Estimating Loss-of-Control: a Data-Based Predictive Control Approach

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Loss-of-control is a major contributor to aircraft fatalities. Recent work has been done to develop quantitative criteria for determining loss-of-control from accident time history data. This work proposes an approach to estimating boundaries on control actions to provide information to pilots and/or control systems to assist in avoiding loss-of-control scenarios. Data-based predictive control theory is used to develop an algorithm that finds the minimum control input that will result in the aircraft exceeding a safe operating envelope at various minimum time estimates. The calculated minimum control inputs become a boundary of a set of safe control inputs. With this information, a pilot could change flying strategy or an autonomous system could schedule controller gains to prevent the vehicle from exceeding the envelope.

Nomenclature

\( A \) = state transition matrix
\( A_1, A_2 \) = convenience variables
\( A_{\text{Lat}} \) = example lateral state transition matrix
\( A_{\text{Lon}} \) = example longitudinal state transition matrix
\( B \) = control matrix
\( B_d \) = disturbance input matrix
\( B_{\text{Lat}} \) = example lateral control matrix
\( B_{\text{Lon}} \) = example longitudinal control matrix
\( C \) = output matrix
\( D \) = direct transmission matrix
\( G, H, K, V \) = gains of the dynamic output feedback predictive control laws
\( J \) = receding horizon cost function
\( K_{\text{lat}}, K_{\text{lon}} \) = pole-placement state feedback gains
\( L_S, M_1, M_2 \) = convenience variables
\( P_1, P_2, W \) = parameters of multi-step ahead input-output predictor model
\( Q \) = output error weighting matrix
\( R \) = control input weighting matrix

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$R_S$ = control smoothing weighting matrix
$f$ = number of distinct disturbance frequencies
$g$ = generic variable representing control input or system output
$n$ = system order
$n_i$ = number of system inputs
$n_o$ = number of system outputs
$s$ = length of prediction window
$t$ = time
$u$ = control input
$u_d$ = disturbance input
$u_{k-1}$ = convenience variable
$w$ = generic variable representing the length of a super-vector
$x$ = system state
$y$ = system output
$z$ = desired future output
$\theta$ = pitch angle
$\phi$ = bank angle
$\beta$ = sideslip angle
$\rho$ = number of past data points, also order of predictive controller
$\omega$ = start of prediction window
$\Gamma$ = convenience variable

1. Introduction

Loss-of-control has been the number one contributing factor to fatal airline accidents, and has resulted in more fatalities than any other factor during the past ten years[1]. Historically, loss-of-control has been determined to be a factor in an accident by qualitative judgement based upon accident investigation experience. Recent work[2] has been done to create quantitative loss-of-control criteria to define loss-of-control events. The quantitative criteria are in the form of five envelopes relating to airplane flight dynamics, aerodynamics, structural integrity, and flight control use. Typical maneuvers exceed at most one of the five envelopes, maneuvers bordering on loss-of-control exceed two envelopes, and maneuvers exceeding three envelopes are considered loss-of-control. These criteria constitute a numerical methodology that can reliably identify key upset characteristics in a flight test or accident/incident time history[2].

Reference 2 provides several loss-of-control examples, most of which involve system malfunction and/or inappropriate control inputs. In many of the loss-of-control examples[2], five or more seconds elapse between exceeding the first envelope and entering a loss-of-control situation. The envelopes are used to quantitatively determine whether an accident involved loss-of-control, which, in turn, may help future pilots learn to avoid those situations.

While it is useful to know whether an accident involved loss-of-control, knowing before hand if loss-of-control is about to occur may help prevent an accident and save lives. This approach was taken for developing a command validation algorithm[3,4] (CVA) which monitors commands generated by a fly-by-wire (FBW) system for a rotorcraft. The CVA uses a low-order model to predict future bank and pitch attitude as well as altitude. If the attitude prediction exceeds altitude dependent limits, the command is deemed to be invalid, and the FBW system is automatically disengaged. The CVA was designed for safely testing control algorithms in research aircraft where a safety pilot is present and ready to take over at any moment, but does not provide any information to the pilot other than disengaging the FBW system.

The goal of the present work is to provide information during flight that will assist a pilot or autonomous system to avoid loss-of-control. In particular, the goal of the data-based predictive control approach is to provide a capability of predicting the control actions that result in exceeding the envelope, and how much time remains before the envelope is exceeded. The envelopes proposed in [2] are used for this paper, but it is envisioned that any envelope that describes a boundary in one of the aircraft states could reasonably be used.
II. Data-Based Predictive Control Approach

A. Predictive Control Concept

The safest course in aircraft flight is to operate as far from the edge of the envelope as possible. However, maneuvering near the edge of the envelope may be required in certain circumstances (e.g., recovering after upset, take-off and landing). Furthermore, uncertainties introduced into the system, such as through damage or failure, can change the boundaries of the envelope, and can make certain control inputs more likely to take the aircraft out of the envelope. Control limits can be estimated using data-based predictive control and various minimum time estimates. The control limits can be represented as an n-dimensional hypercube, where n is the number of control inputs. The region within the box can be treated as a safe region.

A concept 2-dimensional display for a system with lateral and longitudinal control inputs is shown in Figure 1. Two notional safe regions are shown for the minimum control input needed to exceed the envelope within 1 second and 5 seconds. The current control input is also shown as being within the green box. The control limits can also be updated periodically, even in real time[5], as the limits change. This information may provide lead time for a pilot to change flying strategy or for an autonomous system to schedule controller gains to prevent the vehicle from exceeding the envelope.

B. State-space and Input-output Representations

Consider an n<sub>x</sub>-input, n<sub>y</sub>-output system with the system state x(k) and output y(k) given by

\[
x(k+1) = Ax(k) + Bu(k) + B_d u_d(k)
\]
\[
y(k) = Cx(k) + Du(k)
\]  \hspace{1cm} (1)

We assume that neither the system model, defined by A, B, B<sub>d</sub>, C, and D, nor the initial state of the system, x(0), are known, but a set of sufficiently rich and long excitation input u(k) and possibly disturbance–corrupted output data y(k) is available. The disturbance input, u<sub>d</sub>(k), if present, is assumed to be a sum of a finite number of unknown harmonics. Only an upper bound of the number of harmonics is known.

The representation of the data history can be simplified by the introduction of “super-vector” notation, defined by

\[
g_w(k) = \begin{bmatrix} g(k) \\ g(k+1) \\ \vdots \\ g(k+w-1) \end{bmatrix}
\]  \hspace{1cm} (2)

where g will generally represent an output or control input (column) vector, and w is the length of the vector.

For the system in Eq. (1), the output y(k) is dependent on the initial state x(0) and the disturbance inputs u<sub>d</sub>(k). Since the disturbance input and the initial state are assumed unknown, it is beneficial to describe the system using a relationship between the excitation input and disturbance-corrupted output that does not explicitly include the terms involving the initial state and the disturbances. In [6], the interaction matrix formulation captures this
input–output relationship, which does not depend on initial states and disturbances. It was shown that the following relationship holds for excitation input and possibly disturbance-corrupted output,

\[ y_s(k + q) = P_1 u_p(k - p) - P_2 y_p(k - p) + W u_{\text{res}}(k) \]  

(3)

when \( p \) is selected such that \( n_o \rho > n + 2f + 1 \) and \( 0 \leq \omega \leq \rho \), where \( n \) is the system order, \( f \) is the number of distinct disturbance frequencies, and the 1 accounts for a constant disturbance if present. In this context, \( p \) is the number of past data points, \( q \) is the start of the prediction window, and \( s \) is the length of the prediction window. A conservative value for \( p \) can be chosen using an upper bound on the order of the system and the number of distinct disturbance frequencies.

C. Model-Predictive Controller Design

A predictive controller can be designed to minimize the receding-horizon cost function

\[
J(k) = \left[ y_s(k + q) - z_s(k + q) \right]^T Q \left[ y_s(k + q) - z_s(k + q) \right] + u_{s+q}^T(k) R u_{s+q}(k)
\]

(4)

where \( z_s(k + q) \) is the desired output trajectory to be tracked. The output error cost is evaluated over the interval from time \( k + q \) to \( k + s + q - 1 \), with a weight matrix of \( Q \). The control input cost is evaluated over the interval from time \( k \) to \( k + s + q - 1 \), with a weight matrix of \( R \). The control smoothing cost, which is the weighted norm of the difference between the control input at time \( k \) and the previous control input at time \( k - 1 \), is evaluated over the interval from time \( k \) to \( k + s + q - 1 \), with weighting matrix \( R_S \).

The future control input history \( u_{s+q}(k) \) that minimizes the resultant cost function can be found by combining Eq. (3) with Eq. (4), and taking the derivative with respect to \( u_{s+q}(k) \), where \( u_{s+q}(k) \) becomes

\[
u_{s+q}(k) = A_1 u_p(k - p) + A_2 y_p(k - p) + M_1 z_s(k + q) + M_2 u_{k-1}
\]

(5)

where

\[
A_1 = -\Gamma^{-1} W^T Q P_1, \quad A_2 = \Gamma^{-1} W^T Q P_2, \quad M_1 = \Gamma^{-1} W^T Q, \quad M_2 = \Gamma^{-1} (1 - L_S)^T R_S,
\]

\[
\Gamma = \left( R + (1 - L_S)^T R_S (1 - L_S) + W^T Q W \right), \quad L_S = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \text{and} \quad u_{k-1} = \begin{bmatrix} u(k-1) \\ \vdots \\ 0_{(s+q-1) \times 1} \end{bmatrix}.
\]

The optimal control law is extracted from the first \( n_t \) rows of Eq. (5), resulting in the dynamic output feedback form shown in Eq. (6).

\[
u(k) = Gu_p(k - p) + Hy_p(k - p) + K z_s(k + q) + V u_{k-1}
\]

(6)

The gains \( G, H, K, \) and \( V \) are the first \( r \) rows of \( A_1, A_2, M_1, \) and \( M_2 \) respectively.

D. Cost Function Modifications for Control Limits

To determine the control boundary, the goal is to find the minimum control input that will result in the vehicle being at the boundary at a specified time \( t \). Rather than specify an entire desired output trajectory \( z_s(k + q) \) from the current time until time \( t \), the start \( q \) of the prediction window can be chosen as \( q = t \), and the length \( s \) of the prediction window can be chosen as \( s = 1 \). Choosing the weighting matrix \( Q \) to be a diagonal matrix with non-zero elements only for the \( k+t \) time step can produce the same effect, but results in much more computation in practice. Including
only the output error cost and control input cost results in a first control input \(u(k)\) that is very conservative, and not representative of the boundary of the control. By including the control smoothing cost, we can cause the control input to be nearly constant from the current time until the end of the prediction window. Also, by setting the first \(n_t\) diagonal elements of \(R_S\) to zero, the controller is allowed to select any control input, and afterwards remain nearly constant. It can also be shown that setting the first \(n_t\) diagonal elements of \(R_S\) to zero results in \(V_{t(k-1)}\) being identically zero, thus simplifying the computation of the control input. The resulting control is the amplitude of a step input that, if applied, would result in the vehicle being at the boundary of the envelope at time \(t\). It is important to note that an under-damped second order system will have overshoot, and for a time \(t\) that is chosen beyond the peak overshoot, the system will leave the envelope before \(t\), and return to the boundary at time \(t\). Therefore, the boundary generated is an estimate, not a guarantee of the minimum control that will result in the vehicle leaving the envelope. However, multiple algorithms each having a unique time estimate \(t\) can be used to create multiple limits, and the innermost box could be considered the safe region of operation.

E. Numerical Example

The operation of the algorithm is demonstrated with a simple model of a generic transport aircraft\[7\] with decoupled lateral-directional and longitudinal dynamics. The longitudinal dynamics use elevator as the input and pitch angle and pitch rate as the states, with state-space matrices:

\[
A_{Lon} = \begin{bmatrix} 0 & 1 \\ -2.6923 & -0.7322 \end{bmatrix}, \quad B_{Lon} = \begin{bmatrix} 0 \\ -3.5352 \end{bmatrix}.
\]

(7)

The lateral-directional dynamics use aileron and rudder as inputs and roll rate, yaw rate, bank angle and sideslip angle as states, with state-space matrices:

\[
A_{Lat} = \begin{bmatrix} -1.752 & 0.509 & 0 & -10.1076 \\ 0.024 & -0.3842 & 0 & 2.7698 \\ 1 & 0.0506 & 0 & 0 \\ 0.0475 & -0.9907 & 0.0404 & -0.1578 \end{bmatrix}, \quad B_{Lat} = \begin{bmatrix} 6.33 & 1.735 \\ 0.0351 & -2.2464 \\ 0 & 0 \\ 0.0042 & 0.0476 \end{bmatrix}.
\]

(8)

A second lateral-directional model is also used as an off-nominal model. For this example, the off-nominal model is a damaged aircraft with 25\% tip loss of the left wing. The same longitudinal matrices are used, and the modified lateral dynamics are

\[
A'_{Lat} = \begin{bmatrix} -1.134 & 0.4811 & 0.0001 & -9.5195 \\ 0.0636 & -0.4107 & 0 & 2.9274 \\ 1 & 0.0892 & 0 & 0 \\ 0.086 & -0.9903 & 0.0404 & -0.1451 \end{bmatrix}, \quad B'_{Lat} = \begin{bmatrix} 3.3953 & 1.7538 \\ -0.0069 & -2.3722 \\ 0 & 0 \\ 0.0054 & 0.0495 \end{bmatrix}.
\]

(9)

State-feedback control is applied to the longitudinal dynamics with gain \(K_{Lon} = [-1.0064 \quad -0.7928]\), to give the closed-loop longitudinal transfer function a frequency of 2.5 rad/s and damping ratio 0.707. State feedback is also applied to the lateral-directional channel with gain

\[
K_{Lat} = \begin{bmatrix} -0.0209 & 0.5414 & 0.2241 & -1.6017 \\ 0.0482 & -1.7509 & 0.0131 & 0.117 \end{bmatrix}.
\]

(9)

to give the closed-loop lateral-directional transfer function spiral and roll modes time constants of 1 and 0.2857 seconds, respectively, and to give the dutch roll mode a frequency of 1.2 rad/s and damping ratio 0.707. For simplicity, this example assumes the sideslip angle (\(\beta\)) command is always zero, and only bank angle is available to the predictive control algorithm. The envelope boundaries are set at \(\theta = -10^\circ\) and \(25^\circ\) and \(\phi = \pm 45^\circ\), and the control position limits are set at \(-20^\circ\) and \(30^\circ\) for elevator deflection and \(\pm 20^\circ\) for rudder and aileron deflection. The system is discretized with sampling frequency 100hz.

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For this example, separate model-predictive control algorithms are designed for the longitudinal and lateral-directional axes. For the longitudinal axis, the order of the system is 2, the number of outputs m is 1, and there are no disturbances, so the minimum p that can be used for the longitudinal channel is p=3. For the lateral axis, the order of the system is 4, the number of outputs is 1, and the minimum p that can be used is p=5. If only an estimate of the order of the system and an upper bound on the number of disturbance frequencies is known, then p can be chosen conservatively. For this example, p is selected to be 10 for both axes as such a conservative choice. As discussed previously, the length s of the prediction window is selected as s=1, and the values for q used correspond to 1 and 3 seconds, or q=100 and 300. The output error weighting matrix Q is a single value and chosen to be Q=1, and the control input weighting matrix R is a diagonal matrix with diagonal elements 0.000001. The control smoothing weighting matrix Rₜ is a diagonal matrix with first diagonal value 0, and subsequent values 10. A rich set of control inputs and system outputs was collected for the system, and the controller gains G, H, and K were computed for each value of q.

Both the nominal and off-nominal systems were then simulated with the same pre-recorded sequence of pilot inputs applied to each system. Figures 2 through 4 show a series of snapshots of the control inputs and control limits during six seconds of simulation. A two-second trace of the control history is shown as a continuous black line, with the earliest point marked as a black dot and the latest point the time at which the snapshot was taken. The boundaries shown are also the boundaries for the time at which the snapshot was taken. In Figure 2, the aircraft is flying wings-level, and the reduced control boundary for the off-nominal condition is clearly visible. The 1 second control boundary for the off-nominal aircraft is less than the 3 second control boundary. This is consistent with the behavior of an underdamped system. Although the expectation is that a longer time horizon requires less control input to reach the boundary, in an underdamped system the aircraft will overshoot before settling back at a level which is within the boundaries of loss-of-control. This illustrates the importance of having multiple boundaries, as the vehicle would actually leave the safe boundary in one second, and return to the boundary after three seconds. In Figure 3, the aircraft has rolled slightly to the right, and the 1 second boundary has now saturated at full left lateral control input, thus indicating that the plane will not leave the boundary within one second, even with full left stick deflection. Figure 4 shows the condition after the aircraft has rolled to the left, showing the reduced control boundary and indicating that the aircraft is nearing the boundary.

III. Conclusion

This work allows estimation of the control boundaries that permit safe operation of an aircraft. This work is envisioned to assist pilots or autonomous systems in predicting and preventing loss-of-control events. Because the algorithm is based on input-output data, the boundaries can be updated for the current aircraft and flight condition. Alternatively, the gains of the algorithm can be precomputed and scheduled with the flight control gains. The control boundaries may help to reduce inappropriate pilot actions, and may help alert the pilot to reduced control authority.

References

Figure 2. Control input history and control limits, t=0 to 2 seconds.

The aircraft is flying wings-level, and the reduced control boundary for the off-nominal condition is clearly visible.
Figure 3a.  Control input history and control limits, $t=2$ to 4 seconds.
The aircraft has rolled slightly to the right, and the 1 second boundary has now saturated at full left lateral control input, thus indicating that the plane will not leave the boundary within one second, even with full left stick deflection.

Figure 3b.  Aircraft Response, $t=2$ to 4 seconds.
The aircraft roll angle phi versus time, showing the aircraft has rolled slightly to the right
Figure 4a.  Control input history and control limits, t=4 to 6 seconds.
After the aircraft has rolled to the left, showing the reduced control boundary and indicating that the aircraft is nearing the boundary.

Figure 4b.  Aircraft Response, t=4 to 6 seconds.
The aircraft roll angle phi versus time, showing the aircraft has rolled to the left