# A Model-Based Probabilistic Inversion Framework for Characterizing Wire Fault Detection Using TDR

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Abstract—Time-domain reflectometry (TDR) is one of the standard methods for diagnosing faults in electrical wiring and interconnect systems, with a long-standing history focused mainly on hardware development of both high-fidelity systems for laboratory use and portable hand-held devices for field deployment. While these devices can easily assess distance to hard faults such as sustained opens or shorts, their ability to assess subtle but important degradation such as chafing remains an open question. This paper presents a unified framework for TDR-based chafing fault detection in lossy coaxial cables by combining an S-parameter based forward modeling approach with a probabilistic (Bayesian) inference algorithm. Results are presented for the estimation of nominal and faulty cable parameters from laboratory data.

Index Terms—S-parameters, TDR, wiring, Bayesian, fault detection

#### I. INTRODUCTION

THE Federal Aviation Administration (FAA), Naval Sys-L tems Air Command (NAVAIR) and National Aeronautics and Space Administration (NASA) have all identified wire chafing as the largest factor contributing to electrical wiring and interconnect system failures in aging aircraft [1]. Furthermore, the detection of wire chafing is important because it leads to more significant problems such as opens and shorts. This article provides a technically extended discussion of results initially published in [2] on a new general method for characterizing wiring chafe detectability using time domain reflectometry (TDR). Our approach combines physics-based modeling for signal propagation through the system and fault, with a probabilistic inference method for recovering key system parameters, including fault location and size, from measured data. The method further provides clear uncertainty information regarding the estimated parameters, without relying on linear model approximation techniques. Finally, it is flexible enough to apply to a variety of wiring types, measurement conditions, and arbitrary input interrogation signals.

TDR is an industry standard method for diagnosing faults in wiring systems. Intuitively, it works by applying an input signal (*e.g.*, step, Gaussian pulse, pseudo-noise) to the wire under test, which propagates as a wave along the line. When the main wavefront passes over a fault on the line, part of it is reflected and travels back to the input where it can be measured. Finally, the measured response is diagnosed, either by eye or using automated software, for signal variation caused by potential faults.

Wiring fault detection using TDR has a long history, where the detection of chafing is considered significantly more difficult than hard failures such as opens and shorts [3]. Over the last decade, many time-domain reflectometry (TDR), frequency-domain reflectometry (FDR) and time- and frequency-based investigations [4] were published. Among these investigations and many others the primary mechanism for automated fault detection is the application of a sliding correlator, or matched filter, to detect fault location and size [3]–[8]. In addition, knowledge of the wire material parameters such as permittivity and conductivity along with measurement setup and impedance matching conditions are usually either assumed known in advance, or fixed from baseline measurements.

Unfortunately, these methods generally fail to detect small faults in practice for at least a couple reasons. First, when baseline measurements are available, they are often unreliable because of the constantly changing material properties and measurement conditions in the field. Second, matched filter based detection techniques are optimal only after characterizing and accounting for the channel. For the wire fault detection problem, the channel depends not only on the same changing material properties and measurement conditions that affect the baseline, but also on the location of the fault. Even high quality cable exhibits loss and dispersion effects that significantly change the shape of the propagating signal wave as a function of the propagation distance. In essence, one does not reliably know ahead of time the correct matched filter to use. Finally, because correlation based detection methods fail to accurately account for these effects, they also fail to answer basic analysis trade-space questions such as fault detectability versus distance.

The method presented in this article overcomes the difficulties with traditional approaches highlighted in the previous paragraph. We begin in section II by developing a framework based on *scattering parameters* (or *S*-parameters) to build a computationally simple yet effective forward model for how chafed shielding affects signal propagation, and thus the measured TDR response. This model includes the key parameters contributing to signal loss and dispersion effects such as dielectric permittivity, finite conductor conductivity, and input source impedance mismatch. In section III, this forward model is then combined with a general Bayesian probabilistic inversion procedure, which enables robust fault parameter estimation in the presence of measurement noise and initial model parameter uncertainty (*i.e.*, uncertainty in permittivity,

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conductivity, impedance mismatch, etc). In fact, this method simultaneously estimates not only fault location and size, but an entire set of key parameters affecting the measured TDR response along with the corresponding joint uncertainty information, which in turn enables a reliable characterization of trade-space issues. Finally, in section IV example results characterizing fault detectability in RG58 coaxial cable are presented.

To keep the presentation clear and concrete, our method is explained in terms of a simple example involving a single chafing fault in coaxial cable. However, it should be clear throughout that this example is easily generalized to handle a wide variety of wire types and fault conditions, simply by replacing the individual S-parameter models for the coaxial case, with the S-parameter models required for the general case. In addition, a new effective TDR hardware model is derived that may be common to many systems.

Finally, before moving on, we admit up front that the fault parameter retrieval method presented here is not intended for practical application in the field, because it is computationally far too slow. However, the method is important because it enables a general characterization of fault detectability in a wide variety of wiring systems using virtually any TDR hardware measurement setup and input interrogation signal. As such, it establishes fundamental limits on fault detection performance in advance of further hardware and software development cost.

#### II. FORWARD MODEL FOR TDR

This section describes our systematic approach to building a computationally efficient forward model for the interrogation of a chafed coaxial cable using TDR. The modeling method of choice is the S-parameter formalism; the reader is referred to [9], [10] for a refresher. Specifically, each cable segment is treated as a two-port device with a  $2 \times 2$  matrix of S-parameters. These S-parameters are then combined in cascade to obtain the overall response of the system. In this process, one is aided by the formula:

$$\Gamma_1 = S_{11} + \frac{S_{12}S_{21}\Gamma_2}{1 - S_{22}\Gamma_2},\tag{1}$$

which relates the reflection coefficients seen looking into port 1 ( $\Gamma_1$ ) and out of port 2 ( $\Gamma_2$ ) of a two-port device within a network (see Figure 2).

## A. Coaxial Cable

For nominal (i.e., unfaulted) segments of the cable, one has

$$S_{11} = S_{22} = 0,$$
  

$$S_{12} = S_{21} \equiv S_0(l),$$

where the dependence of the relevant S-parameters on the cable length l has been indicated explicitly for later convenience. Adopting the standard textbook model for a coaxial transmission line (see, for instance, [11], p. 551) one obtains

$$S_0(l) = e^{-jk(\omega)l},\tag{2}$$



Fig. 1: Impedance Step.

where

$$k(\omega) \simeq \omega \sqrt{\mu_0 \epsilon_d} + \frac{1}{2\ln(b/a)} \sqrt{\frac{\omega \epsilon_d}{j\sigma_c}} \left(\frac{1}{a} + \frac{1}{b}\right).$$
(3)

In (3), *a* and *b* respectively denote the radius of the core and the (inner) radius of the shield, both of which are assumed to have a (finite) conductivity  $\sigma_c$ , while  $\epsilon_d$  denotes the permittivity of the insulator separating the two conductors, and  $\mu_0$  is the vacuum permeability. We will also need the characteristic impedance of the cable, which is given by

$$Z_0 = \frac{\ln(b/a)}{2\pi} \frac{k(\omega)}{\omega\epsilon_d}.$$
(4)

The above formulation relates the key cable parameters ( $S_0$  and  $Z_0$ ) directly to the "constitutive" parameters ( $\sigma_c$  and  $\epsilon_d$ ), and is therefore preferable to the distributed RLCG parameter model that is more commonly found in textbook treatments.

#### B. Impedance Step

In this section a model for an impedance step in the system is derived. Figure 1 illustrates the problem in a generic setting. The task is then to determine  $\Gamma_1$  given  $\Gamma_2$ ,  $Z_2$ , and  $Z_1$ . First we define the reflection coefficient caused by the impedance step (for waves moving to the right):

$$\Gamma_s = \frac{Z_2 - Z_1}{Z_1 + Z_2} = \left[\frac{V_1^-}{V_1^+}\right]_{V_2^- = 0}$$

Using a voltage loop it is easy to see that the transmission coefficient must then be  $1 + \Gamma_s$ . Combining these two facts we can write the following two equations for the voltage waves entering and exiting the impedance step:

$$\begin{array}{rcl} V_1^- &=& \Gamma_s V_1^+ + (1 - \Gamma_s) V_2^- \\ V_2^+ &=& (1 + \Gamma_s) V_1^+ - \Gamma_s V_2^-, \end{array}$$

and from these two equations the desired result is easily obtained:

$$\Gamma_1 = \frac{\Gamma_s + \Gamma_2}{1 + \Gamma_s \Gamma_2}.$$
(5)

## C. Chafing Fault

A simple yet accurate model for the S-parameters of a chafed coaxial cable is now presented using an approach that is generalizable to other types of wiring. The situation of interest is depicted in Figure 2, where a segment of length d and width w is chafed on a coaxial cable with characteristic impedance  $Z_0$ . The chafed segment is modeled as having a constant (*i.e.*, z- and  $\omega$ -independent) characteristic impedance  $Z_F$ .



Fig. 2: (Left) A constant-impedance model for a chafed cable segment. (Right) Cross section of chafed coax.

To derive  $S_{11}$  of the chafe, we conceptually match the coax impedance on the right and define,

$$\Gamma_2 = \frac{Z_0 - Z_F}{Z_0 + Z_F},$$

as the reflection coefficient for a wave traveling into the second impedance discontinuity. Any wave transmitted through this discontinuity will never return because the impedance is matched. Now, if we move d meters to the left of the second impedance discontinuity to a position just after the first impedance discontinuity the input reflection coefficient will be:

$$\Gamma_2 e^{-j\omega \, 2t_d},$$

where  $t_d$  is the one-way travel time from the first to the second discontinuity ( $2t_d$  seconds pass as the incident wave travels this distance, reflects from the second discontinuity, and travels back). Clearly,  $t_d = d/v_p$ , where  $v_p$  is the wave propagation velocity inside the fault, which for our model will equal the propagation velocity of the nominal cable. The only remaining step is to cross the first impedance discontinuity. To do this we use equation (5) from §II-B, where  $\Gamma_s = -\Gamma_2$ . Thus,

$$S_{11} = \Gamma_1 = \frac{\Gamma_2(e^{-j\omega 2t_d} - 1)}{1 - \Gamma_2^2 e^{-j\omega 2t_d}},$$
(6)

where the first equality follows because the output impedance is matched.

Next we derive an approximation for  $S_{21}$  by simply noting  $(1-\Gamma_2)$  times the incident voltage wave is transmitted through the first impedance discontinuity, delayed by  $t_d$ , and  $(1 + \Gamma_2)$ is transmitted through the second discontinuity. Thus,

$$S_{21} \approx (1 - \Gamma_2^2) e^{-j\omega t_d}.$$
(7)

This is an approximation because there are additional reflections that ring within the fault.

The exact expression is derived by tracing the voltage waves as they reflect within the fault, and adding the transmitted parts of the delayed reflections together in an infinite series. This procedure produces,

$$S_{21} = \frac{(1 - \Gamma_2^2)e^{-j\omega t_d}}{1 - \Gamma_2^2 e^{-j\omega 2t_d}}.$$
(8)

Note, this exact expression does produce noticeably better results when the fault magnitudes are also small (and they usually are). Finally, since this chafe model is symmetric, we have  $S_{11} = S_{22}$ , and  $S_{21} = S_{12}$ .



Fig. 3: Source Connection.

We must next relate the hitherto unknown parameters  $Z_F$ and  $v_p$  to the geometry of the chafe's cross section shown in Figure 2. This geometry depends on the conductor radius a, inside shield radius b, and outside shield radius c which are considered known (but cable type dependent) constants. The fault impedance  $Z_F$  and velocity of propagation  $v_p$  are both functions of principally the chafe width w and dielectric permittivity  $\epsilon_d$ . These functions are determined numerically by building lookup tables using a standard finite difference method to solve for  $Z_F$  and  $v_p$  over a grid of different values for w and  $\epsilon_d$ . The theoretical underpinnings and the numerical implementation of this approach are presented in  $[12]^1$ . We have found that this simple rectangular chafe geometry and lookup table based approach are remarkably accurate for modeling practical chafes, which are typically elliptical in shape.

## D. Source Connection

In this section a simple source connection model is derived. The situation is presented in Figure 3. Using the equations shown on the schematic and a little algebra it is easy to show:

$$\frac{V^{-}}{V_{S}} = \frac{Z_{0}\Gamma}{Z_{0}(1+\Gamma) + Z_{S}(1-\Gamma)}$$
(9)

$$\frac{V}{V_S} = \left(\frac{1+\Gamma}{\Gamma}\right)\frac{V^-}{V_S} = \frac{Z_0(1+\Gamma)}{Z_0(1+\Gamma) + Z_S(1-\Gamma)} (10)$$

An important subtlety is the net voltage V is measured in the characteristic impedance  $Z_0$ , after any possible impedance mismatch with the source. Since most TDR systems measure voltage with respect to the source impedance rather than the line impedance, this important case is treated in the next section.

# E. TDR Hardware

A general model for the TDR hardware is shown in Figure 4. In this figure, the "down-stream network" represents any wiring system that is defined by a characteristic impedance  $Z_0$  and a reflection coefficient  $\Gamma_0$  at the system input. The goal is to determine the experimentally measured voltage  $V_M$ in terms of the TDR source voltage  $V_S$ .

<sup>&</sup>lt;sup>1</sup>Note the method presented in [12] assumes  $v_p$  is equal to the nominal velocity of propagation on the cable, which is  $\simeq 1/\sqrt{\mu_0 \epsilon_d}$ . While this is not theoretically true, the assumption seems to work reasonably well in practice for the small chafe faults considered in this paper.

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Good models for TDR hardware should incorporate three practical effects: (1) the frequency-dependent impedance mismatch between the source and the cable, (2) a measurement delay time needed to account for signal propagation *within* the TDR unit, and (3) a gain factor to account for a typically small mis-calibration between the modeled and measured TDR response voltages. The equation below for the net transfer function captures these effects:

$$H(\omega) = \frac{V_M}{V_S} = \frac{G}{2} \left( 1 + \frac{\Gamma_S + \Gamma_0}{1 + \Gamma_S \Gamma_0} e^{-j2\omega t_M} \right), \qquad (11)$$

where  $\Gamma_S = (Z_0 - Z_S)/(Z_0 + Z_S)$  accounts for the port impedance mismatch,  $t_M$  represents the one-way internal delay, and G is the gain factor used to account for possible calibration issues. The key parameters for the TDR unit are thus seen to be the source impedance  $Z_S$ , the internal delay  $t_M$ , and gain factor G.

Equation (11) is derived by processing the schematic from right to left. To start, we need to deal with the fact that  $\Gamma_0$ is specified with reference to  $Z_0$ , while the TDR voltage measurement is made with respect to  $Z_S$ . In other words, there is a possible impedance mismatch between the TDR port and the downstream network. This is easily accomplished by using the impedance step model presented earlier with  $\Gamma_S = (Z_0 - Z_S)/(Z_0 + Z_S)$ . The updated reflection transfer function after the impedance step is then:

$$\Gamma_0' = \frac{\Gamma_S + \Gamma_0}{1 + \Gamma_S \Gamma_0}.$$
(12)

The next step is to incorporate the delay block which represents a time lag between the voltage measurement and the TDR port. This is also easy to do using the fact that a delay in time is equivalent to the following in the frequency domain:

$$\Gamma_M = \Gamma'_0 e^{-j\omega 2t_M}.$$
(13)

Finally, we need to convert  $\Gamma_M$  which specifies the transfer function between the forward and reverse voltage waves into the transfer function between the net source signal  $V_S$  and the measured net response signal  $V_M$ . With respect to Figure 3,  $\Gamma$  is the reflection coefficient looking into the characteristic impedance  $Z_0$ . Since equation (12) already took care of the TDR port impedance mismatch,  $\Gamma_M$  is looking into  $Z_S$ , so we can set  $Z_0 = Z_S$  for this final step in our development of the TDR hardware model. Thus equation (10) above simplifies to:

$$\frac{V_M}{V_S} = \frac{1 + \Gamma_M}{2}.$$
 (14)



Fig. 4: TDR hardware model.

Combining equations (12), (13), and (14) produces the TDR hardware model given by (11) after multiplying by the system gain factor G.

### F. Model Synthesis

The pieces discussed separately above are now put together to obtain the system model shown in Figure 5. The model is analyzed from right to left, starting with the load reflection coefficient  $\Gamma_L = (Z_L - Z_0)/(Z_L + Z_0)$ . By repeated application of equation (1), we obtain

$$\Gamma_2 = S_0^2(l_2)\Gamma_L, \tag{15}$$

$$\Gamma_1 = S_{11} + \frac{S_{12}S_{21}\Gamma_2}{1 - S_{22}\Gamma_2}, \tag{16}$$

$$\Gamma_0 = S_0^2(l_1)\Gamma_1, \tag{17}$$

where  $S_0(l)$  is given in (2), and  $S_{ij}$  are given in (6) and (8).

Inserting these equations into (11), we obtain an *analytical* relationship between the TDR input and output signals, which explicitly contains the various physical system and fault parameters discussed above. (The derivation is straightforward, but the result is too unwieldy to include here.) Rewriting (11) in the time domain<sup>2</sup>, we have

$$v_M(t) = \int_0^t h(t - t'; \theta) \, v_S(t') \, \mathrm{d}t', \tag{18}$$

where the dependence of the impulse response h on the set  $\theta$  of key model parameters has been indicated to motivate the discussion in §III. Typically, equation (18) is computed numerically using the Fast Fourier Transform (FFT) algorithm.

We note in passing that this modeling approach can be generalized readily to a cable with chafes (or other kinds of faults) at multiple locations, and in fact to arbitrary wiring networks. Most importantly, as the number of wiring and interconnect components grows, the computational effort needed to evaluate the model grows only linearly, and the memory resources needed stays roughly *fixed*.

## G. Full S-parameter Model

In the previous section we showed how to derive the frequency dependent input reflection coefficient for a faulted

<sup>&</sup>lt;sup>2</sup>In taking the inverse Fourier transform of  $H(\omega)$  to obtain h(t), one must respect the frequency dependence of the various S-parameters and impedances in the model, which has been suppressed throughout for notational simplicity.



Fig. 5: S-parameter representation of a chafed coaxial cable.

$$\begin{array}{c|c} V^+(\omega,0) & V^+(\omega,l) \\ \hline \\ \hline \\ V^-(\omega,0) & V^-(\omega,l) \end{array}$$

Fig. 6: Block representation of a faulted coax cable.

wire with a source input impedance discontinuity. In this section, we derive the full S-parameter model for the same setup.

Let us begin with the setup shown in Figure 6, which represents a fault with two sections of possibly lossy coax cable attached to it. To derive the S-parameters for this system, the chain scattering matrix approach is used [10]. With this approach the chain scattering matrix for the system  $T_{sys}$  is readily specified as:

$$T_{sys} = \begin{bmatrix} (A_{21}F_{21}B_{21})^{-1} & -A_{21}^{-1}F_{22}B_{21}F_{21}^{-1} \\ A_{21}F_{22}B_{21}^{-1}F_{21}^{-1} & A_{21}B_{21}(F_{21}^2 - F_{22}^2)F_{21}^{-1} \end{bmatrix},$$

where  $A_{mn}$ ,  $F_{mn}$ , and  $B_{mn}$  are the S-parameters for the first section of cable, the fault, and the second section of cable, respectively. Converting the chain scattering parameters  $T_{sys}$  back to S-parameters we get:

$$S_{sys} = \begin{bmatrix} A_{21}^2 F_{22} & A_{21} F_{21} B_{21} \\ A_{21} F_{21} B_{21} & F_{22} B_{21}^2 \end{bmatrix}.$$
 (19)

Now that we have the S-parameters for the faulted cable, the next step is to derive expressions for the S-parameters after source and load impedance discontinuities. The situation is shown in Figure 7. By definition of the S-parameters for the system inside the impedance discontinuities we have:

$$b_1 = S_{11}a_1 + S_{12}a_2$$
  

$$b_2 = S_{21}a_1 + S_{22}a_2.$$
(20)

The goal is to derive the S-parameters for the outer system defined by:

$$\overline{b}_1 = \overline{S}_{11}\overline{a}_1 + \overline{S}_{12}\overline{a}_2 \overline{b}_2 = \overline{S}_{21}\overline{a}_1 + \overline{S}_{22}\overline{a}_2,$$

given we know the S-parameters for the internal system and all characteristic impedances  $Z_0, Z_1$  and  $Z_2$ . To do this we make use of the following boundary conditions for the voltage waves traveling into and out of the impedance discontinuities on both sides of the internal system:

$$a_{1} = (1 + \Gamma_{1})\overline{a}_{1} - \Gamma_{1}b_{1} \qquad a_{2} = (1 - \Gamma_{2})\overline{a}_{2} - \Gamma_{2}b_{2}$$
  

$$\overline{b}_{1} = (1 - \Gamma_{1})b_{1} + \Gamma_{1}\overline{a}_{1} \qquad \overline{b}_{2} = (1 + \Gamma_{2})b_{2} - \Gamma_{2}\overline{a}_{2}.$$
(21)

Note, these are the same boundary conditions derived in §II-B.

To solve for the S-parameters of the outer system we start by solving equations (21) for  $a_1$ ,  $a_2$ ,  $b_1$ , and  $b_2$  in terms of  $\overline{a}_1$ ,  $\overline{a}_2$ ,  $\overline{b}_1$ , and  $\overline{b}_2$ . Next, these results are substituted into the equations for the internal S-parameters (20), and solved for the external S-parameters to give:

$$\overline{S}_{11} = \frac{S_{11} + \Gamma_1 - \Gamma_1 \Gamma_2 S_{22} - (S_{11} S_{22} - S_{12} S_{21}) \Gamma_2}{1 + S_{11} \Gamma_1 - S_{22} \Gamma_2 - (S_{11} S_{22} - S_{12} S_{21}) \Gamma_1 \Gamma_2}$$
(22)

$$\overline{S}_{12} = \frac{S_{12}(1-\Gamma_1)(1-\Gamma_2)}{1+S_{11}\Gamma_1 - S_{22}\Gamma_2 - (S_{11}S_{22} - S_{12}S_{21})\Gamma_1\Gamma_2}$$
(23)



Fig. 7: Block representation of a faulted coax cable.

$$\overline{S}_{21} = \frac{S_{21}(1+\Gamma_1)(1+\Gamma_2)}{1+S_{11}\Gamma_1 - S_{22}\Gamma_2 - (S_{11}S_{22} - S_{12}S_{21})\Gamma_1\Gamma_2}$$
(24)  
$$\overline{S}_{22} = \frac{S_{22} - \Gamma_2 - \Gamma_1\Gamma_2S_{11} + (S_{11}S_{22} - S_{12}S_{21})\Gamma_1}{1+S_{11}\Gamma_1 - S_{22}\Gamma_2 - (S_{11}S_{22} - S_{12}S_{21})\Gamma_1\Gamma_2}$$
(25)

To verify the equation for  $\overline{S}_{11}$ , consider the case where  $\Gamma_2 = \Gamma_L$  and  $\Gamma_1 = 0$ . After substituting these values, equation (22) becomes:

$$\overline{S}_{11} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L},$$

which is the same as equation (1) for the input reflection coefficient given load impedance  $Z_L$  and matched source impedance (as it should be).

Finally, with these system S-parameters it is easy to derive the input/output voltage relationships with mismatched source and load impedances. This case occurs frequently in practice whenever the wire impedance mismatches the source impedance. Since the mismatch is now taken care of *within* the S-parameter block we can consider attaching the source and load to *matched* impedances. This means we can simply write:

$$V(0)^{+} = (1/2)V_{S} \text{ (since } Z_{1} = Z_{S})$$

$$V(0) = V(0)^{+} + V(0)^{-} = \frac{1}{2}(1 + \overline{S}_{11})V_{S}$$

$$V(l)^{-} = 0 \text{ (since } Z_{2} = Z_{L})$$

$$V(l) = V(l)^{+} + V(l)^{-} = \frac{1}{2}\overline{S}_{21}V_{S}$$

#### **III. PROBABILISTIC INVERSION**

#### A. Bayesian Framework

In this section, a probabilistic framework is presented for inferring the fault parameters from measured TDR data. Starting with a sampled version of (18), the measurement process is modeled in the usual way as

$$y = F(x;\theta) + \nu, \tag{26}$$

where  $x \in \mathbf{R}^n$  is the interrogation signal injected by the TDR unit into the cable under test,  $\theta \in \mathbf{R}^m$  is the set of unknown model (*i.e.*, system and fault) parameters, the function  $F(x;\theta) : \mathbf{R}^n \times \mathbf{R}^m \to \mathbf{R}^n$  represents the forward model,  $\nu \in \mathbf{R}^n$  is a vector of additive random measurement noise, and  $y \in \mathbf{R}^n$  is a time series of voltage samples forming the measured TDR signal.

Two probability distributions functions (pdf) are now introduced for the construction of a Bayesian inversion framework: (1) the *prior* distribution  $\mathbf{Pr}(\theta)$ , which describes our state of knowledge regarding the unknown model parameters before any measurements are made, and (2) the *likelihood* distribution  $\mathbf{Pr}(y|\theta)$ , which specifies the probability of observing a particular measurement for a given set of model parameters. Bayes' theorem then gives the *posterior* distribution for  $\theta$  in the form [13]

$$\mathbf{Pr}(\theta|y) = \frac{\mathbf{Pr}(y|\theta) \, \mathbf{Pr}(\theta)}{\int \mathbf{Pr}(y|\theta') \, \mathbf{Pr}(\theta') \, d\theta'}.$$
 (27)

The maximum *a posteriori* estimate  $\theta^*$  is found by solving the optimization problem

maximize 
$$\mathbf{Pr}(\theta|y)$$
. (28)

Furthermore, the shape and the spread of the posterior distribution around  $\theta^*$  indicate how confident we are in this estimate. There are two typical approaches to quantifying this shape for general distributions like  $\mathbf{Pr}(\theta|y)$ , which depends heavily on the nonlinear forward model (among other things). The first is to assume the distribution is approximately Gaussian around the optimal estimate  $\theta^*$ , and to use the inverse of the Hessian of  $-\log \mathbf{Pr}(\theta^*|y)$  as an approximation for the covariance matrix which quantifies the spread of the distribution [13, Ch. 3]. The second approach relies on the remarkable fact that one can sample random vectors directly from the posterior distribution  $\mathbf{Pr}(\theta|y)$ , and use the spread of the samples to quantify the distribution shape, without making any additional Gaussian assumptions. This is the approach we take up in the next section.

## B. Markov-Chain Monte Carlo Estimation

Finding the optimal estimate and quantifying the uncertainty associated with it are computationally challenging tasks when the forward model F is nonlinear in  $\theta$ , as in the present case. Furthermore, in cases where the forward model is an algorithm (rather than a closed-form expression), it can be prohibitively expensive to compute the gradient and the Hessian of the cost function, which are needed to solve the optimization problem (28) using traditional methods. Thus, a natural approach for this type of problem is the application of Markov-Chain Monte Carlo (MCMC) methods to obtain a set of random samples drawn directly from the posterior distribution, which are used to estimate the desired quantities by applying the law of large numbers. The underlying premise for this approach is that, for sufficiently large N, a set of samples

$$\theta_k \sim \mathbf{Pr}(\theta|y), \quad i = 1, 2, \dots, N,$$
(29)

adequately captures the essential features of the posterior distribution. Specifically, the sample  $\theta_k$  that maximizes the posterior distribution provides us with a globally optimal estimate, while the spread of the N samples around  $\theta_k$  may be taken as a measure of our uncertainty about this estimate. More generally the law of large numbers guarantees:

$$\frac{1}{N}\sum_{k=1}^{N}f(\theta_{k}) \to \mathbf{E}f(\theta) = \int f(\theta) \mathbf{Pr}(\theta|y) \, d\theta, \qquad (30)$$

as  $N \rightarrow \infty$ . Thus, the samples can be used to estimate the expected value of almost any event or function. Standard

examples include the mean  $f(\theta) = \theta$ , and variance  $f(\theta) = (\theta - \mathbf{E} [\theta])^2$ .

There are many different MCMC-based algorithms one might implement to achieve the above sampling. The results presented in §IV were obtained using a relatively new method called *nested sampling*. This algorithm is a natural fit for solving the estimation problem posed by equations (27) and (28), while also estimating other relevant quantities such as the integral in the denominator of (27), which can be used for model selection (*i.e.*, choosing the best among competing forward-modeling schemes). Like many other MCMC methods, this one also tends to be slow: it took around 8-10 hours to solve the estimation examples discussed in §IV on a 32-bit 1.8-GHz Linux PC. The interested reader is referred to [13] for details on the nested-sampling algorithm.

## **IV. RESULTS**

This section presents a couple example results on system parameter estimation and chafing fault detection for a 7-m long RG58 coaxial cable with an open load condition (*i.e.*,  $Z_L = \infty$ ), along with a simulated result highlighting the more complex nature of detecting particularly small faults.

## A. Problem Setup

Laboratory measurements were obtained using an Agilent 54754A digital TDR unit. The elements of the measurement noise vector  $\nu$  were assumed to be independent and identically distributed normal random variables with zero mean and a standard deviation of  $\sigma_M = 1$  mV, a value roughly equal to the residual error standard deviation between the measured data and the optimal model fit. Under these assumptions the likelihood distribution is

$$\mathbf{Pr}(y|\theta) = (2\pi\sigma_M^2)^{-n/2} \exp\left\{-\frac{1}{2\sigma_M} \|y - F(x;\theta)\|^2\right\},$$
(31)

where  $\theta = (l_1, d, w, \epsilon_d, \sigma_c, Z_S, l, t_M, G)$  is our vector of key model parameters, all of which were carefully defined throughout section II.

The prior information is summarized in Table I, where  $\mathcal{U}(x,y)$  denotes the uniform distribution on the interval [x,y], and  $\mathcal{N}^+(\mu,\sigma)$  denotes the normal distribution restricted to positive values with pdf,

$$f_X(x) = \begin{cases} \frac{e^{-(x-\mu)/2\sigma^2}}{\Phi(\mu/\sigma)\sqrt{2\pi\sigma^2}}, & x \ge 0\\ 0, & \text{otherwise,} \end{cases}$$
(32)

where  $\Phi(z)$  represents the cdf of a standard normal random variable. This prior specification represents information and uncertainty regarding the known wire material properties and TDR equipment specifications in a *reasonable* way. For example the nominal values of  $\sigma_c$  and  $\epsilon_d$  are typically supplied by the cable manufacturer, but the parameters of a *particular* cable may deviate appreciably from the "batch" values, a fact captured by the specified prior probability distributions; and the same argument holds for the TDR hardware parameters as well.

Before any measurements are made, our prior knowledge of each key parameter is assumed independent of the other

TABLE I: Parameter Prior Information

Param.	Distribution	Description	
$l_1$	$\mathcal{U}(0,l)$ m	distance to fault	
d	$\mathcal{U}(0,50)$ mm	fault length	
w	$\mathcal{U}(0,2b)$ mm	fault width	
$\epsilon_d$	$\mathcal{N}^+(2.25, 0.2)$	relative dielectric permittivity	
$\sigma_c$	$\mathcal{N}^+(3,2)  imes 10^7$ S/m	conductor conductivity	
$Z_S$	$\mathcal{N}^+(50,2) \ \Omega$	source impedance	
l	$\mathcal{N}^+(7, 0.1)$ m	cable length	
$t_M$	$\mathcal{N}^+(0.5,0.2)$ ns	measurement time delay	
G	$\mathcal{N}^{+}(1, 0.1)$	system gain	

parameters, except for fault location  $l_1$  and cable length l which are jointly distributed according to

$$\mathbf{Pr}(l_1, l) = \mathbf{Pr}(l_1|l) \, \mathbf{Pr}(l) = \mathcal{U}(0, l) \times \mathcal{N}^+(7, 0.1).$$
(33)

Thus, the pdf of the prior parameter vector  $\mathbf{Pr}(\theta)$  is simply the product of the distributions listed in Table I. With the likelihood and prior pdfs now defined, equation (27) provides the posterior distribution. Note, even though standard likelihood and prior distributions were assumed, the nonlinear nature of the model function makes the posterior distribution nonstandard: not Gaussian, uniform, or any other typical distribution. In fact the general posterior distribution can be multi-modal, and this fact has important consequences for fault detection. The final section provides an example.

## B. Chafing Fault Detection Example

As an example, our estimation procedure is applied to simultaneously retrieve all parameters from a single measured TDR response collected from the 7 m long RG58 coaxial cable with a single  $10 \times 3$  mm chafe at a distance of 6 m from the source<sup>3</sup>. The optimal estimates are shown in Table II, along with their corresponding standard deviations. Also, Figure 8 characterizes joint estimation performance between pairs of parameters. Note, the correlation between fault width and fault length is expected since changing these parameters in the forward model have roughly the same effect on the TDR response. The model also explains the very strong correlation between cable length *l* and dielectric permittivity  $\epsilon_d$  since both affect the total propagation time through the cable (*i.e.*,  $\epsilon_d$ affects propagation velocity).

Finally, with all the key model parameters inferred from data, we now use the optimal parameter estimates and the known source voltage profile  $v_S(t)$  to compute the modelpredicted TDR signal,  $v_M(t)$ . The result presented in Figure 9 shows near perfect agreement with the laboratory measurement, thus validating the effectiveness of the forward model.

#### C. A Multi-modal Example

The example in the previous section highlighted a case where the fault signature was small, but visible by eye. As

TABLE II: Parameter Estimates  $\pm 1$  Standard Deviation

Param.	Estimate	Param.	Estimate
$l_1$	$6.010 \pm 0.036 \text{ m}$	d	$14.5\pm2.9~\mathrm{mm}$
w	$2.71\pm0.14~\mathrm{mm}$	$\epsilon_d$	$2.242\pm0.027$
$\sigma_c$	$3.019 \pm 0.016 \times 10^7$ S/m	$Z_S$	$48.9\pm0.3~\Omega$
l	$7.02\pm0.04~\mathrm{m}$	$t_M$	$0.552\pm0.01~\mathrm{ns}$
G	$0.991\pm0.000$		



Fig. 9: Model fit to the measured TDR signal using the optimal estimate for  $\theta$ . The fit captures the variation in the measured signal to within a standard deviation of 0.43 mV, and includes both the primary and the secondary reflections from the chafing fault.

shown in Figure 8, the posterior samples in that case are well modeled by a multivariate Gaussian distribution; and that *is* the standard treatment for a posterior distribution built around general nonlinear models, as is the present case. This approach however, is not always appropriate. For example, Figure 10 presents the posterior samples from a *simulated* TDR response to a  $3 \times 2$  mm fault located 2 m from the source. In this case, the fault signature is buried in the measurement noise (not shown), and the estimation procedure yields posterior samples that cluster around various possible fault locations along the cable. Clearly, these samples are not well described by a multivariate Gaussian distribution, so the standard approach would yield very misleading results. Thus, we can conclude the standard method is inaccurate for assessing fault detectability at or near the limits of detection.

The MCMC parameter estimation approach presented in this article naturally reveals the proper multi-modal distribution by treating the full nonlinear model without further approximation. In this particular case, Figure 10 shows the primary mode of the distribution provides good estimates for the fault location and length, but that is not known in advance. Given the available measured data, prior information, and model, one can conclude only that the most likely fault is at 2 m, but other locations are also somewhat probable. In fact, that is just what the posterior samples provide: a set of probable parameter values given the assumed model and measured data.

<sup>&</sup>lt;sup>3</sup>Lab measurements were made using a tape measure for distance-to-fault and cable length, and digital calipers for fault length and width. These measurements are all subject to some inaccuracy



Fig. 8: Example parameter estimation results and uncertainty analysis. The star marks the most probable estimate, while the confidence ellipse is the minimum area ellipse enclosing 95% of the most likely samples from the posterior distribution.



Fig. 10: A small chafing fault detection example. In this case the posterior samples, which represent possible fault locations and lengths, reveal an underlying multi-modal posterior distribution. While the most likely estimate, marked by the star, provides a good estimate of fault location and size, the posterior samples indicate a number of other possibilities that can not be ruled out given the measured TDR data.

## V. CONCLUSION

This article presented an effective forward model for the TDR response of chafed cable, given the input signal and a set of model parameters. The novelty in our approach lies not in the application of S-parameter based signal modeling or electromagnetics, but in the identification of the important model parameters and system structure needed to accurately represent the actual hardware measured TDR response in the simplest possible way. This was in fact the direct result of a long process of trial and error with lab measured data a full description of which the reader has been spared. The resulting model incorporates key effects caused by practical non-idealities such as source impedance mismatch, measurement delays, signal loss and dispersion, changing material properties, and even some degree of mis-calibration. These issues are not specific to the coaxial chafing fault detection example this article focused on, and are all important to the general application of TDR based wiring fault detection methods in the field.

The forward model was then combined with a Bayesian inversion framework to formulate and solve the problem of optimal fault detection and performance characterization using MCMC based techniques. Although this method is computationally slow, it handles general nonlinear models without further approximation; and this leads to an accurate characterization of estimation uncertainty that traditional methods can fail to provide. The results section highlighted this effect through two simple chafing fault detection examples. Furthermore, this method is optimal in the sense that given the measured TDR response, no other detection method can find a more likely fault location and size, under equivalent conditions (i.e., same measurement hardware, input signal, noise, wiring system etc.). Finally, the inversion approach is easily generalized to handle a variety of parametric models, since the model itself is viewed simply as an input to the inversion procedure. Thus, we have presented a truly generalized framework applicable to the characterization of TDR based fault detection for a large variety of TDR hardware, wiring types, and network topologies.

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