An Event-based Approach to Distributed Diagnosis of Continuous Systems

Matthew Daigle [1], **Indranil Roychoudhury** [2], **Gautam Biswas** [3], **and Xenofon Koutsoukos** [3]

[1] *University of California, Santa Cruz, NASA Ames Research Center, Moffett Field, CA, 94035, USA*
matthew.j.daigle@nasa.gov
[2] *SGT Inc., NASA Ames Research Center, Moffett Field, CA, 94035, USA*
indranil.roychoudhury@nasa.gov
[3] *Institute for Software Integrated Systems, Dept. of EECS, Vanderbilt University, Nashville, TN, 37235, USA*
{gautam.biswas,xenofon.koutsoukos}@vanderbilt.edu

ABSTRACT

Distributed fault diagnosis solutions are becoming necessary due to the complexity of modern engineering systems, and the advent of smart sensors and computing elements. This paper presents a novel event-based approach for distributed diagnosis of abrupt parametric faults in continuous systems, based on a qualitative abstraction of measurement deviations from the nominal behavior. We systematically derive dynamic fault signatures expressed as event-based fault models. These models are used for designing local event-based diagnosers for the system based on global diagnosability analysis. The local diagnosers each generate globally correct diagnosis results locally, without a centralized coordinator, and by communicating a minimal number of measurements between themselves. The proposed approach is applied to a multi-tank system, and results demonstrate a marked improvement in scalability compared to a centralized approach.

1 INTRODUCTION

The complexity of modern engineering systems warrants the adoption of fault diagnosis capabilities to ensure system safety, reliability, and availability. Faults must be quickly isolated so that corrective actions may be taken while the system is still in operation and loss of function minimized. As systems become more complex, it is correspondingly more difficult to develop and deploy centralized diagnosis solutions. Further, such centralized schemes have single points of failure, do not scale as the size of systems increases, and have large computational and memory requirements. This, along with the increased pervasiveness of distributed, networked components, fuels the need for distributed diagnosis frameworks.

In previous work, we have developed a centralized framework for qualitative event-based diagnosis for parametric faults in continuous systems (Daigle *et al.*, 2009). Deviations of measured behavior from predicted nominal behavior, termed *fault signatures*, are captured qualitatively using magnitude and slope symbols, forming the basis of the qualitative fault isolation scheme (Mosterman and Biswas, 1999). The orders in which these deviations manifest, termed *relative measurement orderings*, are also used for fault isolation, thus forming event-based descriptions of fault-induced behavior. This diagnostic information may be computed from the system model and used to build event-based diagnosers similar to those used for discrete-event systems (Sampath *et al.*, 1996). However, this approach, being centralized, scales poorly, because as the number of faults and measurements increases, the possible number of event traces increases as well.

To address the problems of centralized diagnosis, we apply the distributed diagnoser design methodology presented in (Roychoudhury *et al.*, 2009a) to the formal event-based framework developed in (Daigle *et al.*, 2009). This distributed diagnoser design approach of (Roychoudhury *et al.*, 2009a) is based on global diagnosability analysis, where the local diagnosers generated are designed to provide globally correct diagnosis results through local analysis, without a centralized coordinator, and by communicating the minimal number of measurements among themselves. The approach does not incorporate measurement orderings, but the addition of measurement orderings improves diagnosability, allowing the local diagnosers to be more efficient.

This paper presents, using a multi-tank system as a case study, how a global event-based diagnoser may be decomposed into several independent local event-based diagnosers, each of which leverage measurement orderings for diagnosis. Distributed diagnoser design results demonstrate the vast reduction in diagnoser size that may be obtained using this approach, resulting in, for each subsystem, a small, compact local diagnoser capable of providing globally correct diagnoses of local faults. Results demonstrate the improved scalability of the distributed approach.

The paper is organized as follows. Section 2 formulates the diagnosis problem. Section 3 reviews qualitative fault isolation and event-based fault modeling. Section 4 defines diagnosability in the event-based framework. Section 5 describes the distributed

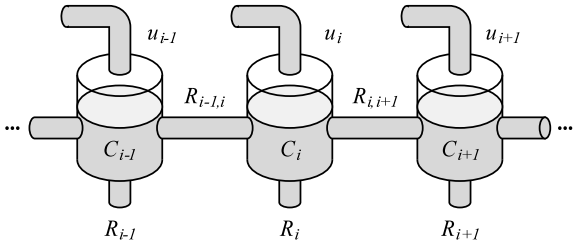


Figure 1: Tank system schematic.

diagnoser design problem. Section 6 discusses the global and local diagnoser construction, and Section 7 demonstrates the approach in simulation, and provides scalability results. Section 8 concludes the paper.

2 PROBLEM FORMULATION

We consider the problem of single fault diagnosis in continuous systems. We assume the system, $\mathcal{S}$, is described by

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \boldsymbol{\theta}(t), \mathbf{u}(t)) + \mathbf{v}(t)$$
$$\mathbf{y}(t) = \mathbf{h}(\mathbf{x}(t), \boldsymbol{\theta}(t), \mathbf{u}(t)) + \mathbf{n}(t),$$

where $\mathbf{x}(t) \in \mathbb{R}^{n_x}$ is the state vector, $\boldsymbol{\theta}(t) \in \mathbb{R}^{n_\theta}$ is the parameter vector, $\mathbf{u}(t) \in \mathbb{R}^{n_u}$ is the input vector, $\mathbf{v}(t) \in \mathbb{R}^{n_v}$ is the process noise vector, assumed to be zero-mean Gaussian, $\mathbf{f}$ is the state equation, $\mathbf{y}(t) \in \mathbb{R}^{n_y}$ is the output vector, $\mathbf{n}(t) \in \mathbb{R}^{n_n}$ is the measurement noise vector, assumed to be zero-mean Gaussian, and $\mathbf{h}$ is the output equation.

We denote a measurement as m, which is a time-varying signal of $\mathbf{y}(t)$ obtained from an associated sensor. A measurement set is denoted as M.

We consider single, abrupt, parametric faults, where faults are modeled as unexpected step changes in system parameter values. We name faults by the associated parameter and the direction of change, i.e., θ^+ denotes a fault defined as an increase in the value of parameter θ, and θ^- denotes a fault defined as a decrease in the parameter value. We denote a fault as f and a set of faults as F.

The diagnosis problem is to isolate single faults of F using the measurements of M. In distributed diagnosis, the problem is decomposed into smaller subtasks to achieve this overall goal.

Throughout the paper, we will use a multi-tank system as a running example. The tanks are connected serially as shown in Fig. 1, and we will consider a variable number of tanks. For tank i, u_i denotes the input flow, C_i denotes the capacitance, and R_i denotes the drain pipe resistance. For tanks i and j, R_{ij} denotes the resistance of the connecting pipe. For an n-tank system, the pressure of tank $i = 1$ is described by

$$\dot{p}_i = \frac{1}{C_i}\Big(u_i - \frac{1}{R_i}(p_i) - \frac{1}{R_{i,i+1}}(p_i - p_{i+1})\Big),$$

of tanks $i = 2, \ldots, n-1$ by

$$\dot{p}_i = \frac{1}{C_i}\Big(u_i + \frac{1}{R_{i-1,i}}(p_{i-1} - p_i) - \frac{1}{R_i}(p_i) - \frac{1}{R_{i,i+1}}(p_i - p_{i+1})\Big),$$

and of tank $i = n$ by

$$\dot{p}_i = \frac{1}{C_i}\Big(u_i - \frac{1}{R_i}(p_i) - \frac{1}{R_{i-1,i}}(p_{i-1} - p_i)\Big).$$

The complete fault set consists of $F = \{C_i^-, C_i^+, R_i^-, R_i^+ : i = 1, \ldots, n\} \cup \{R_{i,i+1}^-, R_{i,i+1}^+ : i = 1, \ldots, n-1\}$. The complete measurement set is defined as $M = \{q_i : i = 1, \ldots, n\}$, where q_i describes the output flow of tank i, i.e.,

$$q_i = \frac{1}{R_i}(p_i).$$

3 QUALITATIVE EVENT-BASED DIAGNOSIS FRAMEWORK

We develop an event-based, qualitative diagnosis framework. Faults are viewed as unobservable events, manifesting as abrupt changes in system parameter values. These faults cause transients in the system behavior, causing deviations in observed measurement values from nominal measurement values. In this section, we first review the theoretical framework for qualitative fault isolation, followed by a formal framework for event-based fault modeling.

3.1 Qualitative Fault Isolation

Measurement deviations from nominal values caused by faults are abstracted using qualitative $+$, $-$, and 0 values to form *fault signatures* (Mosterman and Biswas, 1999). Fault signatures represent these deviations as the immediate change in magnitude and the first nonzero derivative change.

Definition 1 (Fault Signature)**.** A *fault signature* for a fault f and measurement m is the qualitative magnitude and slope change in m caused by the occurrence of f, and is denoted by $\sigma_{f,m} \in \Sigma_{f,m}$.

In general, ambiguities may exist in the fault signatures, so $\sigma_{f,m}$ may not be unique.

In addition to fault signatures, we also capture the temporal order of measurement deviations, termed *relative measurement orderings* (Daigle *et al.*, 2007c), based on the intuition that fault effects will manifest in some parts of the system before others. Measurement orderings are based on analysis of the transfer functions from faults to measurements (Daigle *et al.*, 2007c).

Definition 2 (Relative Measurement Ordering)**.** If fault f manifests in measurement m_i before measurement m_j, then we define a *relative measurement ordering* between m_i and m_j for fault f, denoted by $m_i \prec_f m_j$. We denote the set of all measurement orderings for f as $\Omega_{f,M}$.

The fault signatures and measurement orderings can be computed manually or from a system model. One method is to use a temporal causal graph (TCG) representation that is derived from the system model, along with a forward propagation algorithm to predict qualitative effects of faults on measurements and their possible sequences of deviations (Mosterman and Biswas, 1999; Daigle, 2008). TCGs can be derived automatically from a bond graph model of the system (Mosterman and Biswas, 1999; Narasimhan, 2002).

Table 1: Signatures and Measurement Orderings for the Three-tank System

Fault	q_1	q_2	q_3	Measurement Orderings
C_1^-	$+-$	$0+$	$0+$	$q_1 \prec q_2, q_1 \prec q_3, q_2 \prec q_3$
C_2^-	$0+$	$+-$	$0+$	$q_2 \prec q_1, q_2 \prec q_3$
C_3^-	$0+$	$0+$	$+-$	$q_2 \prec q_1, q_3 \prec q_1, q_3 \prec q_2$
R_1^+	$-+$	$0+$	$0+$	$q_1 \prec q_2, q_1 \prec q_3, q_2 \prec q_3$
R_2^+	$0+$	$-+$	$0+$	$q_2 \prec q_1, q_2 \prec q_3$
R_3^+	$0+$	$0+$	$-+$	$q_2 \prec q_1, q_3 \prec q_1, q_3 \prec q_2$
R_{12}^+	$0+$	$0-$	$0-$	$q_2 \prec q_3$
R_{23}^+	$0+$	$0+$	$0-$	$q_2 \prec q_1$

The fault signatures and measurement orderings for a three-tank system with $F = \{C_1^-, C_2^-, C_3^-, R_1^+, R_2^+, R_3^+, R_{12}^+, R_{23}^+\}$ and $M = \{q_1, q_2, q_3\}$ are shown in Table 1. For example, a decrease in the capacitance of tank 1, denoted C_1^-, causes a discontinuous increase in the tank 1 output flow, q_1, followed by a smooth decrease. This measurement deviation manifests first. This is followed by smooth increases in q_2 and then q_3. The tanks provide natural delays of the propagation of fault effects, which manifest in the computed measurement orderings.

3.2 Event-based Fault Modeling

Fault signatures combined with relative measurement orderings provide event-based information for diagnosis. For a given fault, the combination of all fault signatures and measurement orderings yields all the possible ways a fault can manifest in the measurements. We denote each of these possibilities as a *fault trace*.

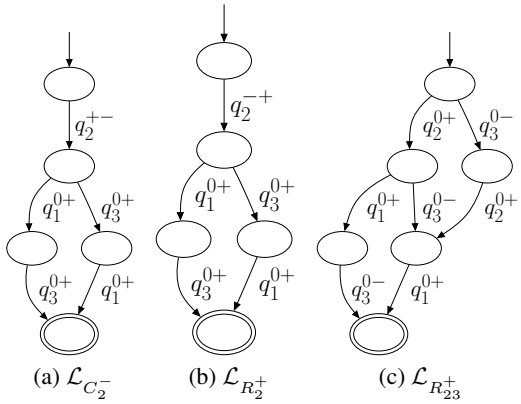
Definition 3 (Fault Trace)**.** A *fault trace* for a fault f over measurements M, denoted by $\lambda_{f,M}$, is a string of length $\leq |M|$ that includes, for every $m \in M$ that will deviate due to f, a fault signature $\sigma_{f,m}$, such that the sequence of fault signatures satisfies $\Omega_{f,M}$.

Note that the definition implies that fault traces are of maximal length, i.e., a fault trace includes deviations for all measurements affected by the fault. We group the set of all fault traces into a *fault language*. The *fault model*, defined by a *finite automaton*, concisely represents the fault language.

Definition 4 (Fault Language)**.** The *fault language* of a fault $f \in F$ with measurement set M, denoted by $L_{f,M}$, is the set of all fault traces for f over measurements M.

Definition 5 (Fault Model)**.** The *fault model* for a fault $f \in F$ with measurement set M, is the finite automaton that accepts exactly the language $L_{f,M}$, and is given by $\mathcal{L}_{f,M} = (S, s_0, \Sigma, \delta, A)$ where S is a set of states, $s_0 \in S$ is an initial state, Σ is a set of events, $\delta : S \times \Sigma \to S$ is a transition function, and $A \subseteq S$ is a set of accepting states.

The finite automata representation allows for the composition of the fault signatures and relative measurement orderings into fault models. The possible fault signatures and measurement orderings can be composed automatically to form the fault models based on the synchronization operation (Daigle *et al.*, 2007a).



(a) $\mathcal{L}_{C_2^-}$ (b) $\mathcal{L}_{R_2^+}$ (c) $\mathcal{L}_{R_{23}^+}$

Figure 2: Fault models for some faults of the three-tank system.

Selected fault models for a three-tank system are shown in Fig. 2. For example, as seen in $\mathcal{L}_{C_2^-}$, the fault C_2^- may manifest as the fault traces are $q_2^{+-} q_1^{0+} q_3^{0+}$ and $q_2^{+-} q_3^{0+} q_1^{0+}$, as implied by the fault signatures and measurement orderings.

4 DIAGNOSABILITY

With the formal fault isolation framework defined, we may now establish the notions of *distinguishability* and *diagnosability* in this framework. Using these definitions, we can then formally define the distributed diagnoser design problem. Distinguishability between faults is characterized as follows.

Definition 6 (Distinguishability)**.** With measurments M, a fault f_i is distinguishable from a fault f_j, denoted by $f_i \nsim_M f_j$, if f_i always eventually produces effects on the measurements that f_j cannot.

Under our framework, one fault will be distinguishable from another fault if it cannot produce a fault trace that is a prefix (denoted by $\sqsubseteq$) of a trace that can be produced by the other fault[1]. If this is not the case, then when that trace manifests, the first fault cannot be distinguished from the second. This is established with the following lemma (Daigle *et al.*, 2009).

Lemma 1 (Distinguishability)**.** *For meausrements M, a fault $f_i \in F$ is* distinguishable *from a fault $f_j \in F$, if there does not exist a pair of fault traces $\lambda_{f_i,M} \in L_{f_i,M}$ and $\lambda_{f_j,M} \in L_{f_j,M}$, such that $\lambda_{f_i,M} \sqsubseteq \lambda_{f_j,M}$.*

We define a system in our framework as follows.

Definition 7 (System)**.** A *system* $\mathcal{S}$ is tuple $(F, M, L_{F,M})$, where $F = \{f_1, f_2, \ldots, f_n\}$ is a set of faults, M is a set of measurements, and $L_{F,M} = \{L_{f_1,M}, L_{f_2,M}, \ldots, L_{f_n,M}\}$ is the set of fault languages.

[1] A fault trace λ_i is a prefix of fault trace λ_j if there is some (possibly empty) sequence of events λ_k that can extend λ_i such that $\lambda_i \lambda_k = \lambda_j$.

If a system is diagnosable, then we can make guarantees about the unique isolation of every fault in the system.

Definition 8 (Diagnosability)**.** A system $\mathcal{S} = (F, M, L_{F,M})$ is *diagnosable* if $(\forall f_i, f_j \in F) f_i \neq f_j \implies f_i \nsim_M f_j$.

If the system is diagnosable, then every pair of faults is distinguishable using the measurements in M. So, each sequence of measurement deviations we observe can be eventually linked to exactly one fault, if measurement deviation events are generated correctly. Hence, we can uniquely isolate all faults of interest. If the fault set is not diagnosable, then ambiguities will remain after fault isolation, i.e., after all possible measurement deviations have been observed.

5 DISTRIBUTED DIAGNOSER DESIGN

Given a system that is *diagnosable*, our objective is to decompose the overall diagnosis task into smaller subtasks performed by local diagnosers with the following properties: *(i)* all single faults of interest in the system can be diagnosed, and *(ii)* the local diagnosis results are *globally* correct[2]. These two properties eliminate the need for a centralized coordinator.

The system $\mathcal{S} = (F, M, L_{F,M})$ is split into n subsystems, $\mathcal{S}_1$, $\mathcal{S}_2$, ..., $\mathcal{S}_n$, where the fault set is partitioned among subsystems such that the complete set of faults $F = F_1 \cup F_2 \cup \ldots \cup F_n$. Each subsystem $\mathcal{S}_i$ is also assigned a subset of M, M_i for isolation of F_i, i.e., $\mathcal{S}_i = (F_i, M_i, L_{F_i,M_i})$.

Subsystems may be locally diagnosable. A locally diagnosable subsystem is one in which its own faults can be uniquely isolated using its own measurements.

Definition 9 (Local Diagnosability)**.** A subsystem $\mathcal{S}_i = (F_i, M_i, L_{F_i,M_i})$ is *locally diagnosable* if $(\forall f_i \in F_i, f_j \in F_i)\ f_i \neq f_j \implies f_i \nsim_{M_i} f_j$. We say two faults $f_i \in F_i$ and $f_j \in F_i$ are *locally distinguishable* if $f_i \nsim_{M_i} f_j$.

Local diagnosability is not sufficient for local diagnosers to achieve globally correct diagnoses. The problem is that for $\mathcal{S}_i$, there may be some $f_i \in F_i$ and for $\mathcal{S}_j$, some $f_j \in F_j$, such that f_j produces the same effects on M_i as f_i does. The result is that, if f_j occurs local diagnoser i will say that f_i has occurred. In general, we may have faults in a subsystem that are distinguishable from faults local to the subsystem, but which may not be distinguishable from faults *outside* the subsystem. For the local diagnosers to achieve globally correct local diagnoses, the subsystems must satisfy a notion of *global diagnosability*.

Definition 10 (Global Diagnosability)**.** A subsystem $\mathcal{S}_i = (F_i, M_i, L_{F_i,M_i})$ belonging to system $\mathcal{S} = (F, M, L_{F,M})$ is *globally diagnosable* if $(\forall f_i \in F_i, f_j \in F) f_i \neq f_j \implies f_i \nsim_{M_i} f_j$. We say two faults $f_i \in F_i$ and $f_j \in F$ are *globally distinguishable* if $f_i \nsim_{M_i} f_j$.

[2]If the system $\mathcal{S}$ is not diagnosable, we can define aggregate faults, where an aggregate fault is a set of faults that are indistinguishable from each other. The diagnosis methodology can be applied to the modified fault set that includes the aggregate faults (Roychoudhury *et al.*, 2009a).

That is, a subsystem $\mathcal{S}_i$ is *globally diagnosable* if all the faults F_i are distinguishable from every other fault $f \in F$ using only the measurements in M_i. If the subsystems can be structured such that each subsystem $\mathcal{S}_i$ is globally diagnosable, then each local diagnoser can independently generate local diagnoses which are globally correct.

For example, consider the three-tank system defined earlier, with $F = \{C_1^-, C_2^-, C_3^-, R_1^+, R_2^+, R_3^+, R_{12}^+, R_{23}^+\}$ and $M = \{q_1, q_2, q_3\}$. Let us define a subsystem for each tank, where for $i = 1, \ldots, n-1$, $\mathcal{S}_i$ is defined by $F_i = \{C_i^-, R_i^+, R_{i,i+1}^+\}$ and $M_i = \{q_i\}$, and for $i = n$, $\mathcal{S}_i$ is defined by $F_i = \{C_i^-, R_i^+\}$ and $M_i = \{q_i\}$. Consider tank 1. If 0+ is observed for q_1, then that may be the result of local fault R_{12}^+ or any of the remote faults (see Table 1). Clearly, $\mathcal{S}_1$ is not globally diagnosable. Note that it is locally diagnosable, as the three local faults each produce a different effect on the sole measurement of the subsystem, q_1.

Different design problems may be defined which determine partitions of the fault set F and/or the assignment of measurements to subsystems (Roychoudhury *et al.*, 2009a). In each case, the end result must be a set of subsystems, each of which are globally diagnosable. In this paper, we focus on the problem where the system is already partitioned into subsystems, where each subsystem may not be globally diagnosable. We define the distributed diagnoser design problem as the problem of determining, for each subsystem, the minimal number of measurements to pull in from other subsystems to achieve global diagnosability. Formally, the problem can be defined as follows.

Problem (Partitioned System Diagnoser Design)**.** Given n subsystems, where $\mathcal{S}_i = (F_i, M_i, L_{F_i,M_i})$, construct, for each subsystem, a measurement set $M_i{}^+ \subseteq M$ such that *(i)* $M_i^+ - M_i$ is minimal, and *(ii)* $\mathcal{S}_i' = (F_i, M_i^+, L_{F_i,M_i^+})$ is globally diagnosable.

This problem is a variation of the *measurement selection* problem (Narasimhan *et al.*, 1998), which is an instance of the set covering problem, which is known to be NP-complete. Our goal, while designing the local diagnosers, is to minimize the sharing of measurements across subsystems in order to limit the size of the local diagnosers and communication requirements between them. We simplify the measurement search using measurement orderings as a guide, based on the intuition that measurements that deviate before others are more helpful. Further, these measurements provide the fastest diagnosis. To do this, for each fault that is not globally distinguishable, we determine the measurements that deviate first by looking at the measurement orderings, and this set of measurements over all the faults forms the current working measurement set, i.e., measurements with which we try to resolve global diagnosability. This heuristic simplifies the search process, but the algorithm is still exponential in the general case. Additional heuristics may also be used, e.g., the subsystem distance heuristic presented in (Roychoudhury *et al.*, 2009a).

The distributed diagnoser design procedure is given as Algorithm 1. For a diagnosable system $\mathcal{S}$, for each subsystem $\mathcal{S}_i$, we first determine, using diagnosability

Algorithm 1 Distributed Diagnoser Design

Input: $\mathcal{S} = \{\mathcal{S}_i = (F_i, M_i, L_{F_i,M_i}) : i = 1, \ldots, n$
for all $\mathcal{S}_i \in \mathcal{S}$ **do**
  identify $F_i^* = \{f_i^* \in F_i : f_i \nsim_{M_i} f_j$ for $f_j \in F\}$
  $M_i^+ \leftarrow M_i$
  while $F_i^* \neq \varnothing$ **do**
    for all $f_i^* \in F_i^*$ **do**
      $M_{f_i^*} \leftarrow \{m : (m' \prec m) \notin \Omega_{f_i^*, M - M_i^+}\}$
    end for
    identify minimal $M_i^* \subseteq \{M_{f_i^*} : f_i^* \in F_i^*\}$ such that $M_i^+ \cup M_i^*$ isolates maximal $F_i' \subseteq F_i^*$
    $M_i^+ \leftarrow M_i^+ \cup M_i^*$
    $F_i^* \leftarrow F_i^* - F_i'$
  end while
  construct $\mathcal{D}_{F_i, M_i^+}$
end for

analysis, the set of faults $F_i^* \subseteq F_i$ which are not globally distinguishable using M_i. At each iteration, for each fault that is not globally distinguishable using the current measurement set, M_i^+, we compute the set of measurements out of $M - M_i^+$ that may deviate first for the fault, as $M_{f_i^*}$. We then find the minimal set of measurements to add to M_i^+ from the set of measurements found in this way over all f_i^*, and add these to M_i^+. The process repeats until $\mathcal{S}_i$ is globally diagnosable, resulting in the local diagnoser $\mathcal{D}_{F_i,M_i^+}$. We will describe the construction of $\mathcal{D}_{F_i,M_i^+}$ in the next section.

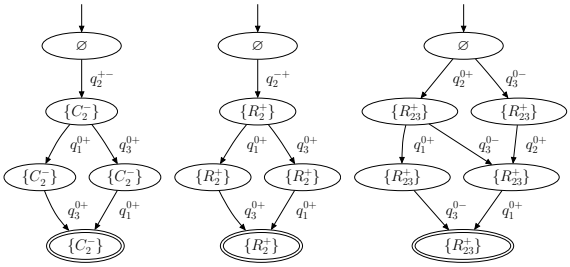
We apply this algorithm to the n-tank system, where for $i = 1, \ldots, n-1$, $\mathcal{S}_i$ is defined by $F_i = \{C_i^-, R_i^+, R_{i,i+1}^+\}$ and $M_i = \{q_i\}$, and for $i = n$, $\mathcal{S}_i$ is defined by $F_i = \{C_i^-, R_i^+\}$ and $M_i = \{q_i\}$. For tank 1, R_{12}^+ is not globally distinguishable. From the measurement orderings, q_2 will deviate before q_3, so $M_i^* = \{q_2\}$. This measurement alone is sufficient to add to M_i^+ to obtain global diagnosability, so no further iteration is necessary. For tank 2, R_{23}^+ is not globally distinguishable, and either q_2 or q_3 may deviate next. Measurement q_3 alone is sufficient to achieve global diagnosability. For tank 3, the subsystem is already globally diagnosable. The new measurement sets are therefore $M_1 = \{q_1, q_2\}$, $M_2 = \{q_2, q_3\}$, and $M_3 = \{q_3\}$.

6 DIAGNOSER IMPLEMENTATION

In this section we describe the construction of the event-based diagnosers. The goal of the event-based diagnoser is, given a sequence of measurement deviation events, to determine which faults are consistent with the observed sequence. We define formally a *diagnosis* and a *diagnoser* in our framework (Daigle *et al.*, 2009).

Definition 11 (Diagnosis)**.** A *diagnosis* $d \subseteq F$ is a set of faults, each of which is consistent with the observations.

Definition 12 (Diagnoser)**.** A *diagnoser* for a fault set F and measurement set M is a tuple $\mathcal{D}_{F,M} = (S, s_0, \Sigma, \delta, A, D, Y)$ where S is a set of states, $s_0 \in S$ is an initial state, Σ is a set of events, $\delta : S \times \Sigma \to S$ is a transition function, $A \subseteq S$ is a set of accepting states, $D \subseteq 2^F$ is a set of diagnoses, and $Y : S \to D$ is a diagnosis map.



(a) $\mathcal{D}_{C_2^-}$ (b) $\mathcal{L}_{R_2^+}$ (c) $\mathcal{D}_{R_{23}^+}$

Figure 3: Diagnosers for some individual faults of the three-tank system.

A diagnoser is a finite automaton extended by a set of diagnoses and a diagnosis map. It takes events as inputs, which, as with fault models, correspond to measurement deviations. From the current state, a measurement deviation event causes a transition to a new state. The diagnosis for that new state represents the set of faults that are consistent with the sequence of events seen up to the current point in time. So, like traditional DES diagnosers, the diagnoser states provide estimates of the system condition, but only after a fault has occurred.

The accepting states of the diagnoser correspond to a fault isolation result. We say that a diagnoser *isolates* a fault if it accepts all possible valid traces for the fault and the accepting states map to diagnoses containing the fault.

Definition 13 (Isolation)**.** A diagnoser $\mathcal{D}_{F,M}$ *isolates* fault $f \in F$ if $\mathcal{D}_{F,M}$ accepts all $\lambda_{f,M} \in L_{f,M}$ and for each $s \in A$ that accepts some $\lambda_{f,M}$, $f \in Y(s)$.

We also would like to achieve unique isolation of faults, which corresponds to system diagnosability. We say that a diagnoser *uniquely isolates* a fault if each accepting state maps to the single fault.

Definition 14 (Unique Isolation)**.** A diagnoser $\mathcal{D}_{F,M}$ *uniquely isolates* fault $f \in F$ if $\mathcal{D}_{F,M}$ accepts all $\lambda_{f,M} \in L_{f,M}$ and for each $s \in A$ that accepts some $\lambda_{f,M}$, $\{f\} = Y(s)$.

Ultimately, we would like to systematically construct a diagnoser for a system S that isolates all $f \in F$. Further, we would like to show that if S is diagnosable, then this diagnoser uniquely isolates all $f \in F$. This procedure has been developed in previous work (Daigle *et al.*, 2009). Here, we briefly review the main points.

First, we construct a diagnoser for each fault f that isolates f, i.e., $\mathcal{D}_{\{f\},M}$. These are shown in Fig. 3 for some of the faults of the three-tank system. They are constructed directly from the fault models $\mathcal{L}_{f,M}$, c.f. Fig. 2. Because the fault model $\mathcal{L}_{f,M}$ accepts the fault language $L_{f,M}$, it is easy to show that this diagnoser isolates f.

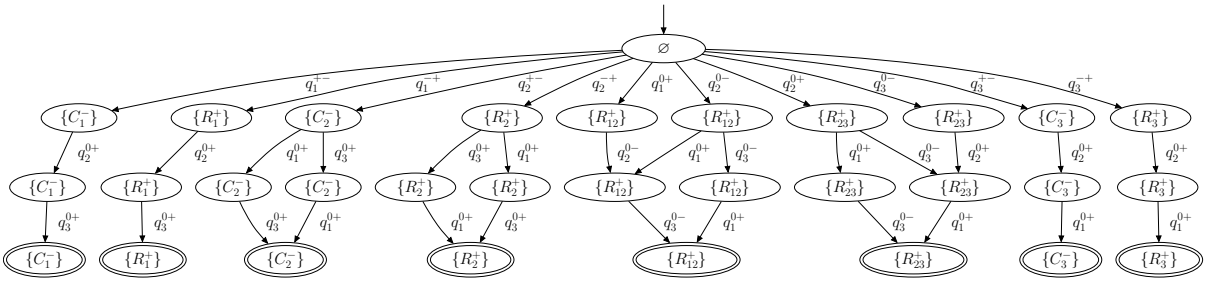
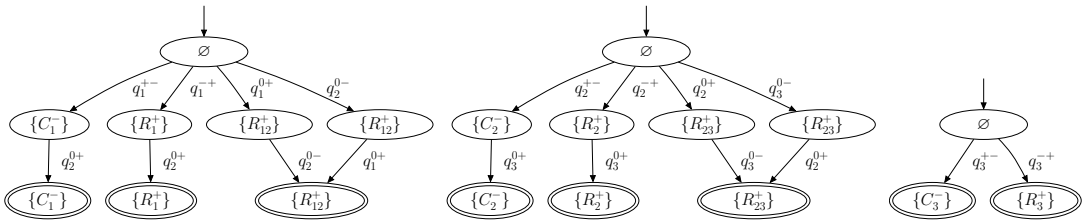


Figure 4: Three-tank system centralized diagnoser for $F = \{C_1^-, C_2^-, C_3^-, R_1^+, R_2^+, R_3^+, R_{12}^+, R_{23}^+\}$ and $M = \{q_1, q_2, q_3\}$



(a) $\mathcal{S}_1$ diagnoser. (b) $\mathcal{S}_2$ diagnoser. (c) $\mathcal{S}_3$ diagnoser.

Figure 5: Local diagnosers for the three-tank system for $F_1 = \{C_1^-, R_1^+, R_{12}^+\}$, $M_1 = \{q_1, q_2\}$, $F_2 = \{C_2^-, R_2^+, R_{23}^+\}$, $M_2 = \{q_2, q_3\}$, $F_3 = \{C_3^-, R_3^+\}$ and $M_3 = \{q_2, q_3\}$.

A composition operator is then defined that composes two diagnosers, such that if each diagnoser isolates its own set of faults, the composed diagnoser will isolate the combined set of faults. We may then compose the individual diagnosers into a global diagnoser that isolates the complete set of system faults. We have shown that the system defined by F and M is diagnosable if and only if the diagnoser constructed in this way uniquely isolates all faults in F (Daigle *et al.*, 2009).

The resulting global diagnoser for the three-tank system described in the earlier sections is given in Fig. 4. It is clear from this figure that the system is diagnosable, as each accepting state has a unique diagnosis. In this case, a unique diagnosis is even known after only a single measurement deviation. The resulting diagnoser may be pruned to reduce diagnoser size by removing states and transitions occuring after a unique diagnosis is known (Daigle, 2008).

6.1 Local Diagnoser Implementation

The design of local diagnosers follows the same procedure as the global diagnoser, i.e., given F_i and M_i for subsystem $\mathcal{S}_i$, we construct $\mathcal{D}_{F_i,M_i}$. The local diagnosers for the distributed diagnoser design example from the previous section are given in Fig. 5. Note that each local diagnoser except the third needs only two measurements, whereas the global diagnoser needs all 3. As n increases, each local diagnoser (except the third) still needs only two measurements, whereas the global diagnoser needs all n measurements, significantly increasing its size.

In terms of scalability, the distributed diagnosis scheme clearly improves on the centralized diagnosis approach. In the worst case, the size of a diagnoser increases factorially with the number of measurements (Daigle *et al.*, 2009). Therefore, the fewer the measurements associated with a diagnoser to achieve local and global diagnosability, the smaller a diagnoser will be. By creating local diagnosers such that each diagnoser uses only a limited number of measurements, each local diagnoser can be significantly smaller than the centralized diagnoser, and the combined size of all local diagnosers can be smaller also.

The distributed diagnosis approach works as follows. Each local diagnoser starts in its initial state. A measurement deviation event is received by all subsystems that include that measurement in their measurement set. If there is a matching event from the current state, a local diagnoser will follow that path to the next state, and remain active. If not, the local diagnoser will block, and its diagnosis result will be $\varnothing$. The process continues until a local diagonser reaches an accepting state. At this point, a globally correct diagnosis is known, if each subsystem was designed to be globally diagnosable. If so, no other local diagnoser may reach an accepting state. Therefore, a globally correct diagnosis result is achieved without the use of a centralized coordinator. If the subsystems are not globally diagnosable, then two or more local diagnosers may both reach an accepting state and a coordinator is needed.

A globally correct diagnosis result may be declared earlier if a local diagonser has not yet reached an accepting state, but has a unique diagnosis, only if all other local diagnosers have blocked. A globally correct diagnosis result may otherwise only be declared when all measurements for a subsystem have deviated (i.e., an accepting state is reached). These conditions correspond directly to those outlined in (Roychoud-

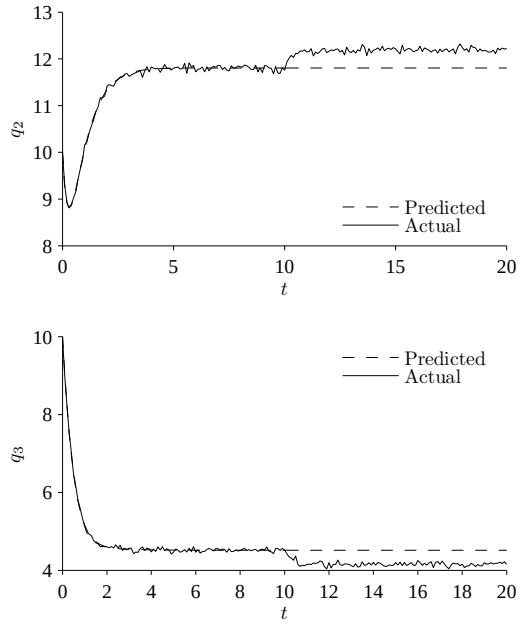


Figure 6: Six tank predicted and observed flow outputs.

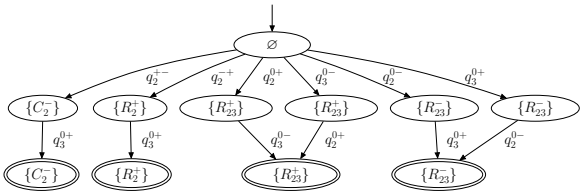
hury *et al.*, 2009a) in the absence of the event-based framework.

7 RESULTS

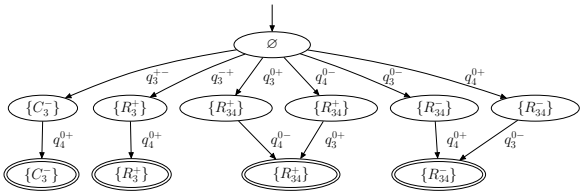
As an example to demonstrate online diagnosis in this framework, consider a six-tank system, with R_{23}^{+} occurring at time 10.0. The plots of q_2 and q_3 are shown in Fig. 6. At time 10.3 a $0-$ is detected in q_3, using the symbol generation mechanism described in (Biswas *et al.*, 2003). Both the local diagnosers for $\mathcal{S}_2$ and $\mathcal{S}_3$ use this measurement and compute this symbol. Partial diagnosers (with some faults omitted) for these diagnosers are shown in Fig. 7. The $\mathcal{S}_2$ diagnoser moves to a state with R_{23}^{+} as the sole candidate, and the $\mathcal{S}_3$ diagnoser moves to a state with R_{34}^{-} as the sole candidate. At time 10.4, a $0+$ is detected in q_2. The $\mathcal{S}_2$ diagnoser moves to an accepting state with R_{23}^{+} as the sole candidate. The $\mathcal{S}_3$ diagnoser does not use this measurement so takes no action. Because the $\mathcal{S}_2$ diagnoser reached an accepting state, a global diagnosis has been achieved.

7.1 Scalability

Here, we consider n-tank systems where for $i = 1, \ldots, n-1$, $F_i = \{C_i^{-}, C_i^{+}, R_i^{+}, R_i^{-}, R_{i,i+1}^{+}, R_{i,i+1}^{-}\}$ and for $i = n$, $F_i = \{C_i^{-}, C_i^{+}, R_i^{+}, R_i^{-}\}$. The same measurement assignment occurs with the distributed diagnoser design algorithm, except tank n needs an additional measurement, i.e., each subsystem pulls in a measurement from an adjacent subsystem. The local diagnoser for $i = 1, \ldots, n-1$ is always 13 states with 14 transitions for the non-pruned version, and 11 states and 10 transitions for the pruned version. For local diagnoser n, both the non-pruned and pruned versions have 7 states and 6 transitions.



(a) Partial $\mathcal{S}_2$ diagnoser.



(b) Partial $\mathcal{S}_3$ diagnoser.

Figure 7: Some partial local diagnosers for the six-tank system

Table 2: Scalability Results for the Multi-tank System

	Not Pruned				Pruned			
Tanks	$\lvert S\rvert$	$\lvert\delta\rvert$	$\Sigma\lvert S_i\rvert$	$\Sigma\lvert\delta_i\rvert$	$\lvert S\rvert$	$\lvert\delta\rvert$	$\Sigma\lvert S_i\rvert$	$\Sigma\lvert\delta_i\rvert$
2	19	20	20	20	17	16	18	16
3	69	96	33	34	37	28	29	26
4	113	148	46	48	55	42	40	36
5	205	284	59	62	73	76	51	46
6	335	484	72	76	91	96	62	56
7	579	840	85	90	109	116	73	66
8	845	1264	98	104	127	136	84	76
9	1181	1812	111	118	145	156	95	86
10	1595	2500	124	132	163	176	106	96

The results demonstrating the scalability of the approach as compared to a centralized approach are shown in Table 2. For both non-pruned and pruned diagnosers, we report the number of states, $|S|$, and number of transitions, $|\delta|$. For the local diagnosers, we sum the number of states over each diagnoser, $\Sigma|S_i|$, and the number of transitions, $\Sigma|\delta_i|$. The sum of the local diagnoser sizes increase linearly, whereas the size of the centralized diagnoser increases exponentially, demonstrating a clear improvement in scalability. In the case of the pruned diagnosers, the centralized diagnoser size increases linearly as well, although its size is still larger than for the local diagnosers. This result is not general but arises here because of the structure imposed by the measurement orderings.

8 CONCLUSIONS

We developed a formal framework for event-based qualitative diagnosis of continuous systems. Global and local diagnosers are automatically derived from fault signatures and relative measurement orderings, which, in turn, may be derived automatically from a system model. This results in a distributed diagnosis framework that eliminates the single point of failure associated with centralized diagnosis frameworks or

distributed frameworks that require the use of a centralized coordinator, while the local diagnosers still obtain globally correct diagnoses. The approach may be naturally applied to systems with clear subsystem boundaries. The distributed approach also scales well with an increase in the number of subsystems, particularly in comparison to a centralized diagnoser.

The event-based framework presented here has clear connections to discrete-event diagnosis methods, e.g., (Sampath *et al.*, 1996; Zad *et al.*, 2003), and also distributed discrete-event diagnosis methods such as (Debouk *et al.*, 2000). Our particular approach may be viewed as an implementation of Protocol 3 in (Debouk *et al.*, 2000), in which we solve the design problem to achieve the conditions for a coordinator-free approach. The use of measurement orderings is also similar to the work of (Meseguer *et al.*, 2008; Puig *et al.*, 2005), where signatures are derived from analytical redundancy relations (ARRs) and do not utilize the rich symbol framework for fault signatures used here. In (Bayoudh *et al.*, 2006), a similar approach is applied to hybrid systems, where the events are defined as changes in ARR values due to mode changes.

In future work, we will be extending the approach to multiple faults based on previous work in (Daigle *et al.*, 2007b), and to hybrid systems, based on preliminary results presented in (Roychoudhury *et al.*, 2009b). We will also investigate alternative distributed design algorithms.

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