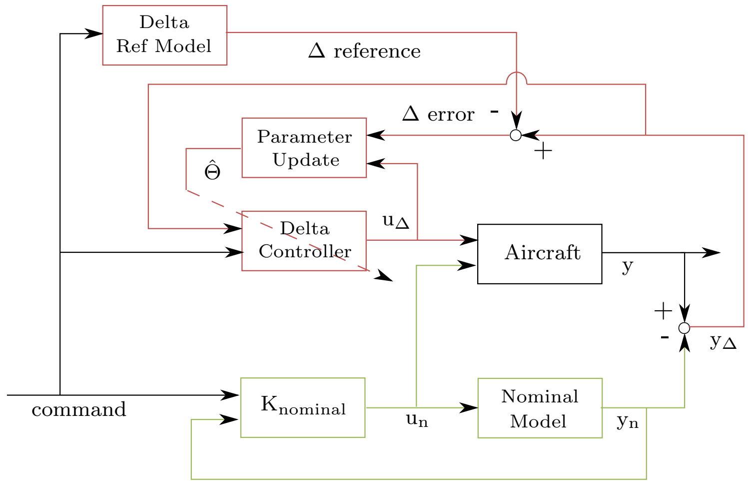

Figure 1: Block diagram for partitioned MRAC structure, nominal control system in green, delta control system in red

Without further specification concerning the control laws this type of partitioned structure can facilitate any number of outputs and two or more inputs. However, the delta law to be used is restricted to use with plants having the same number of inputs and outputs. Consider instead a 2×2 system.

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}. \tag{3}$$

Allow u_1 to be assigned to the nominal control law and u_2 to the delta law. Due to the anticipated single input MRAC design for the delta system, assignment of only one delta output is permissible. Arbitrarily selecting the delta output to be y_2 means that only this signal can be partitioned into nominal and delta components as in Eq. (2). For y_1 there is a portion of the signal that is now "unmanaged":

$$\begin{aligned} y_1 &= G_{11}(s)u_1 + G_{12}(s)u_2 \\ &= G_{1,n}(s)u_n + G_x(s)u_\Delta \\ &= y_{1,n} + y_x. \end{aligned} \tag{4}$$

Here $G_x = G_{12}$ and refers to the unmanaged dynamics while y_x is the corresponding unmanaged portion of the output signal. Without the delta law, y_x would not be present and y_1 could be fully controlled by the nominal law. Use of the delta law however has the potential to disrupt y_1's behavior in this scenario and it must be monitored to ensure that the delta law does not induce unacceptable behavior.

The input splitting feature permits the general statement that $y = y_n + y_\Delta$ for outputs that are associated with the delta system. The goal is to implement MRAC on only the delta portion of the system and as such there must be a way to recover y_Δ from the measured output y. This is accomplished by calculating or estimating y_n and subtracting away its contribution. For example, the nominal control can be applied to a known model of the nominal portion of the system $\bar{G}_n(s)$. The nominal output y_n is replaced by the estimated quantity $\bar{y}_n = \bar{G}_n(s)u_n$. The delta output used for feedback in the adaptive system is thus recovered as $y_\Delta = y - \bar{y}_n$. Figure 1 illustrates this feature in the lower feedback loop.

This structure corresponds to a shifted version of MRAC in some ways. Instead of using a full reference model, the reference model is shifted by subtraction of the closed loop nominal model to create the delta reference model. The adaptive control law then attempts to match the delta reference signal. Correspondingly, many of the typical MRAC implementation requirements fall on the delta portion of the system and will be stated explicitly in Section III.B.

A. Nominal Control Law

The transfer function representation of the open-loop nominal system is given as $y_n = G_n(s)u_n$. An estimated model of the nominal system is given by

$$\bar{y}_n = \bar{G}_n(s)u_n. \tag{5}$$

Any type of control that permits the closed-loop nominal model to be expressed as

$$\bar{y}_n = \bar{W}_n(s)r \tag{6}$$

can be used. This structure is intentionally non-specific so that a variety of control methodologies can be used for the nominal law.

B. Delta Control Law

A model-based control formulation for the delta law is appropriate due to the need to compensate for the closed loop nominal dynamics in its design. The use of a user-selected reference model $y_r = W_r(s)r$ in the design permits the closed-loop nominal model $\bar{W}_n(s)$ to be subtracted away leaving a remainder that serves as the delta reference model $W_\Delta(s)$ according to

$$\begin{aligned} y_{\Delta,r} &= y_r - \bar{y}_n \\ &= \left[W_r(s) - \bar{W}_n\right] r \\ &= W_\Delta(s)r. \end{aligned} \tag{7}$$

where the desired delta output is denoted as $y_{\Delta,r}$. The delta MRAC system is therefore designed to force y_Δ to track the output of $W_\Delta(s)$. It is clear that there are input/output size considerations that must be satisfied in order to form the delta reference model. It may be necessary to remove some outputs as previously discussed. Also, note that in practice it may more useful to instead design the delta reference model directly and recover the corresponding full reference model.

The delta control law in this paper makes use of the MRAC design developed in Ref. 2, although some structural changes have been made. The design utilizes a surrogate tracking error quantity to facilitate nonminimum phase plants so long as the nonminimum phase zeros are known. The design does however display some robustness to incorrect estimates of the nonminimum phase zeros.[10] To summarize how the control design will be used in this application, take the delta portion of the plant from control input $u_\Delta(t) \in \mathbb{R}^l$ to output $y_\Delta(t) \in \mathbb{R}^l$ to be

$$G_\Delta(s) = \beta_p(s)\alpha_p^{-1}(s). \tag{8}$$

The numerator structure should be decomposable as $\beta_p(s) = \beta_{du}(s)\beta_s(s)$ where $\beta_{du}(s)$ is an $l \times l$ polynomial matrix with degree n_u and $\beta_s(s)$ is an $l \times l$ monic polynomial matrix with degree $n - n_u - d$. The denominator structure is given by $\alpha_p(s)$ and is a monic $l \times l$ polynomial matrix of degree $n > 0$. Further delta plant requirements are stated in the following list.

Delta Plant Assumptions

P1. Delta plant must have l inputs and l outputs.
P2. Relative degree d must be at least one and known.
P3. Degree n must be known or upper bounded by a known quantity $\bar{n}$.
P4. $\alpha_p(s)$ and $\beta_p(s)$ must be right coprime.
P5. $\beta_{du}(s)$, which can be expressed as

$$\beta_{du}(s) = \sum_{i=1}^{l} \beta_{d_i}\beta_u(s)e_i, \tag{9}$$

must be known and must at least contain the delta plant's nonminimum phase zeros in $\beta_u(s)$. Thus, $\beta_u(s)$ is an $l \times l$ monic polynomial matrix and each β_{d_i} is an $l \times l$ gain matrix. Note that e_i is used to represent the i^{th} column of the $l \times l$ identity matrix.

While requirement **P5** is straightforward in the SISO case ($l = 1$), the MIMO case ($l > 1$) is more complex. Specifically, structural placement information regarding the nonminimum phase zeros would be required in addition to knowledge of their value. Also note, however, that $\beta_{du}(s) = \beta_p(s)$ and $\beta_s(s) = I$ is always a permissible choice even though this requires significant knowledge of the otherwise unknown delta system.

The reference model for the delta system must have the structure

$$W_\Delta(s) = \alpha_m^{-1}(s)\beta_m(s). \tag{10}$$

The input command $r(t) \in \mathbb{R}^l$ should be bounded and piecewise continuous. The numerator structure $\beta_m(s)$ is an $l \times l$ polynomial matrix of degree $n_m - d_m$, where d_m is the delta reference model's relative degree. Requirements on the model are stated subsequently.

Delta Reference Model Assumptions

M1. d_m must be greater that d.

M2. $\alpha_m(s)$ and $\beta_m(s)$ must be left coprime.

M3. $\beta_m(s)$ must have the structure

$$\beta_m(s) = \beta_{d_m}\beta_{du}(s)\beta_r(s) \tag{11}$$

where $\beta_{du}(s)$ is defined by the plant. β_{d_m} is an $l \times l$ gain matrix. If any additional zeros are to appear in the reference model they must be contained in the monic $l \times l$ polynomial matrix $\beta_r(s)$.

M4. The gain matching condition $\beta_{du}(s)N^* = \beta_{d_m}\beta_{du}(s)$ must be satisfied for some $N^* \in \mathbb{R}^{l\times l}$.

The delta control system aims to eliminate the full system tracking error:

$$\lim_{t\to+\infty} z(t) = \lim_{t\to+\infty} \big(y(t) - y_r(t)\big) = 0. \tag{12}$$

The delta control law is given by

$$u_\Delta(t) = \sum_{i=1}^{n_c} L_i(t)\bar{y}_{\Delta,i}(t) + \sum_{i=1}^{n_c} M_i(t)\bar{u}_{\Delta,i}(t) + N(t)r_f(t). \tag{13}$$

where the updated parameters are $L_i, M_i, N : [0,\infty) \to \mathbb{R}^{l\times l}$ for $i = 1 \ldots n_c$. The parameter n_c is user–choice subject to

$$n_c \geq \max(\bar{n}l, n_m - n_u - d). \tag{14}$$

The signals $\bar{y}_{\Delta,i}$ and $\bar{u}_{\Delta,i}$ are each filtered versions of the signals y_Δ and u_Δ obtained using

$$G_{a_{f,i}}(s) = \frac{s^{n_c-i}}{a_f(s)} I_l. \tag{15}$$

The signal r_f is the filtered version of r obtained using

$$G_{a_{f,i}}(s) = \frac{s^{n_c-i}}{a_f(s)} I_l. \tag{16}$$

Here $a_f(s)$ is a user-choice, asymptotically stable, monic polynomial of degree n_c.

The delta control law can be equivalently written as

$$u_\Delta(t) = \theta^T(t)\phi(t) \tag{17}$$

where the updated parameters are contained in $\theta \in \mathbb{R}^{l\times(2n_c+1)l}$ according to

$$\theta(t) = \big[L_1(t)\ldots L_{n_c}(t) \;\; M_1(t)\ldots M_{n_c}(t) \;\; N(t)\big]^T. \tag{18}$$

The filtered signals are contained in the regressor $\phi(t) \in \mathbb{R}^{(2n_c+1)l}$ as

$$\phi(t) = \Big[\bar{y}^T_{\Delta,i}(t)\ldots\bar{y}^T_{\Delta,n_c}(t) \;\; \bar{u}^T_{\Delta,i}(t)\ldots\bar{u}^T_{\Delta,n_c}(t) \;\; r_f^T(t)\Big]^T. \tag{19}$$

This version of the delta control law can then be manipulated such that the unknown parameters appear in the more familiar vector form by stating

$$u_\Delta(t) = \Psi^T(t)\Theta(t) \tag{20}$$

with $\Theta(t) = \text{vec}\,\theta^T(t) \in \mathbb{R}^{(2n_c+1)l^2}$ and $\Psi(t) = \phi(t) \otimes I_l \in \mathbb{R}^{(2n_c+1)l^2 \times l}$.

Finally, the parameter update is specified to complete the design. The vector $\Theta(t)$ is updated according to the least square–type laws

$$\dot{\Theta}(t) = -P(t)\Phi(t)\Omega^{-2}(t)z_s(t) \tag{21}$$

$$\dot{P}(t) = -P(t)\Phi(t)\Omega^{-2}(t)\Phi^T(t)P(t). \tag{22}$$

where $P(t) \in \mathbb{R}^{(2n_c+1)l^2 \times (2n_c+1)l^2}$ acts like a covariance matrix. Note that $P(0) > 0$ is required and, since this is a least squares–type formulation, $P(t)$ must be reset when it becomes too small to avoid slow adaptation. One resetting possibility that will be utilized in this paper is to set $P(t_r^+) = P(0)$ where $t_r = \{t|\lambda_{\min}(P(t)) \leq c_P\}$ for some $c_P > 0$. The matrix $\Omega \in \mathbb{R}^{l\times l}$ acts like a normalizing factor is given by

$$\Omega^2(t) = I_l + \eta\Phi^T\Phi \tag{23}$$

where $\eta \in (0,\infty)$. The update laws also make use of a variety of filtered quantities that are computed using

$$G_{C_{f,1}}(s) = \alpha_m(s)b_f(s)C_f^{-1}(s) \tag{24}$$

$$G_{C_{f,2}}(s) = a_f(s)C_f^{-1}(s)\beta_{du}(s). \tag{25}$$

Here $C_f(s)$ is a user–choice, monic, $l\times l$ polynomial matrix of degree n_c+n_u+d and $b_f(s)$ is an asymptotically stable, monic polynomial of degree $n_c+n_u+d-n_m$. The signal $z_s(t) \in \mathbb{R}^l$ is the so–called surrogate tracking error and is defined as

$$z_s(t) = z_f(t) + \Phi^T(t)\Theta(t) - u_f(t). \tag{26}$$

The filtered tracking error $z_f(t) \in \mathbb{R}^l$ is the filtered version of the tracking error $z(t)$ obtained using $G_{C_{f,1}}(s)$. The transposed filtered regressor $\Phi^T(t) \in \mathbb{R}^{l\times(2n_c+1)l^2}$ is acquired one column at a time by passing each column of $\Psi^T(t)$ through $G_{C_{f,2}}(s)$. The filtered delta control $u_f(t) \in \mathbb{R}^l$ is obtained from $u_\Delta(t)$ also through the use of $G_{C_{f,2}}(s)$. A summary of the proposed delta control system is given as a block diagram in Fig. 2.

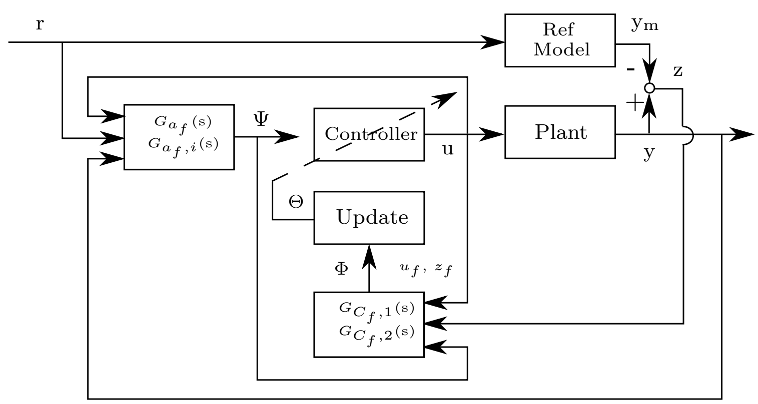

Figure 2: Block diagram of MRAC variant for nonminimum phase plants, used for delta control system

IV. Adaptive MLA Implementation for GTM

A. Aircraft Description

In this study a reduced version of the flexible wing GTM at midcruise conditions is utilized and is shown in Fig. 3. The transport–type aircraft model is modified to have wing stiffness properties scaled to 50% of their nominal values such that flexible motion of the structure must be actively considered. The aircraft is equipped with a novel control actuation system, known as the Variable Camber Continuous Trailing Edge

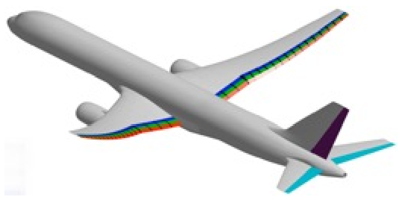

Figure 3: Overview of the GTM

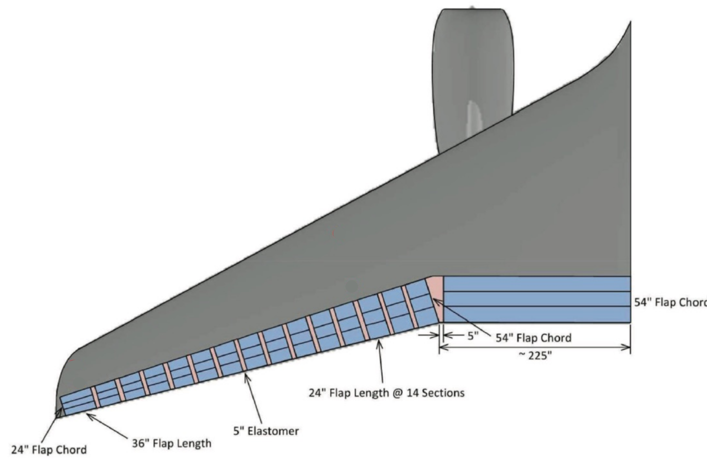

Figure 4: Illustration of 16 segment GTM VCCTEF design

Flap (VCCTEF), along the trailing edge of each wing. This system consists of 16 individually actuated flap segments connected by a flexible elastomer insert to form a smooth surface as is shown in Fig. 4. The VCCTEF can be used to exploit the increased flexibility of the wing by aeroelastically shaping the structure to facilitate performance goals such a drag minimization and load alleviation.[17–19] For simplicity, however, the VCCTEF segments are actuated as a single unit here.

The GTM model consists of the coupled rigid body and elastic dynamics. Only longitudinal motion is simulated and five rigid body states are retained. Additionally, only five elastic modes are retained. Each mode contributes elastic position and velocity states along with three lag states arising from the R.T. Jones method of unsteady aerodynamic approximation used to generate the model. Actuator dynamics are omitted. The reduced model as a total of 35 states. Measured quantities include the rigid body states providing γ and a strain gauge providing M_y. The elevator and trailing edge flap (formed after tying the VCCTEF segments together) are the available control inputs.

B. Control Law Specifications

In this paper the goal is to use the two control laws to track the desired flight path angle γ while reducing the wing root bending moment M_y. For the nominal law, the goal is to generate flight path angle tracking so the elevator is selected as input. Both γ and M_y are assigned as output to facilitate the delta structure, but the nominal controller is not designed to be cognizant of bending moment reduction. For the load-reducing delta control law, the flaps are selected as input and the M_y strain gauge as the only output. A summary of the input and output assignments is provided in Table 1.

Table 1: Control law input and output assignments

	Nominal Control System	**Delta Control System**
Inputs	elevator	flaps
Outputs	γ , M_y	M_y

The left and right flaps are actuated symmetrically in this longitudinal simulation. The delta system is thus a SISO implementation instead of MIMO, and only a scalar u_Δ signal is computed. The input/output arrangement is like the generic 2×2 system discussed in Eq. (3). There will be an unmanaged portion of γ due to actuation of the flaps, and degradation in tracking performance is therefore expected when the delta law is used.

The nominal control law is an LQR design with servomechanism for tracking the γ command. It is designed based on the rigid dynamics of the aircraft only, which are assumed to have full state availability.

The nominal design is made without consideration for the maneuver load. $W_n(s)$, the closed loop nominal system from the γ command to the output γ, is used directly as the estimated nominal system $\bar{W}_n(s)$.

The M_y command is not arbitrarily specified and is instead set to be equal to the M_y trajectory generated by the nominal system in response to the selected γ command. The M_y command is fed to the delta reference model whose parameters are chosen to invert and scale the command as much as is feasible. Then, when the nominal and delta M_y components are combined, the signal components counteract such that the total M_y is of smaller magnitude than M_y from use of the nominal controller alone. However, the greater the reduction in total M_y due to this counteraction, the greater the flap deflection required. Greater flap deflection in the context of the stated design unfortunately disrupts γ tracking. The ability of the delta control system to decrease M_y is limited by how much degradation of γ tracking performance is permissible.

The delta control law utilizes the adaptive control design described in Section III.B along with the SISO structure discussed in Section IV.B. The first task is to design the delta reference model and then use it to recover the full reference model if desired. The delta reference model is formed by choosing $\beta_u(s)$ to contain the single nonminimum phase zero seen at $s = 6.55$ in the transfer function resulting from flaps to M_y. The gain β_d is also recovered from this system. No additional zeros are to be included and thus $\beta_r(s) = 1$. The order of the delta reference model is selected to satisfy assumption **M1** and poles are evenly spaced from $s = -5$ to $s = -5.5$. β_{dm} is varied to explore the γ tracking tradeoff. The selections are then used to construct $W_\Delta(s)$. Finally, the full reference model for the flaps to M_y system is obtained from $W_r(s) = \bar{W}_n(s) + W_\Delta(s)$.

V. Simulation Results

To demonstrate the performance of the proposed design, the control law is simulated in operation on a linear model of the longitudinal GTM dynamics at cruise conditions. The polynomials used in the filter designs are $a_f(s) = (s + 0.1)^{n_c}$, $b_f(s) = (s + 0.1)^{n_c+n_u+d-n_m}$, and $C_f(s) = (s + 0.1)^{n_c+n_u+d}$. The filter degree n_c is selected according to Eq. (14). Initial conditions for all filters are zero. P_0 is set to $1e25I$ and reset to P_0 when $\lambda_{\min}(P(t)) \leq 0.01$. The factor $\eta = 1$ is used to construct $\Omega(t)$. Here the nominal portion of the plant is used directly as the nominal model. The nonminimum phase zeros of the delta portion of the plant are known and used in the delta reference model. The rest of the delta portion is unknown but linear. No disturbances or uncertainties are considered.

Figure 5 demonstrates the load alleviation ability of the control design tracking a series of rounded step changes in γ. Several values of β_{dm} are considered. The plot demonstrates a notable reduction in the magnitude of the wing root bending moment when the delta control law is used with the nominal controller ($\beta_{dm} \neq 0$) versus the case when the nominal controller is used alone (MLA off). Figure 6 shows the flight path angle tracking performance in the same set of cases. Good tracking is obtained in the MLA off case as should be expected for a well designed nominal controller. However, tracking performance is degraded by the addition of the delta controller and its opposing control action. Tracking performance becomes much worse as β_{dm} is increased but M_y reduction improves.

Figure 7 shows the elevator control signal generated by the nominal controller. The control signal remains the same for at all times since the nominal model remains unchanged. Figure 8 shows the flap deflection generated by each version of the delta controller considered in the previous figures.

Finally, Fig. 9 shows the components of the M_y reference signal for each MLA off and on case. Note that $M_{y,r}$ is constructed according to Eq. (7), or $M_{y_r} = \bar{M}_{y,n} + M_{y,\Delta r}$. This graphic makes the competing objectives of the the two control laws clear. The portion of the M_y reference due to the nominal system, $\bar{M}_{y,n}$, is shown in blue. It is a byproduct of the tracking objective and follows a trend in line with the γ command. The remaining lines show the portion of the M_y reference due to the delta reference model, $M_{y,\Delta r}$, for a range of reference model scalings β_{dm}. These purposely follow a trend opposite the γ command to induce the aforementioned counteraction of $\bar{M}_{y,n}$. Note that the magnitude of $M_{y,\Delta r}$ increases as β_{dm} increases which will result in a smaller total M_y reference after the two components are summed. Also note that it is critical to shape the dynamics of the delta reference model so that the extremes of its output are reasonably in phase with the peaks of the nominal controller's action.

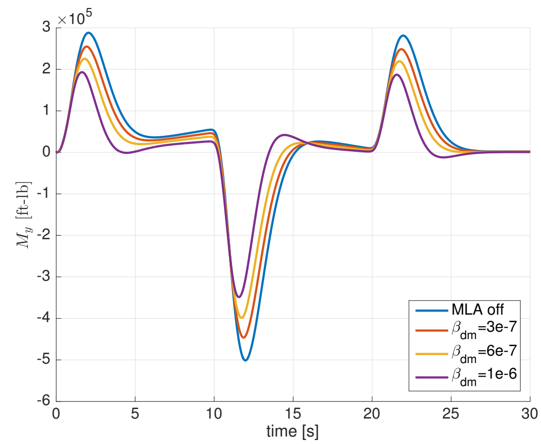

Figure 5: Wing root bending moment for MLA off and multiple MLA on cases

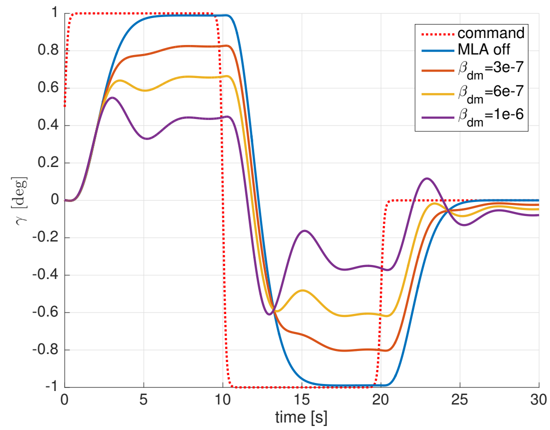

Figure 6: Comparison of γ tracking for MLA off and multiple MLA on cases

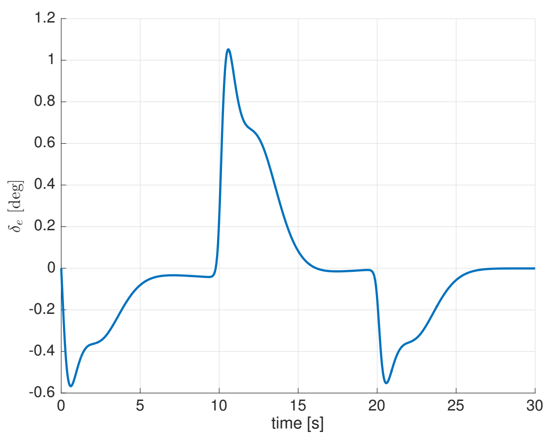

Figure 7: Elevator control signal

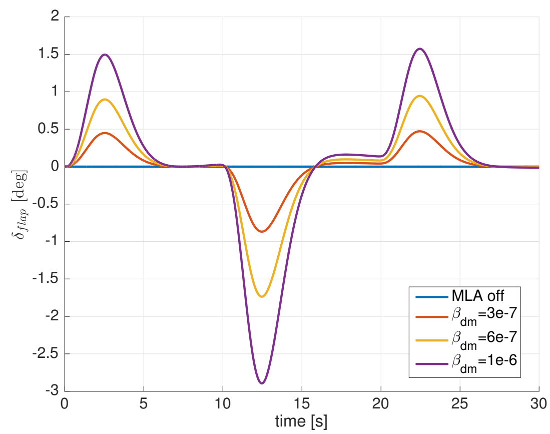

Figure 8: Flap control signal comparison with delta control law for MLA off and multiple MLA on cases

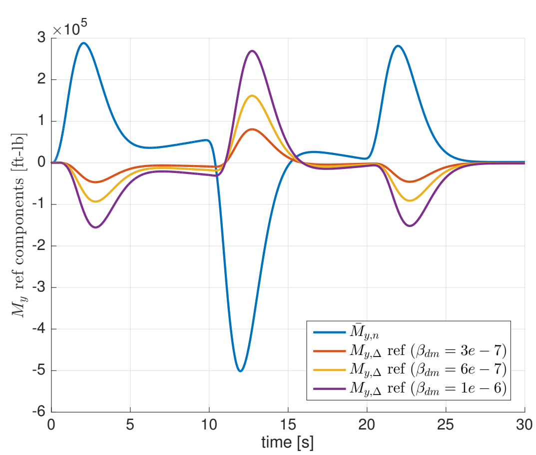

Figure 9: Components of the M_y reference signal for multiple MLA on cases

VI. Conclusions and Future Work

In this paper a partitioned MRAC design is proposed and applied to flight path angle tracking with maneuver load alleviation. The two-part design incorporates both a nonadaptive control law to provide tracking performance and an adaptive control law that facilitates the performance goal. A variant of MRAC that can handle nonminimum phase systems is employed as the adaptive law to accommodate the nonminimum phase behavior of the aircraft. The partitioned design is demonstrated in simulation of a flexible wing transport–type aircraft. In the future, the same design could be used with multiple flap segments available for load alleviation resulting in a MIMO adaptive system and that can make use of the structure's reduced stiffness to shape the wing. Robustness to various types of uncertainty including modeling error and poor knowledge of the nonminimum phase zeros could also be explored.

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