A Model Based Probabilistic Inversion Framework for Wire Fault Detection Using TDR

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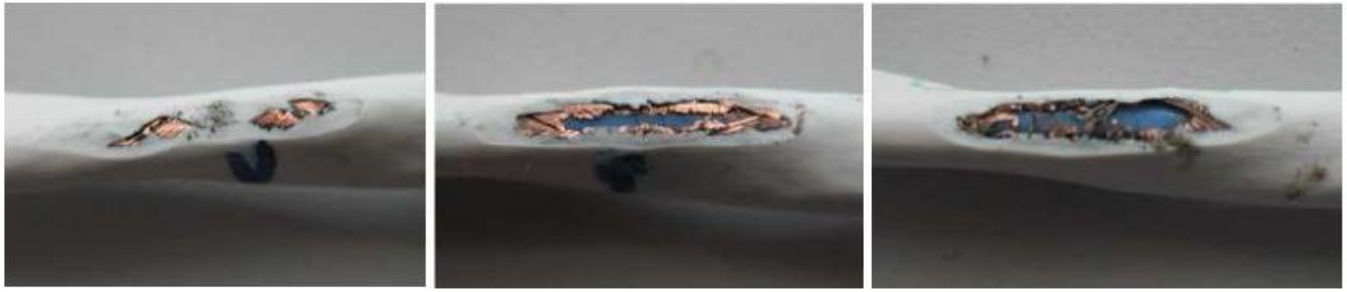
The Problem

Determine ability to detect chafing faults in *shielded wire* using **TDR**

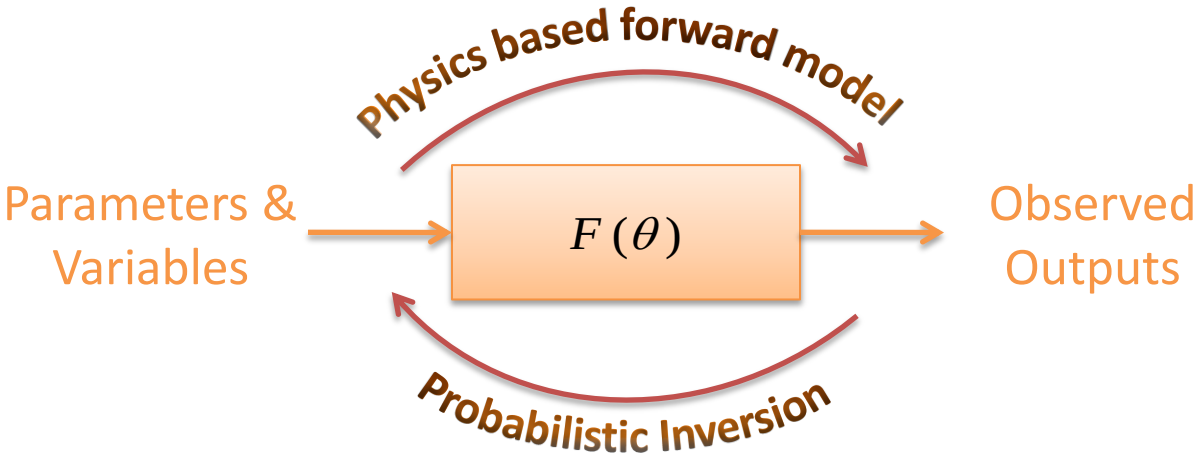


Research goals:

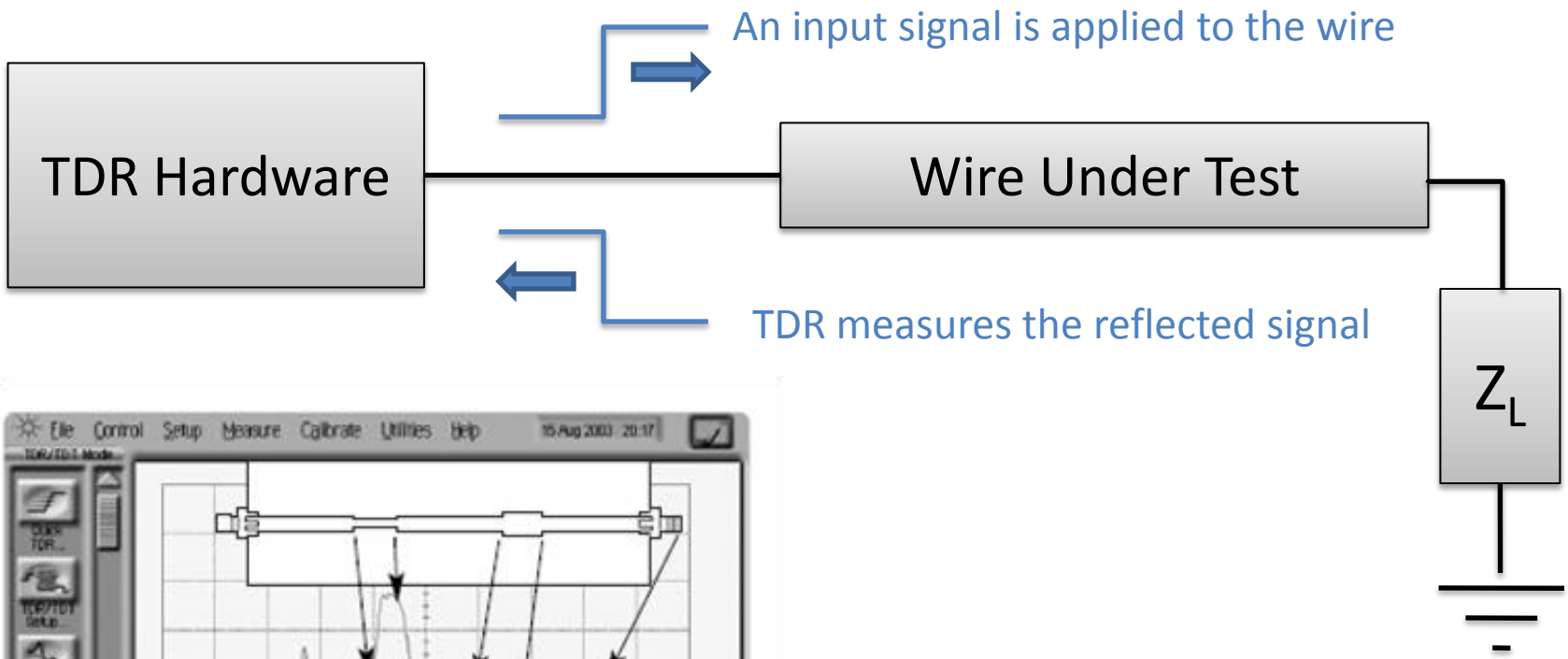
- Characterize ability to detect faults on shielded wire types
- Use results to inform the design of better hardware/software
- **In this presentation we look at chafed coax as an example**
    - The approach presented can be generalized

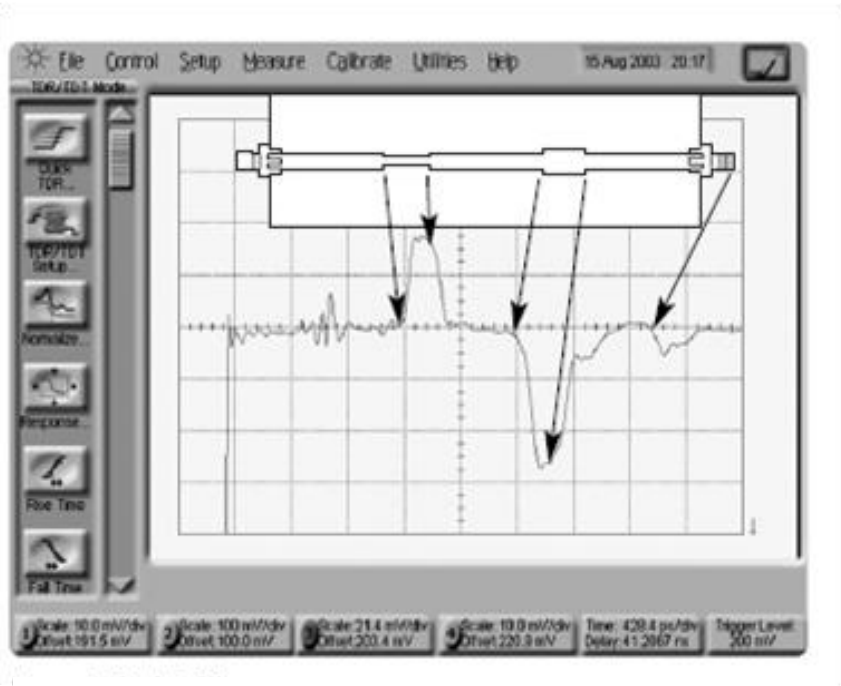
Approach

- Derive computationally efficient physics based forward models
  - Determine the TDR response given wire specs. and fault location
- Develop fault detection software based on a probabilistic approach that accounts for real world uncertainties.
  - Probability theory quantifies effectiveness of a fault detection algorithm



TDR Primer

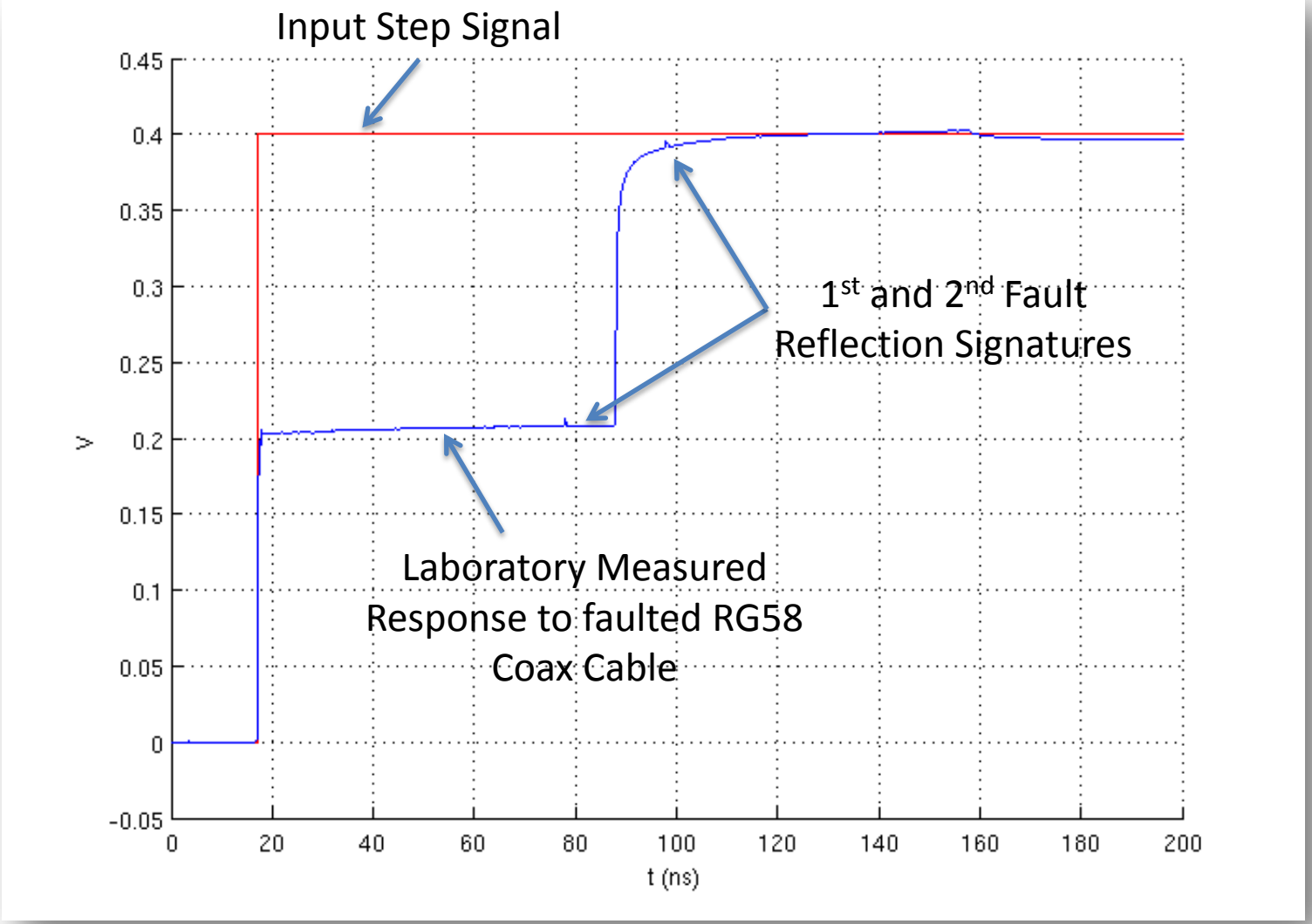




Measured TDR response to a transmission line with 2 geometry discontinuities (faults). Changes in this reflected signature indicate fault location and size.

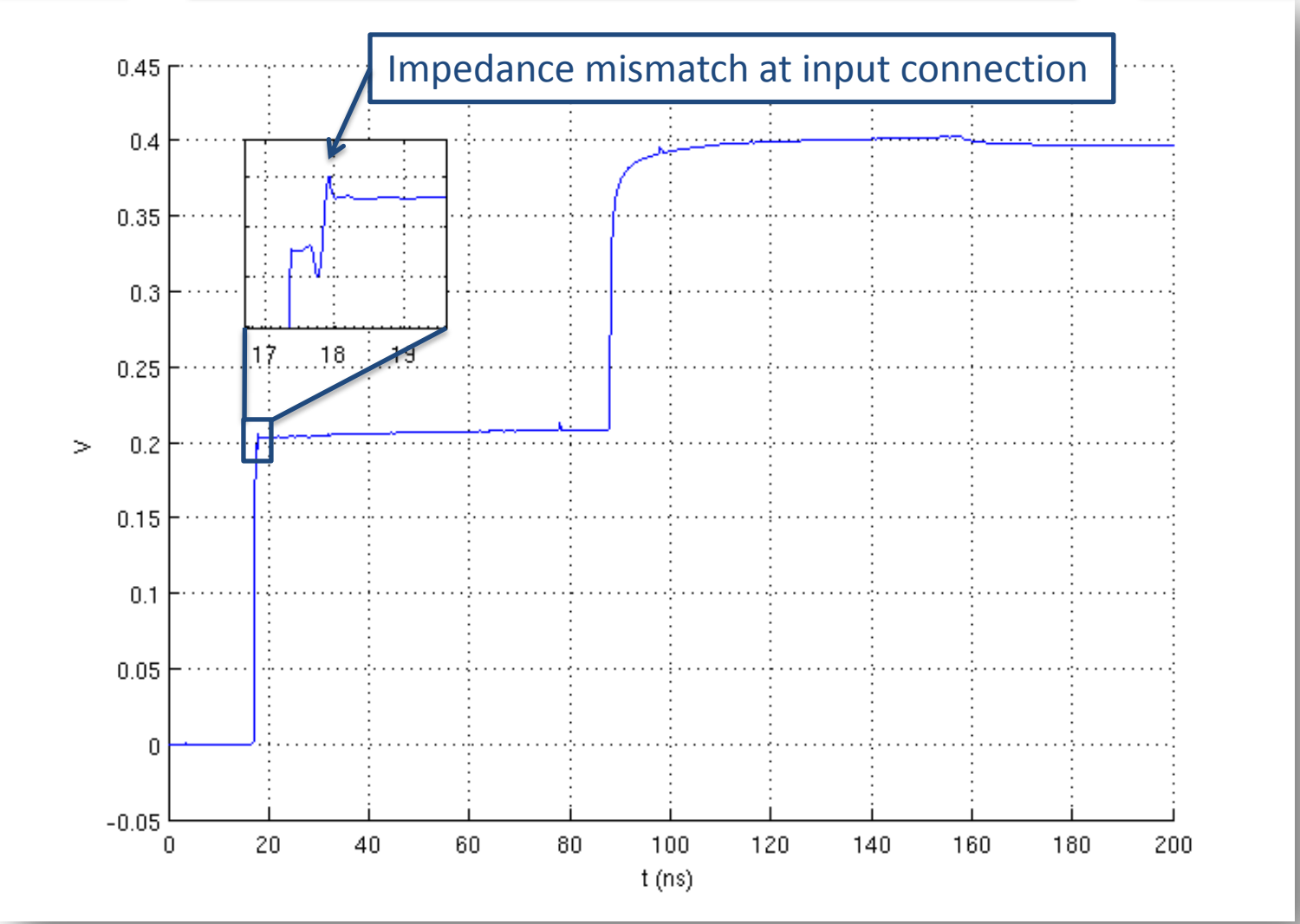
Actual TDR Response for Chafed Cable

TDR — RG58 Coax – 7 m - 11 × 3 mm fault @ 6 m — $Z_L = \infty$



Physical Effects

TDR — RG58 Coax – 7 m - 11 × 3 mm fault @ 6 m — $Z_L = \infty$



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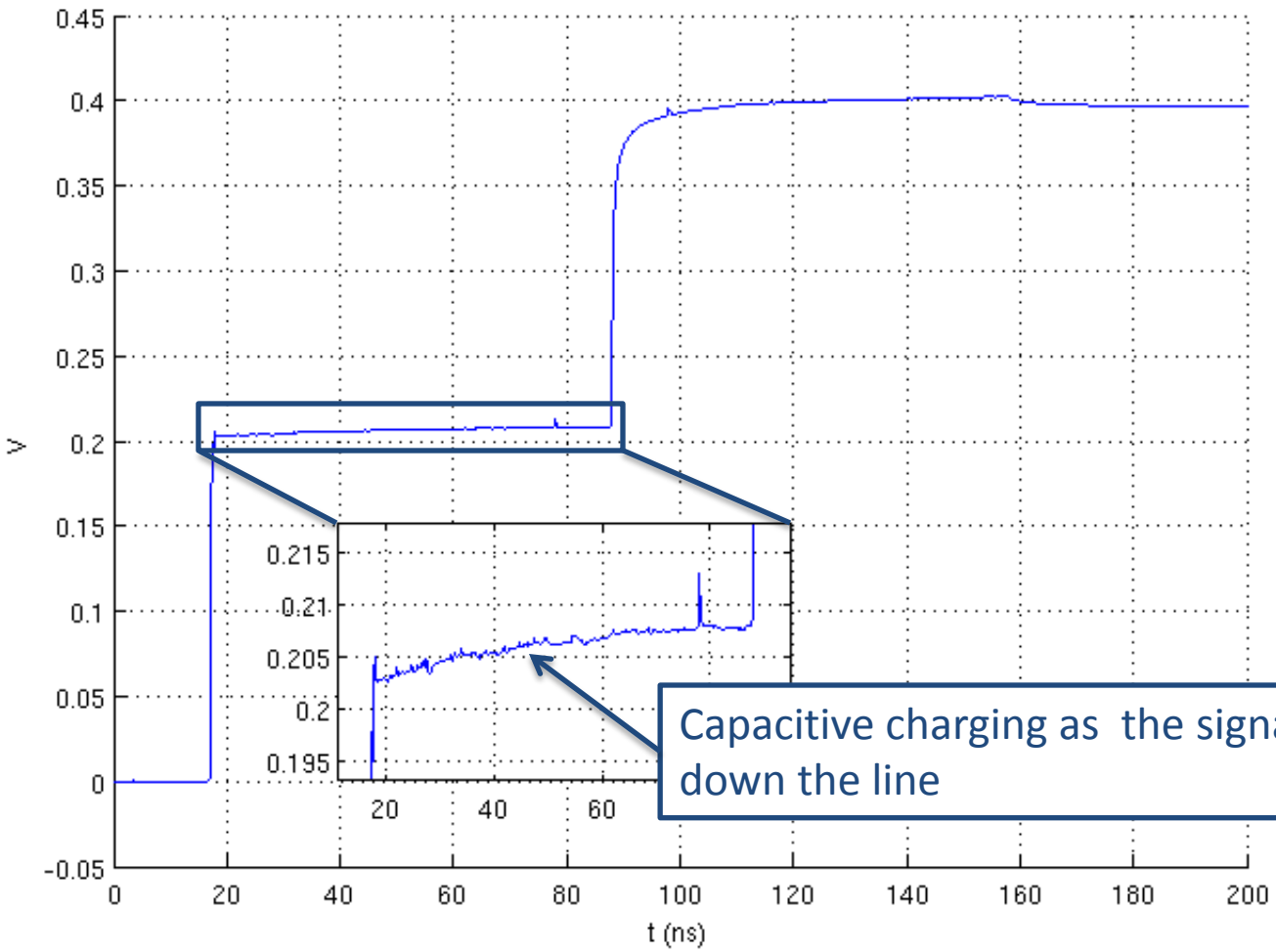
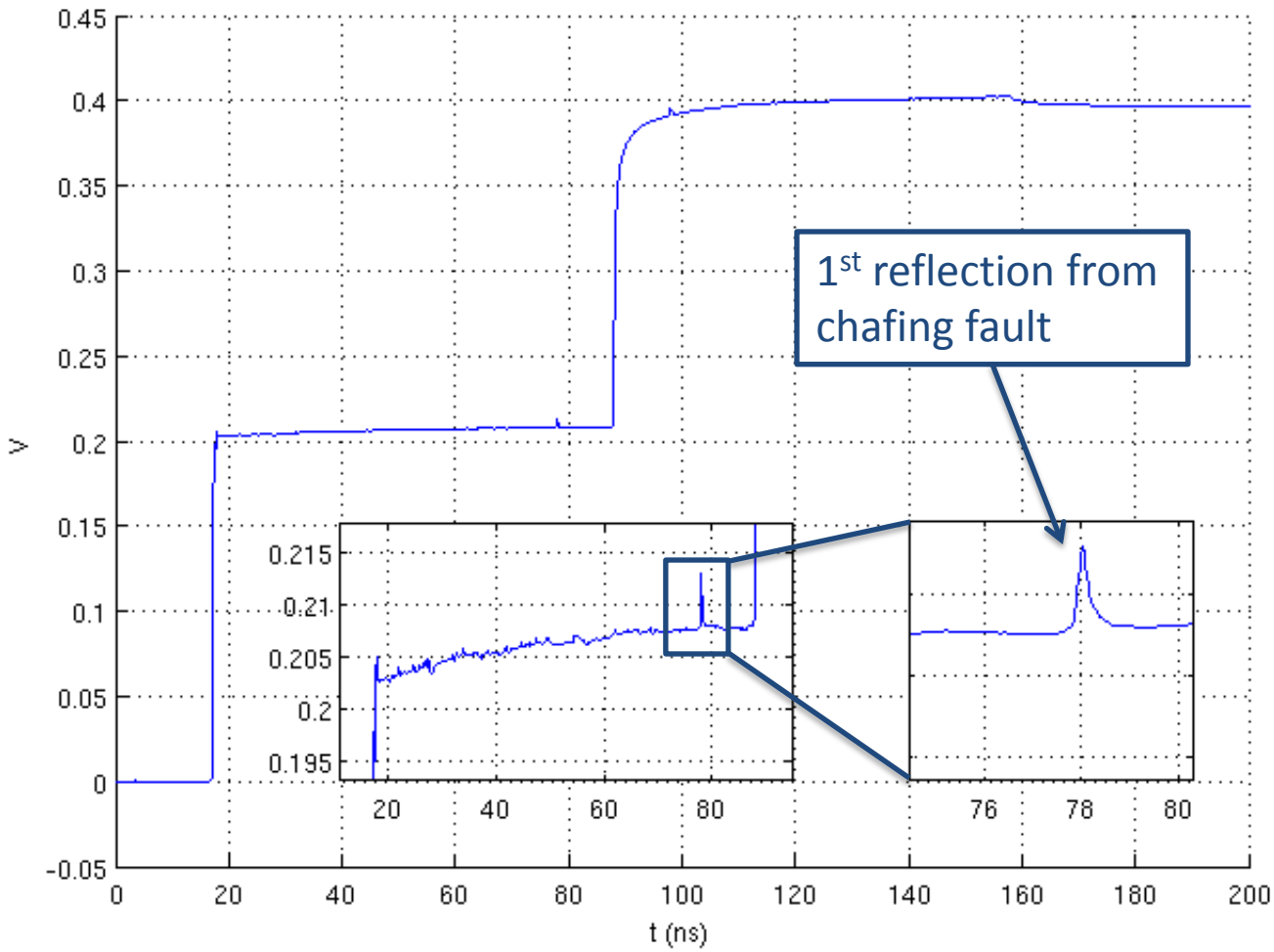


Capacitive charging as the signal flows down the line

Physical Effects

TDR — RG58 Coax – 7 m - 11 × 3 mm fault @ 6 m — $Z_L = \infty$



Physical Effects

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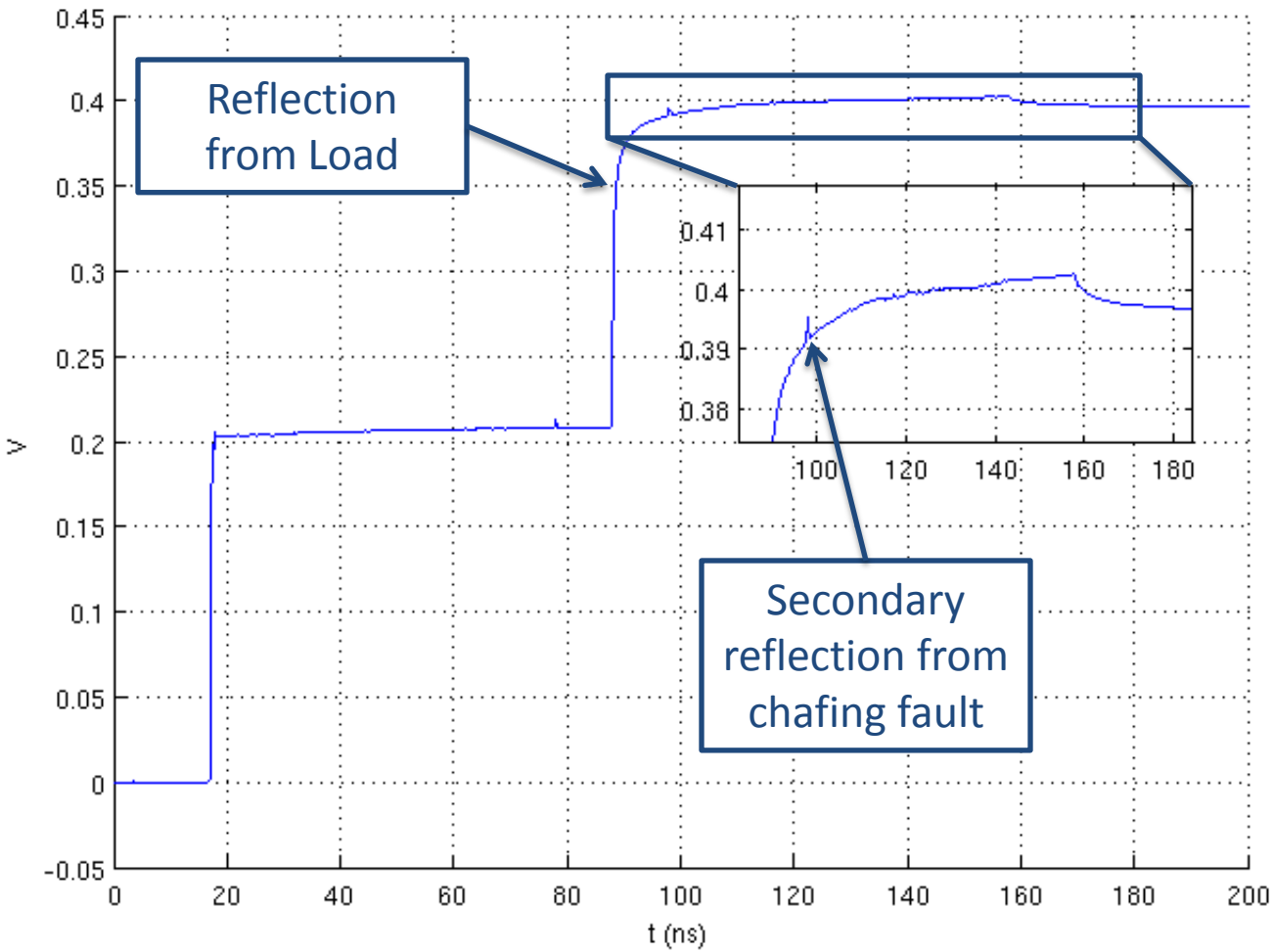
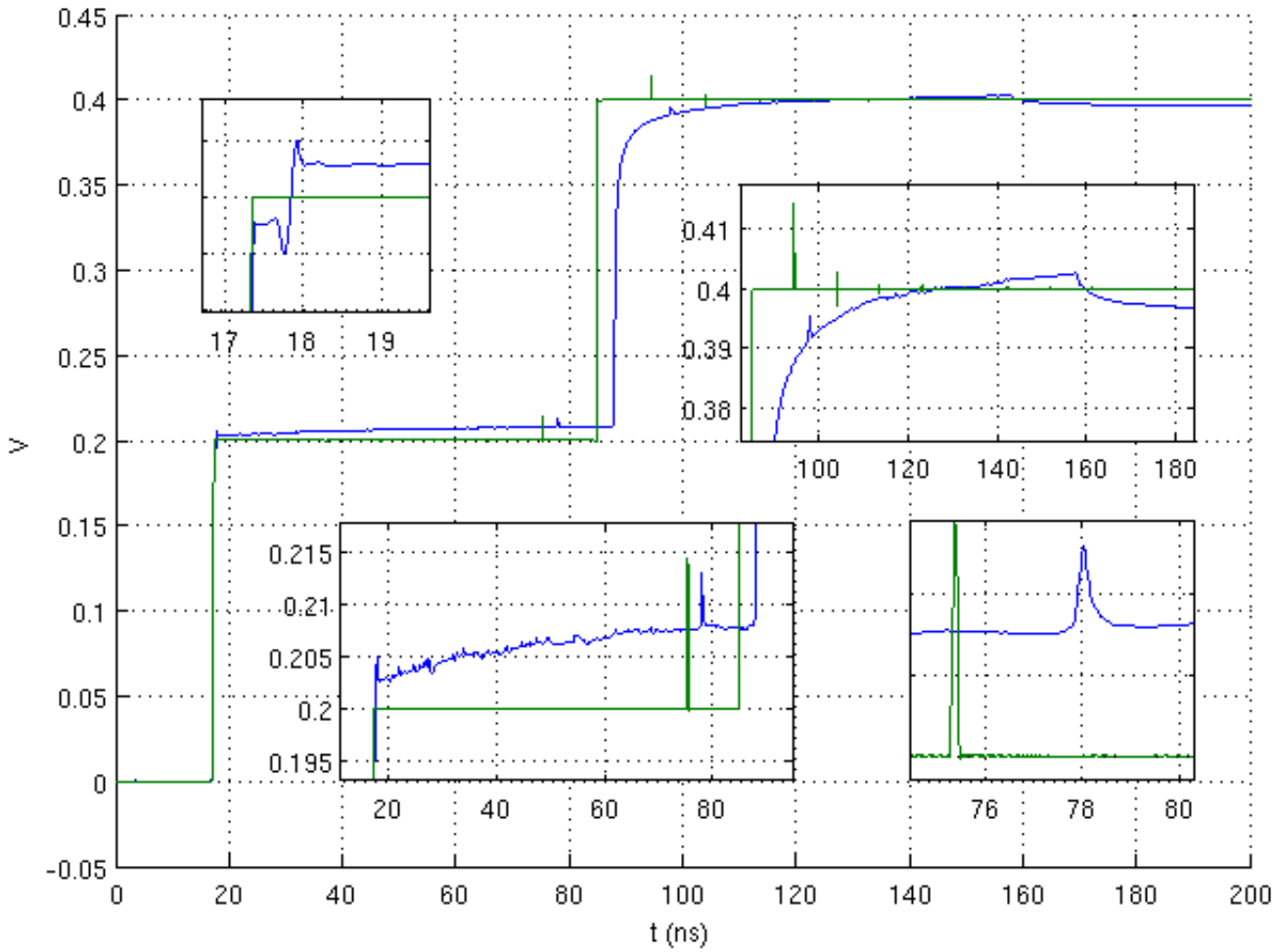


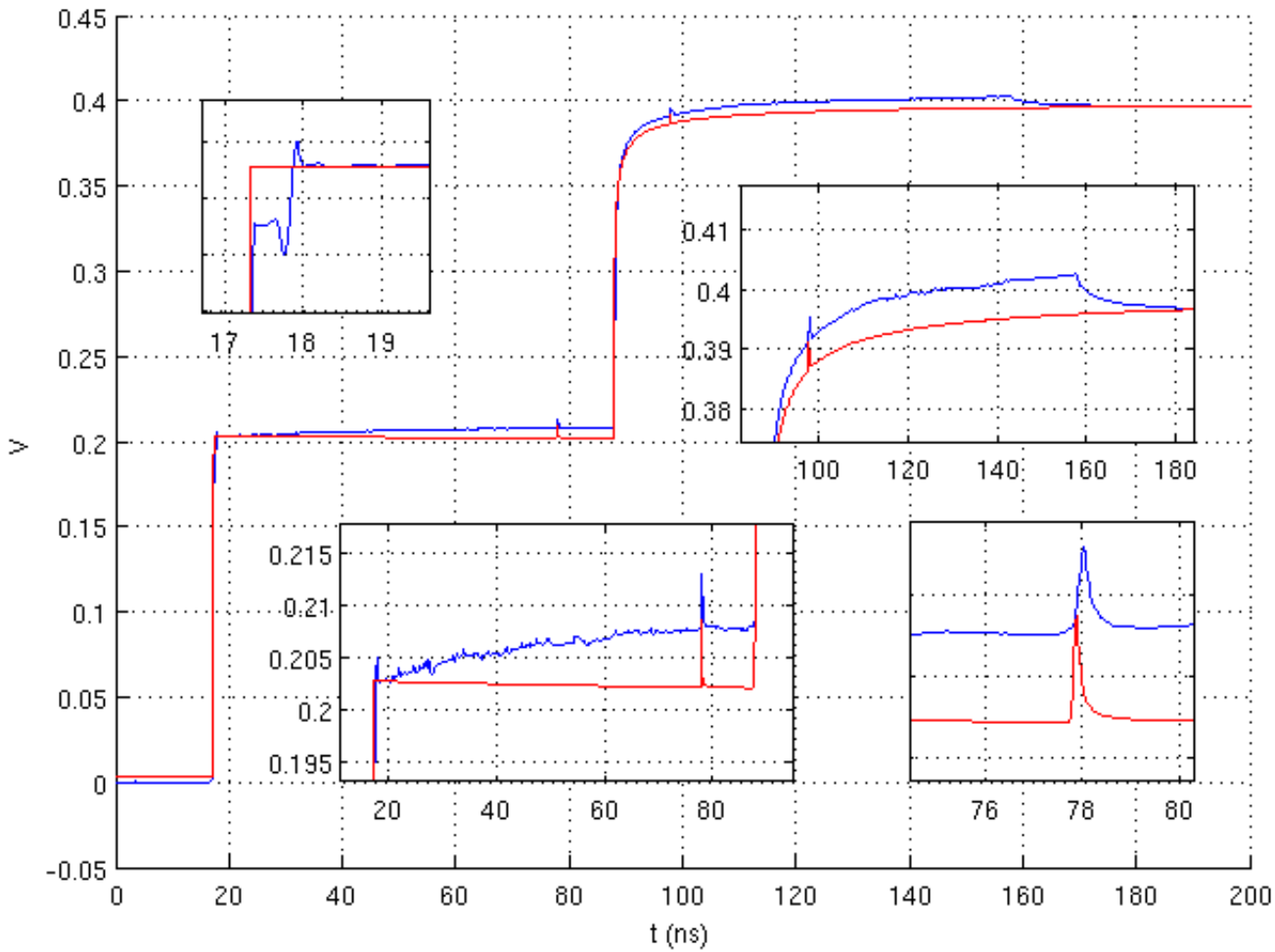
Model Evolution

base model (lossless)



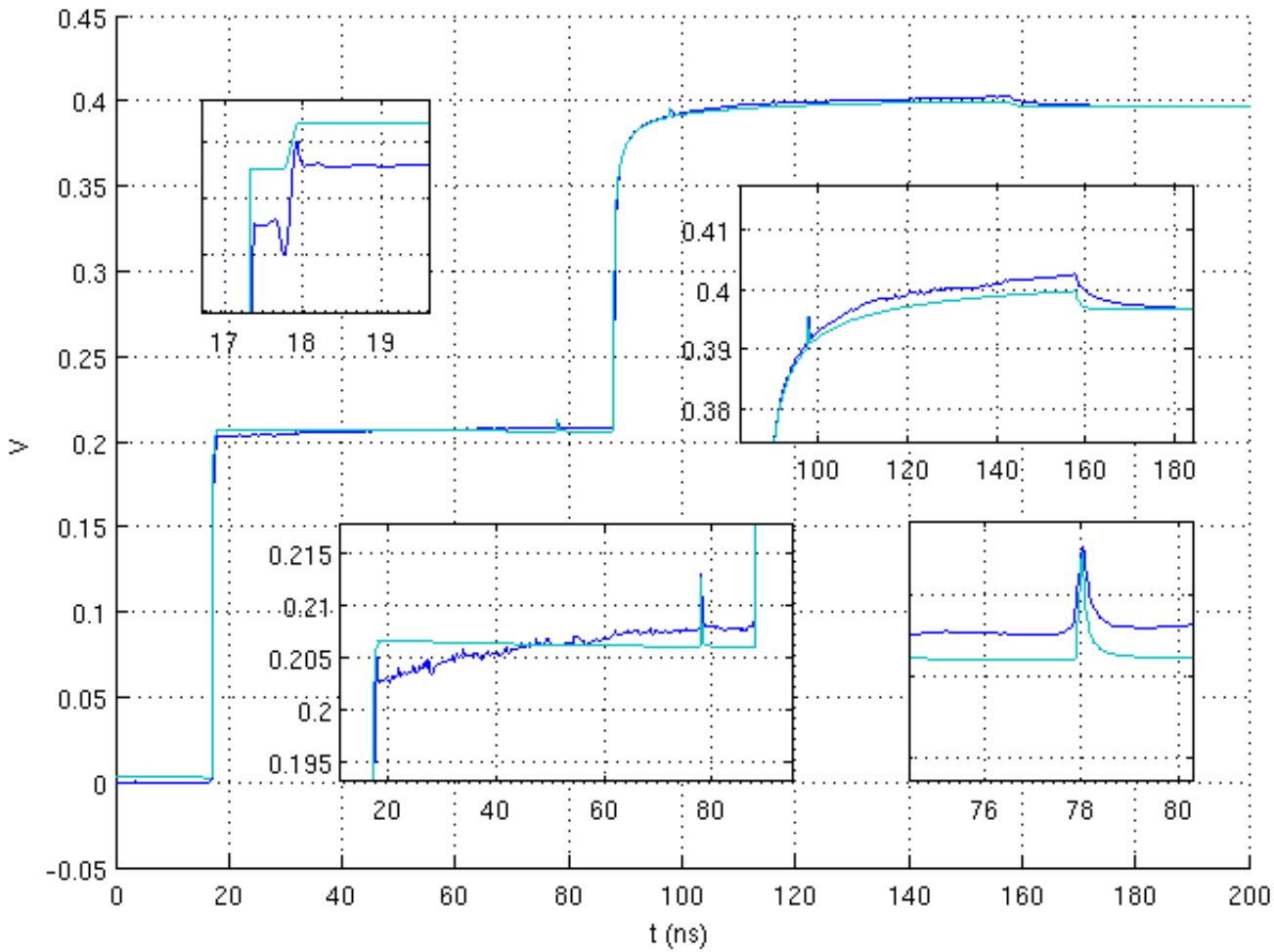
Model Evolution

base model + loss effects ($\sigma_c < \infty$)



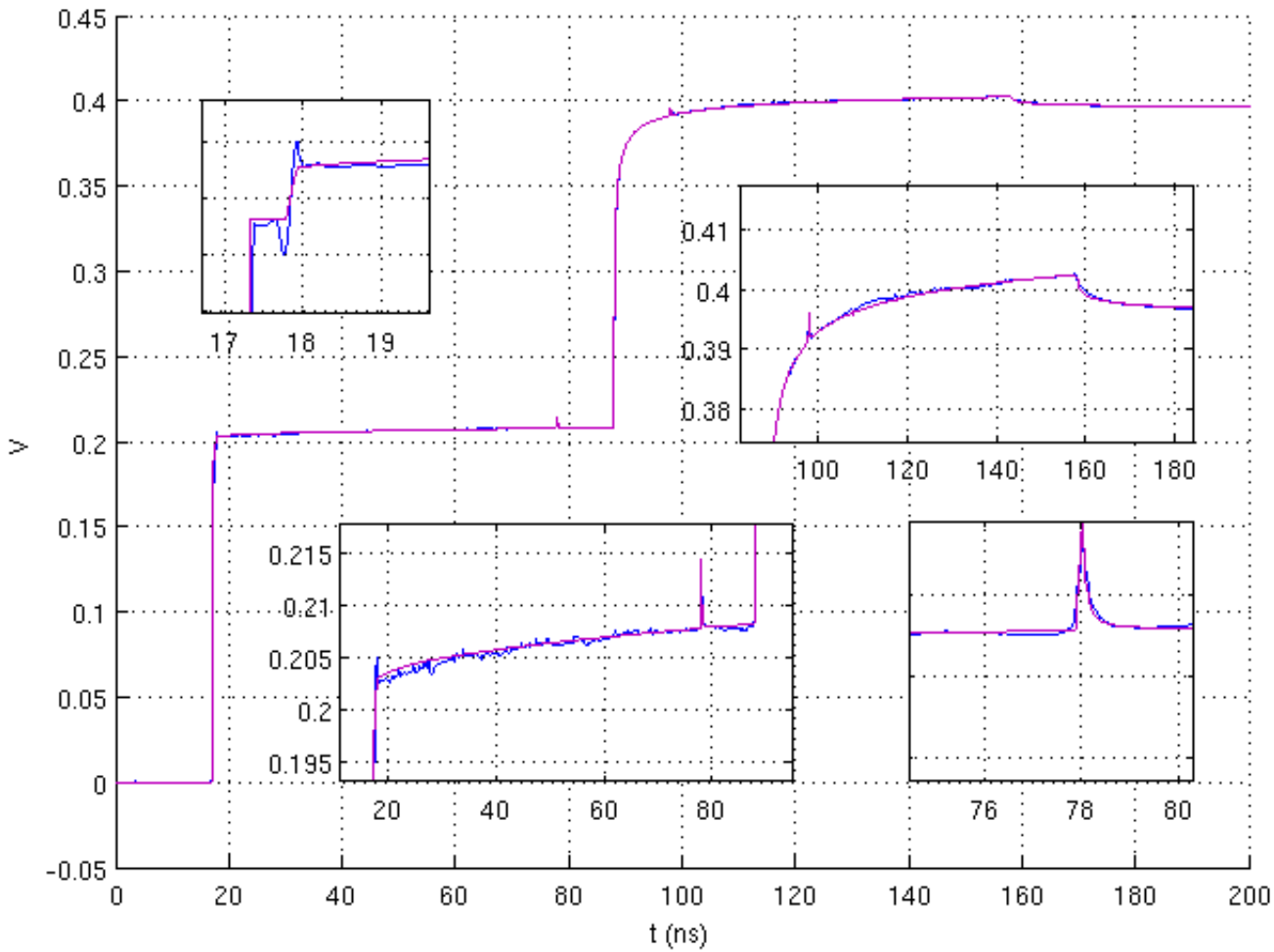
Model Evolution

base model + loss effects ($\sigma_c < \infty$) + constant source mismatch

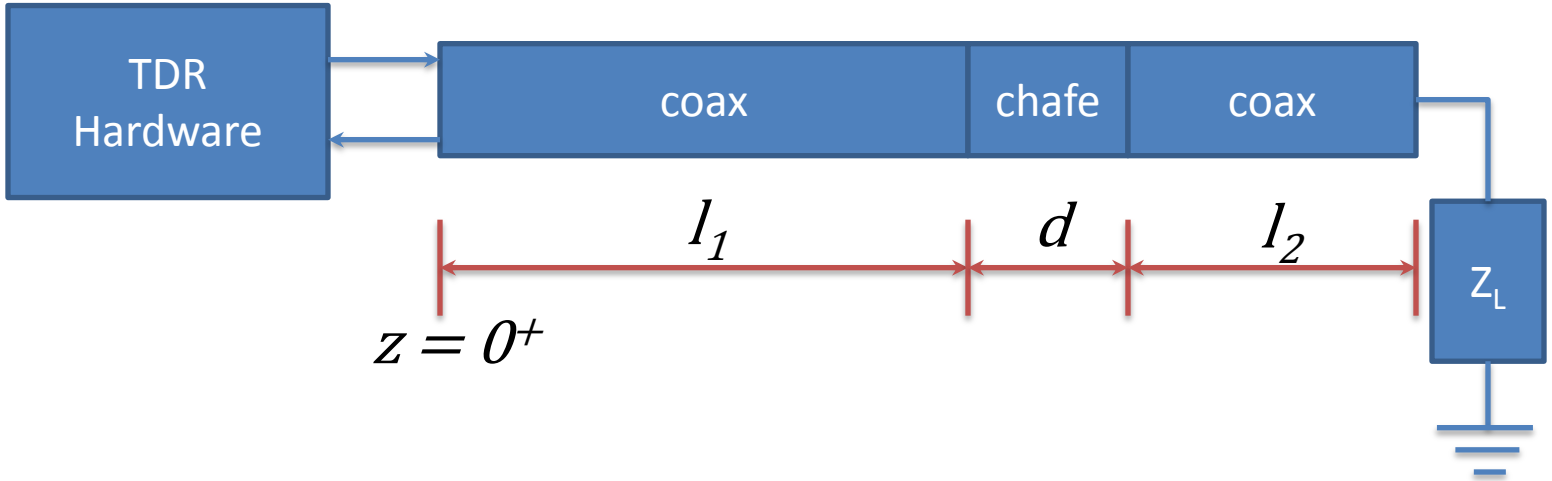


Model Evolution

base model + loss effects ($\sigma_c < \infty$) + freq. dependent source mismatch



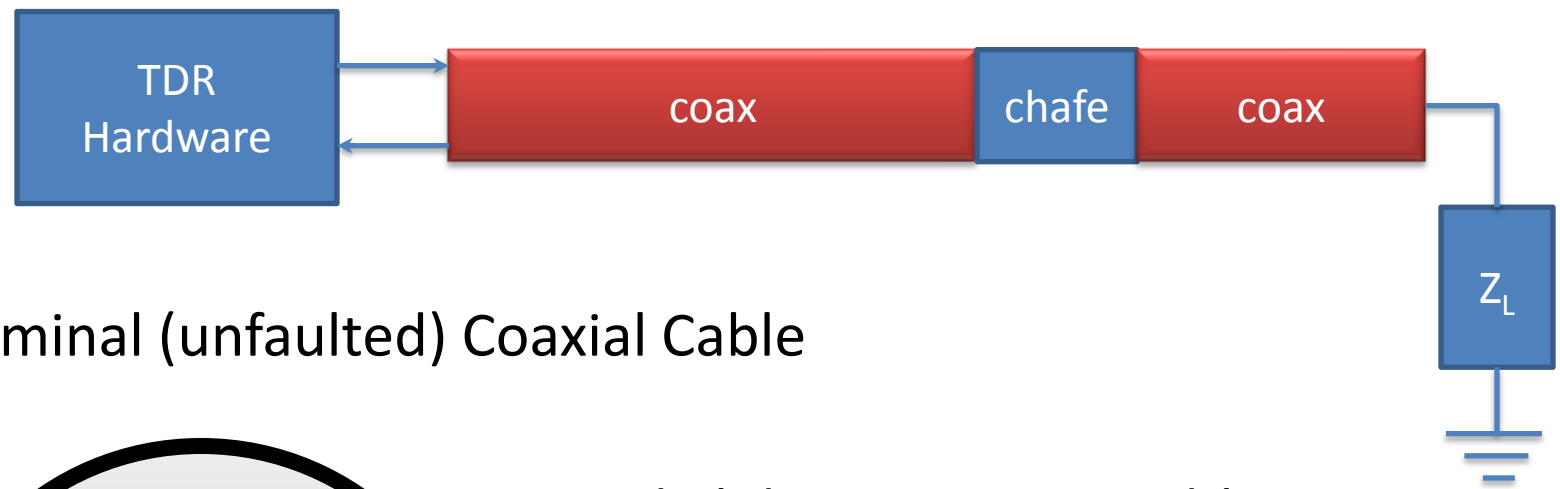
The Forward Model



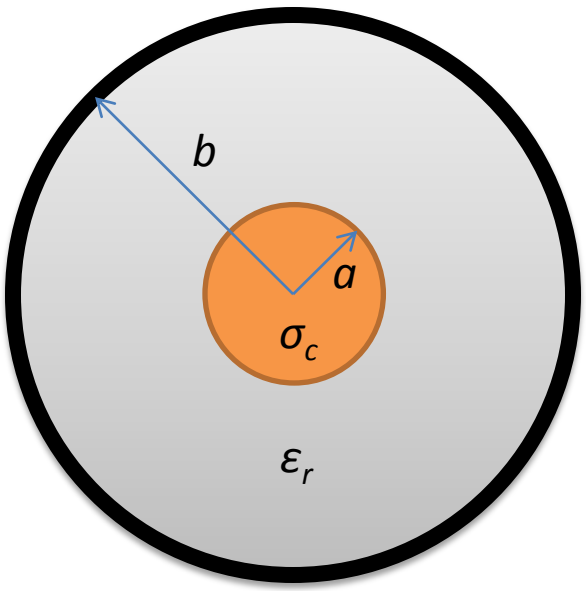
Approach: Use RF fundamentals to model the system piece-by-piece, and then use the pieces to compute the TDR response.

The next few slides cover S-parameter based models for the components in chafed coaxial cable.

Model Components

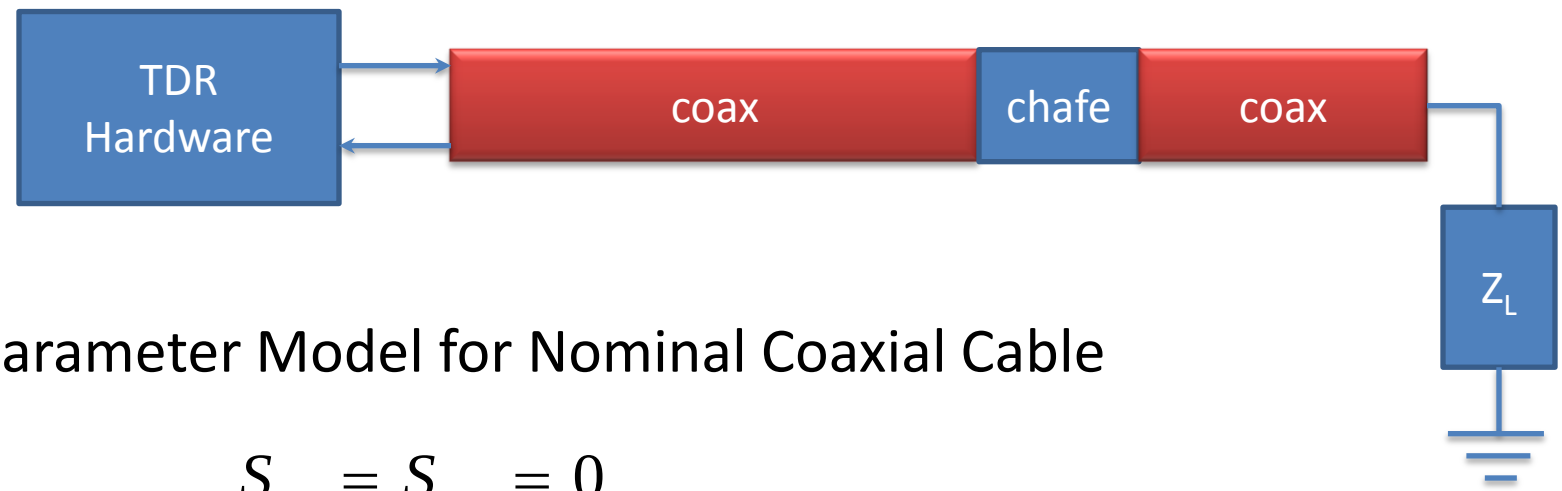


Nominal (unfaulted) Coaxial Cable



- Radial dimensions *a* and *b* are considered **known** constants
- σ_c is the conductance of the inner conductor in [Siemens/meter]
- ε_r is the relative dielectric permittivity
- σ_c and ε_r are considered **unknown** parameters

Model Components



S-Parameter Model for Nominal Coaxial Cable

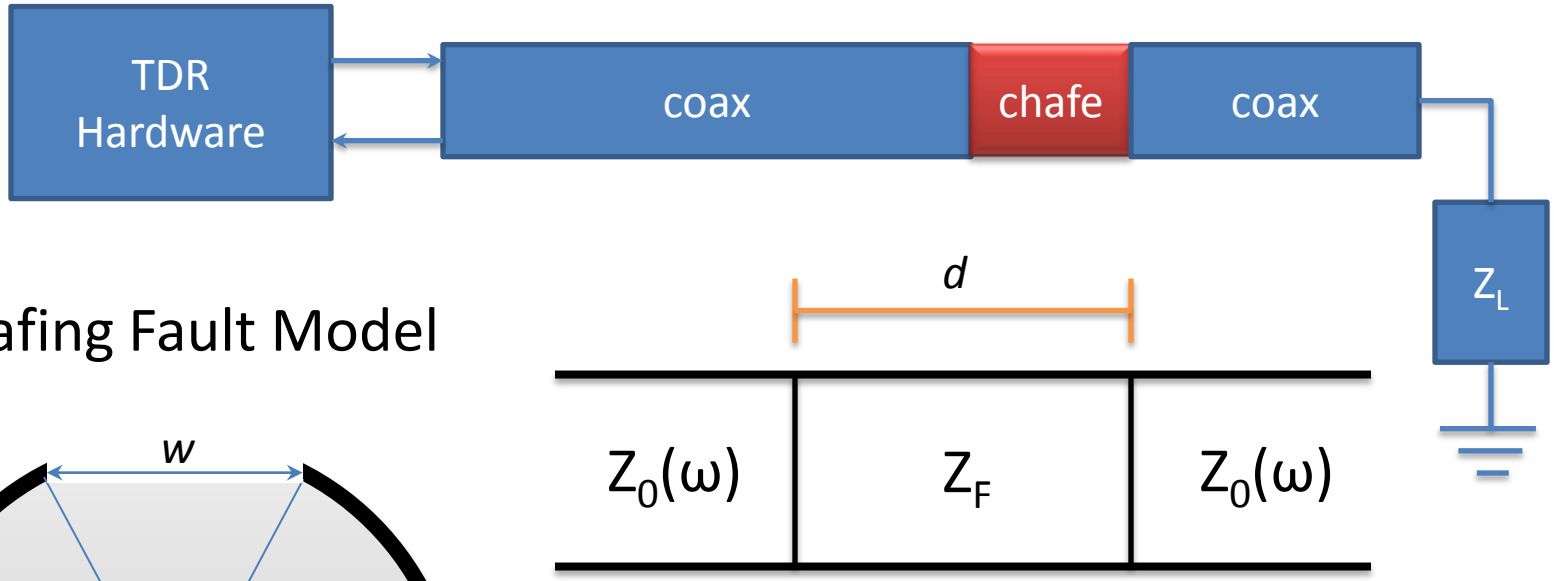
$$S_{11} = S_{22} = 0$$

$$S_{12} = S_{21} \equiv S_0(l) = e^{-jk(\omega)l}$$

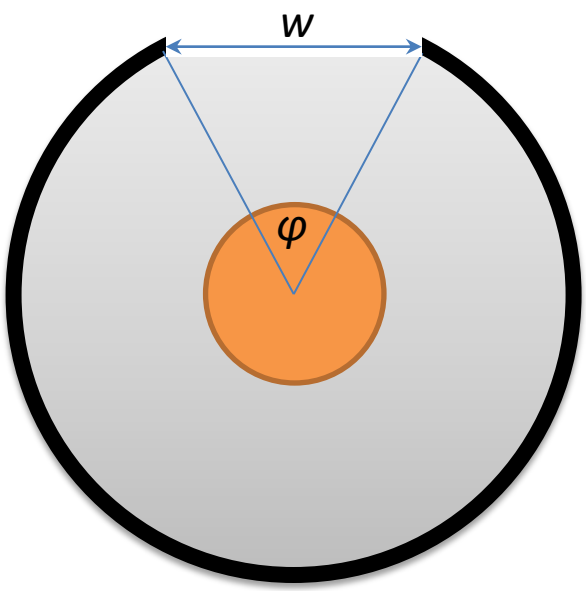
$$k(\omega) \cong \omega\sqrt{\mu_0 \varepsilon_d} + \frac{1}{2\ln(b/a)}\sqrt{\frac{\omega\varepsilon_d}{j\sigma_c}}\left(\frac{1}{a} + \frac{1}{b}\right)$$

$$Z_0(\omega) = \frac{\ln(b/a)}{2\pi}\frac{k(\omega)}{\omega}$$

Model Components

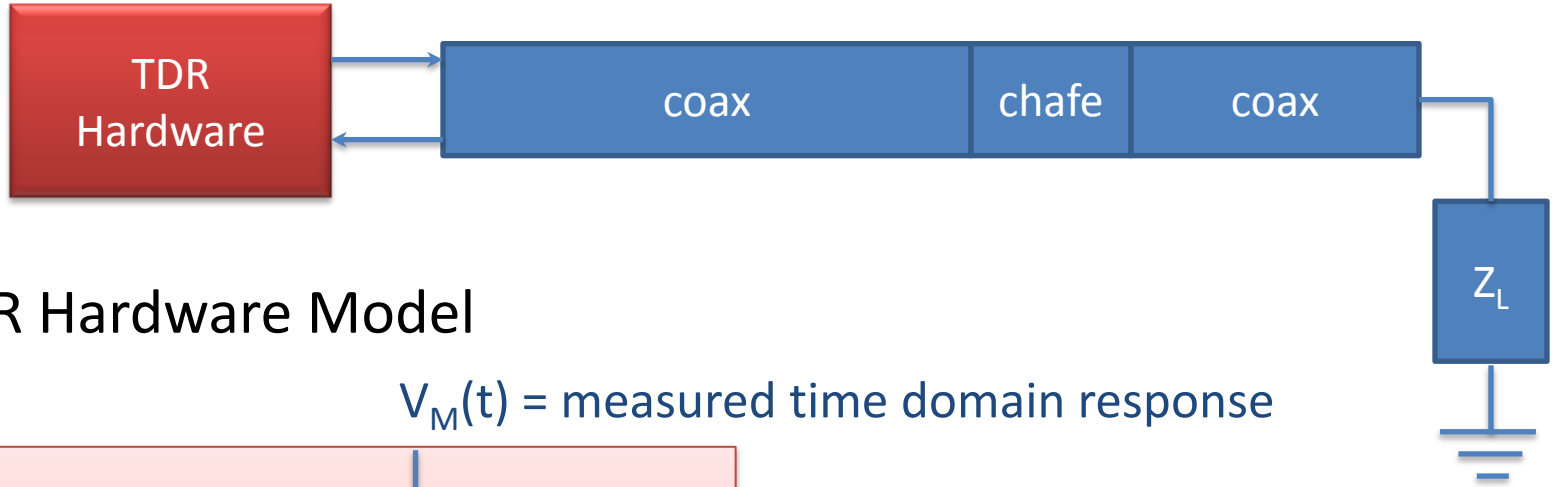


Chafing Fault Model



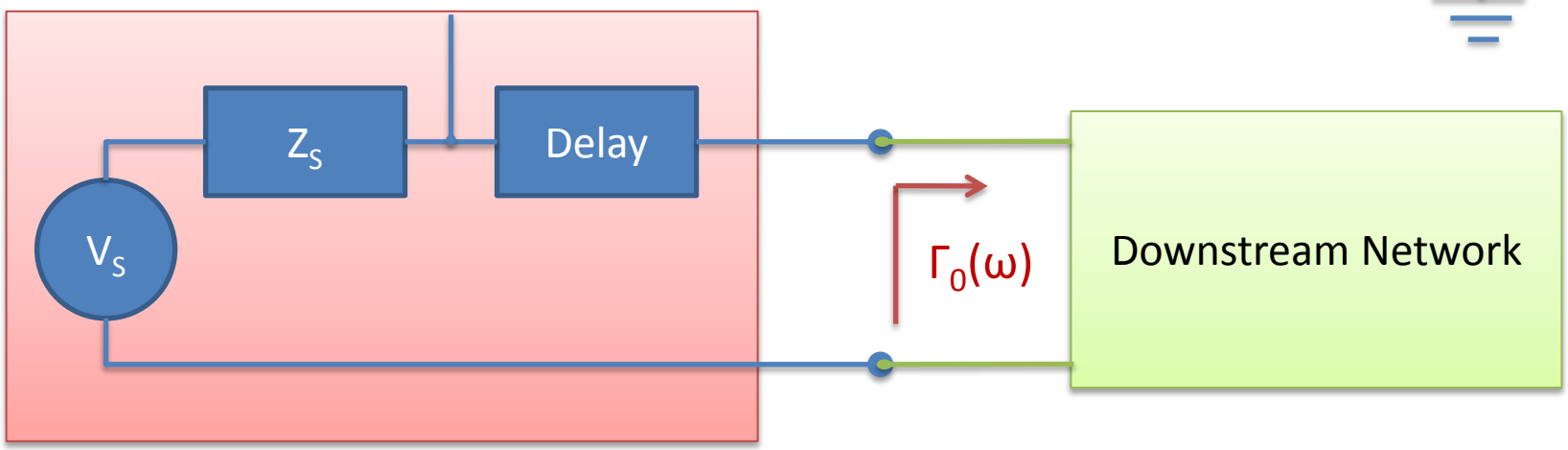
- Finite difference method is used to numerically determine Z_F as a function of w
- S-Parameters are then a function of *d*, Z_F, and $Z_0(\omega)$
- **Unknown** parameters are *w*, and *d*

Model Components



TDR Hardware Model

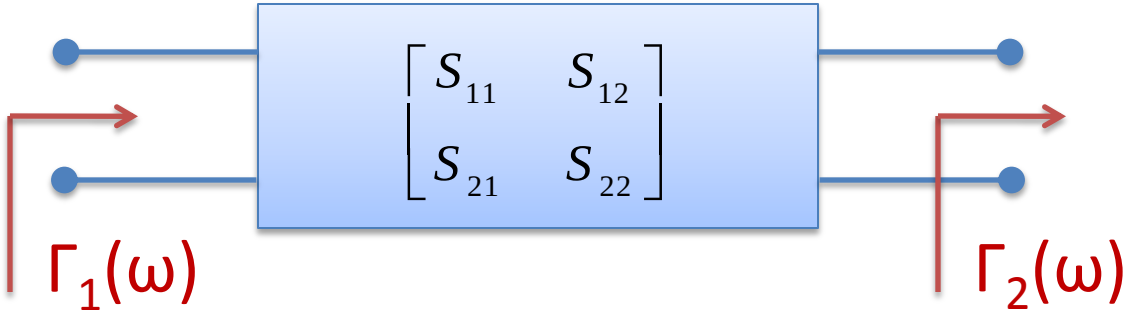
$V_M(t)$ = measured time domain response



- **Unknown** parameters are the measurement delay time t_M, source impedance Z_s, system gain constant G.

TDR Response Calculation for the Composite Model

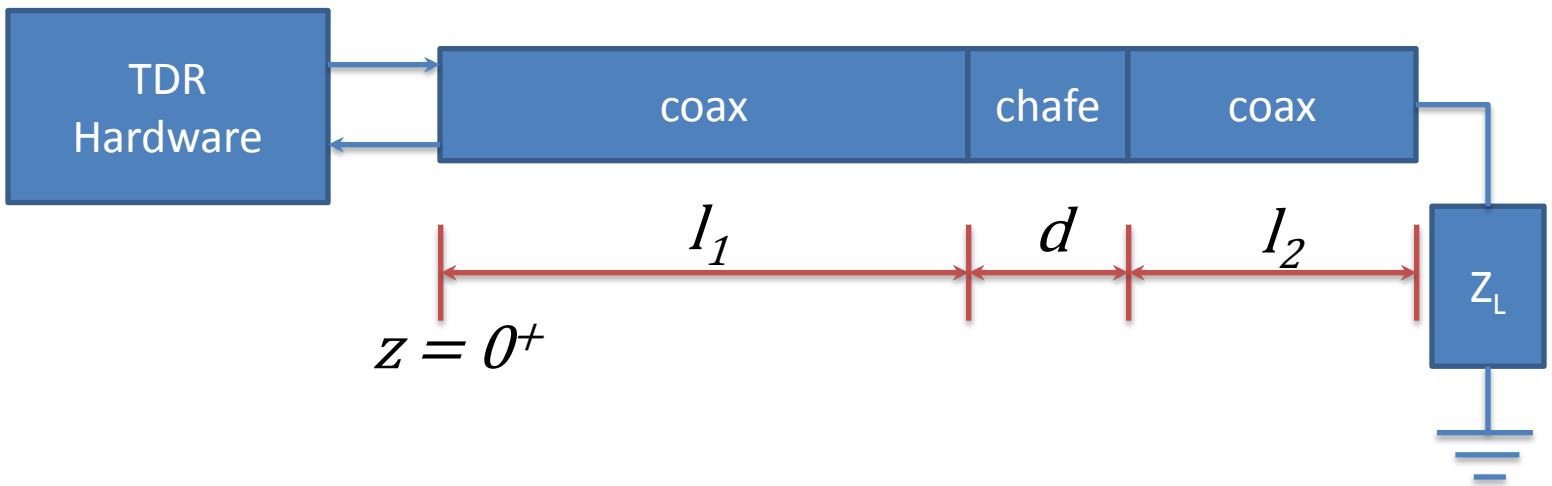
The entire calculation is based on a fundamental S-Parameter Update Equation



$$\Gamma_1 = S_{11} + \frac{S_{12} S_{21} \Gamma_2}{1 - S_{22} \Gamma_2}$$

TDR Response Calculation

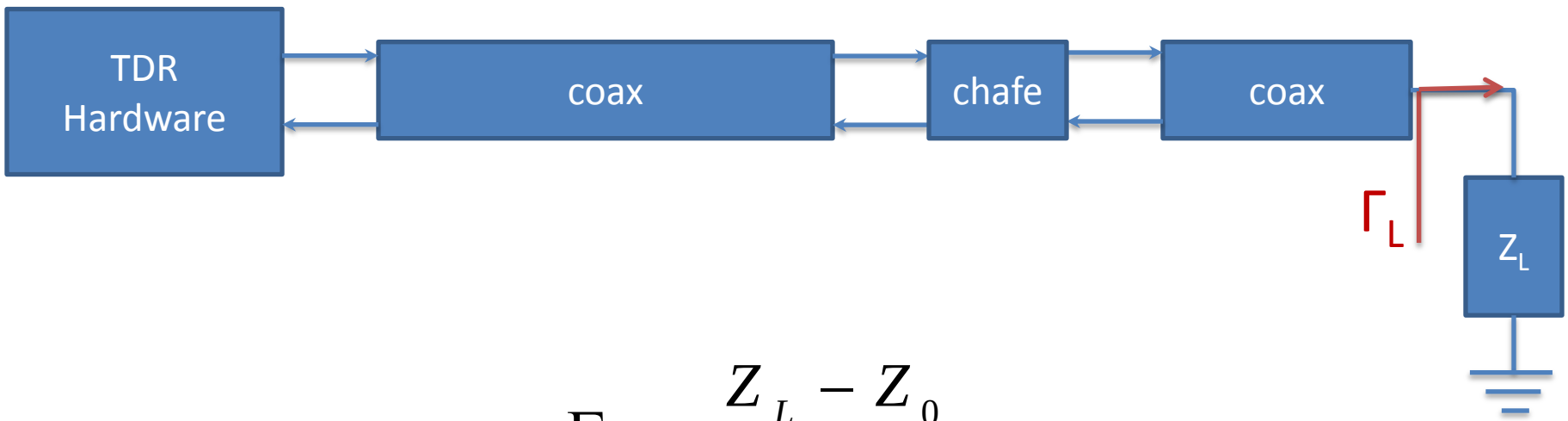
Starting with the composite model, and system parameters compute the TDR response.



System parameters: fault location, length, width, cable dielectric permittivity, conductivity, system gain, port impedance mismatch, and measurement delay

TDR Response Calculation

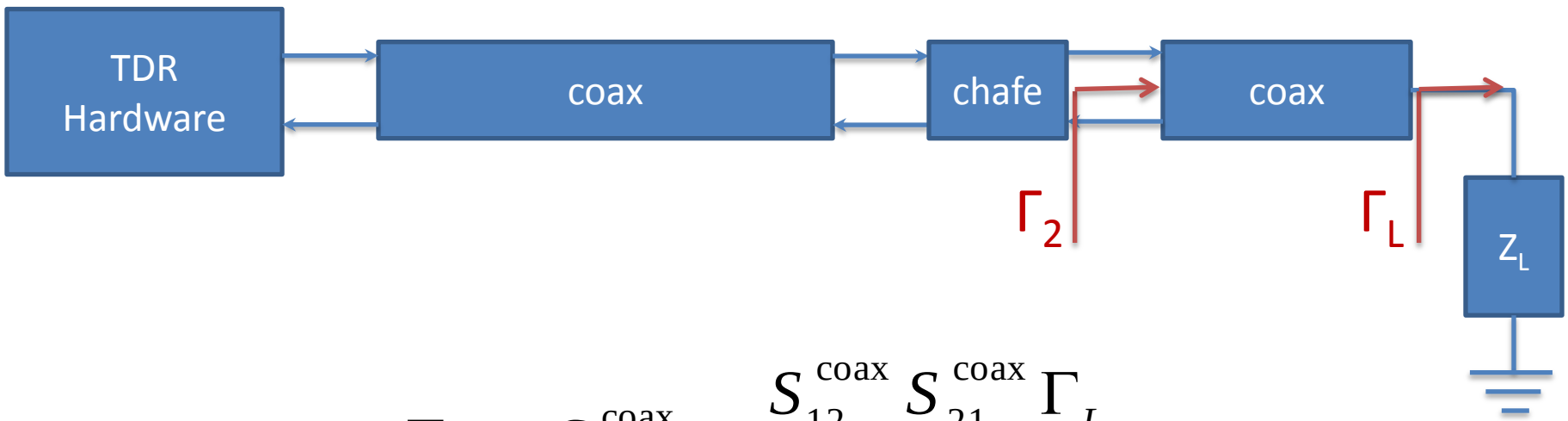
Break it into its components:



$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

TDR Response Calculation

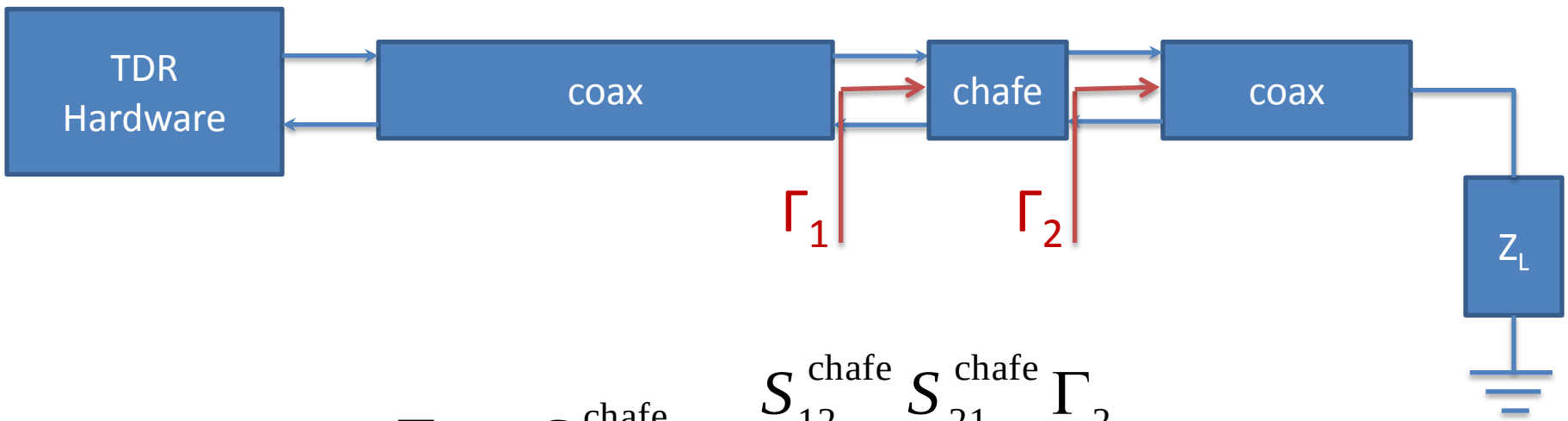
Process the schematic from right to left:



$$\Gamma_2 = S_{11}^{\text{coax}} + \frac{S_{12}^{\text{coax}} S_{21}^{\text{coax}} \Gamma_L}{1 - S_{22}^{\text{coax}} \Gamma_L}$$

TDR Response Calculation

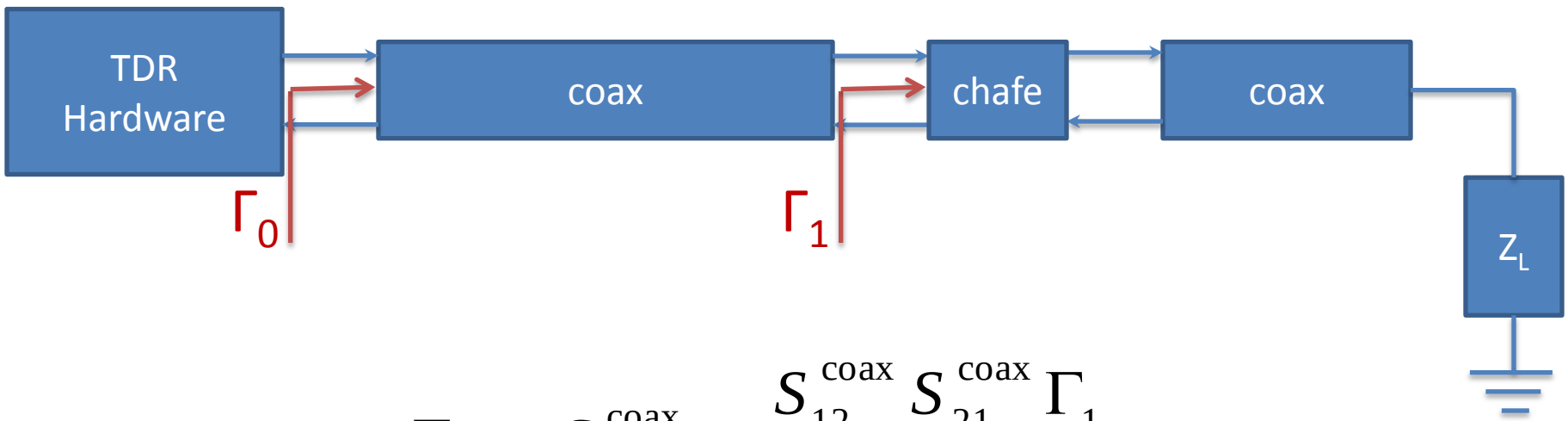
Process the schematic from right to left:



$$\Gamma_1 = S_{11}^{\text{chafe}} + \frac{S_{12}^{\text{chafe}} S_{21}^{\text{chafe}} \Gamma_2}{1 - S_{22}^{\text{chafe}} \Gamma_2}$$

TDR Response Calculation

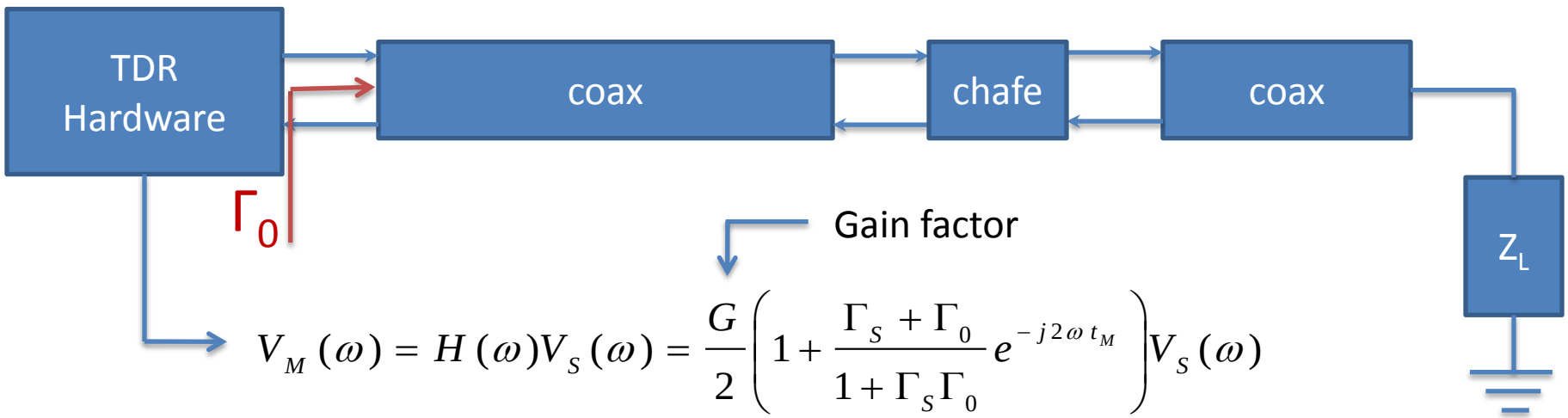
Process the schematic from right to left:



$$\Gamma_0 = S_{11}^{\text{coax}} + \frac{S_{12}^{\text{coax}} S_{21}^{\text{coax}} \Gamma_1}{1 - S_{22}^{\text{coax}} \Gamma_1}$$

TDR Response Calculation

Incorporate the TDR Hardware Model:



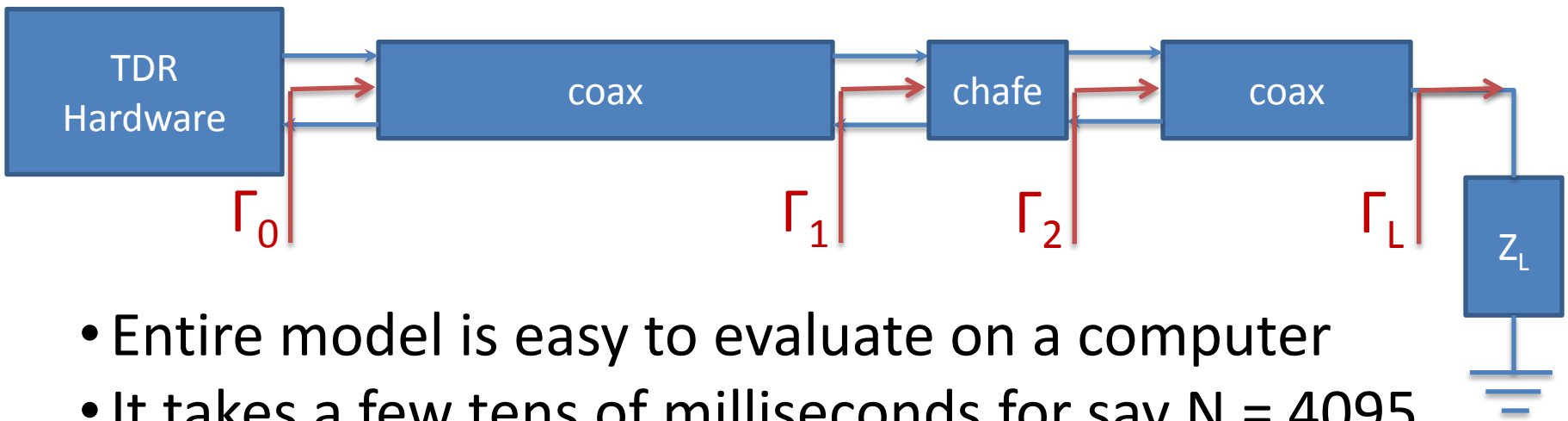
Gain factor

$$V_M(\omega) = H(\omega)V_S(\omega) = \frac{G}{2}\left(1 + \frac{\Gamma_S + \Gamma_0}{1 + \Gamma_S\Gamma_0} e^{-j2\omega t_M}\right)V_S(\omega)$$

$$\Gamma_S = \frac{Z_0 - Z_S}{Z_0 + Z_S}$$

TDR Response Calculation

Summary:



- Entire model is easy to evaluate on a computer
- It takes a few tens of milliseconds for say N = 4095 different frequencies needed for the TDR response
- The model is just a function: $F(\theta)$: $\mathbf{R}^7 \rightarrow \mathbf{C}^N$
- $\theta = (w, d, l_1, \varepsilon_r, \sigma_c, Z_s, t_M, G)$

Probabilistic Inversion

Goal: Find the fault location and size, given the measured TDR response – the inverse of the forward model.

A Probabilistic Approach

Viewed as a function of θ

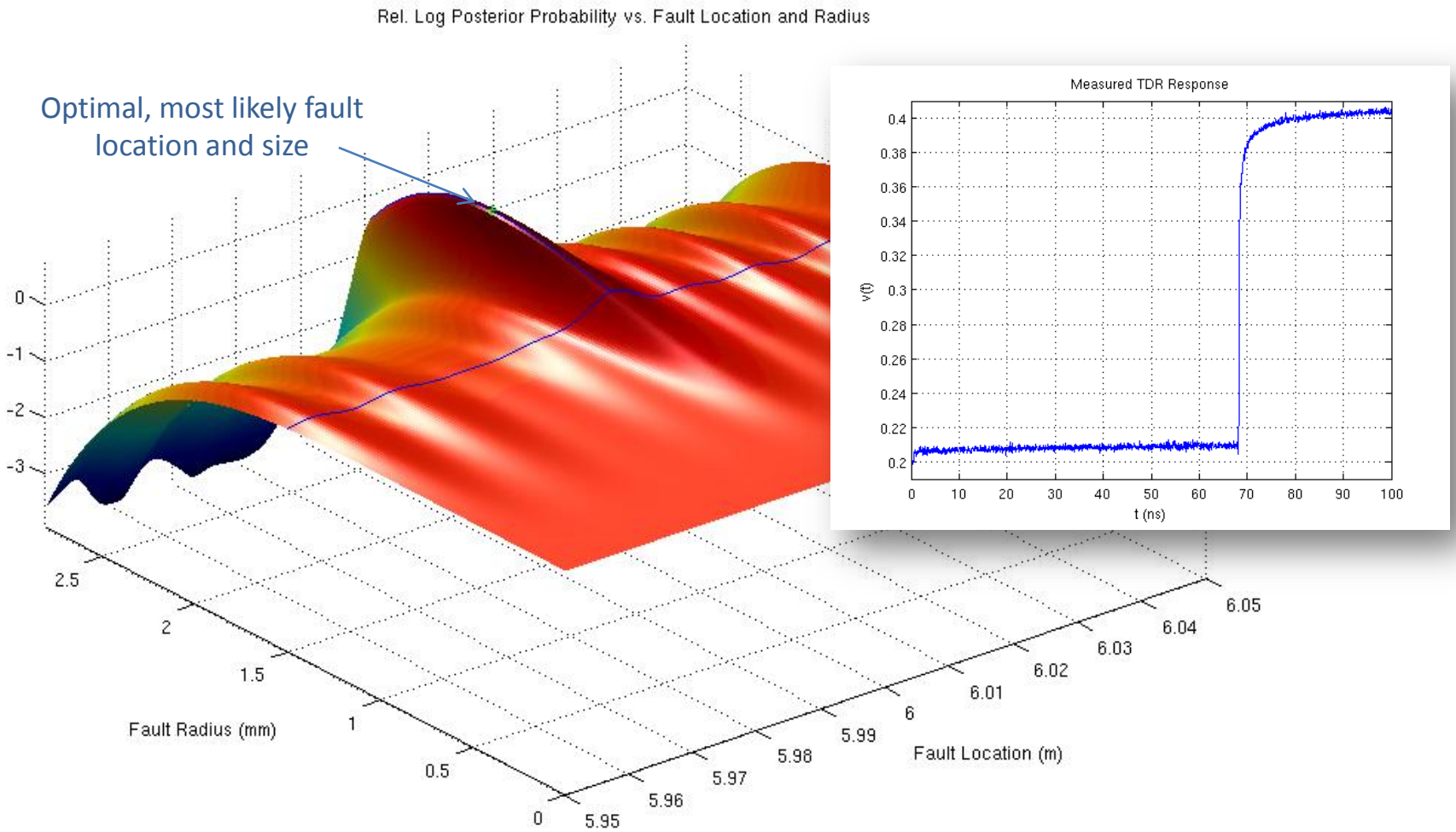
$$\underbrace{\Pr(\theta \mid data, F)}_{\text{posterior}} = \frac{\overbrace{\Pr(data \mid \theta, F)}^{\text{likelihood}}\ \overbrace{\Pr(\theta \mid F)}^{\text{prior}}}{\int \Pr(data \mid \theta', F)\Pr(\theta' \mid F)\, d\theta'} \qquad (1)$$

The inversion problem is then clearly defined as an **optimization problem**:

$$\underbrace{\text{maximize} \quad \Pr(\theta \mid data, F)}_{} \qquad (2)$$

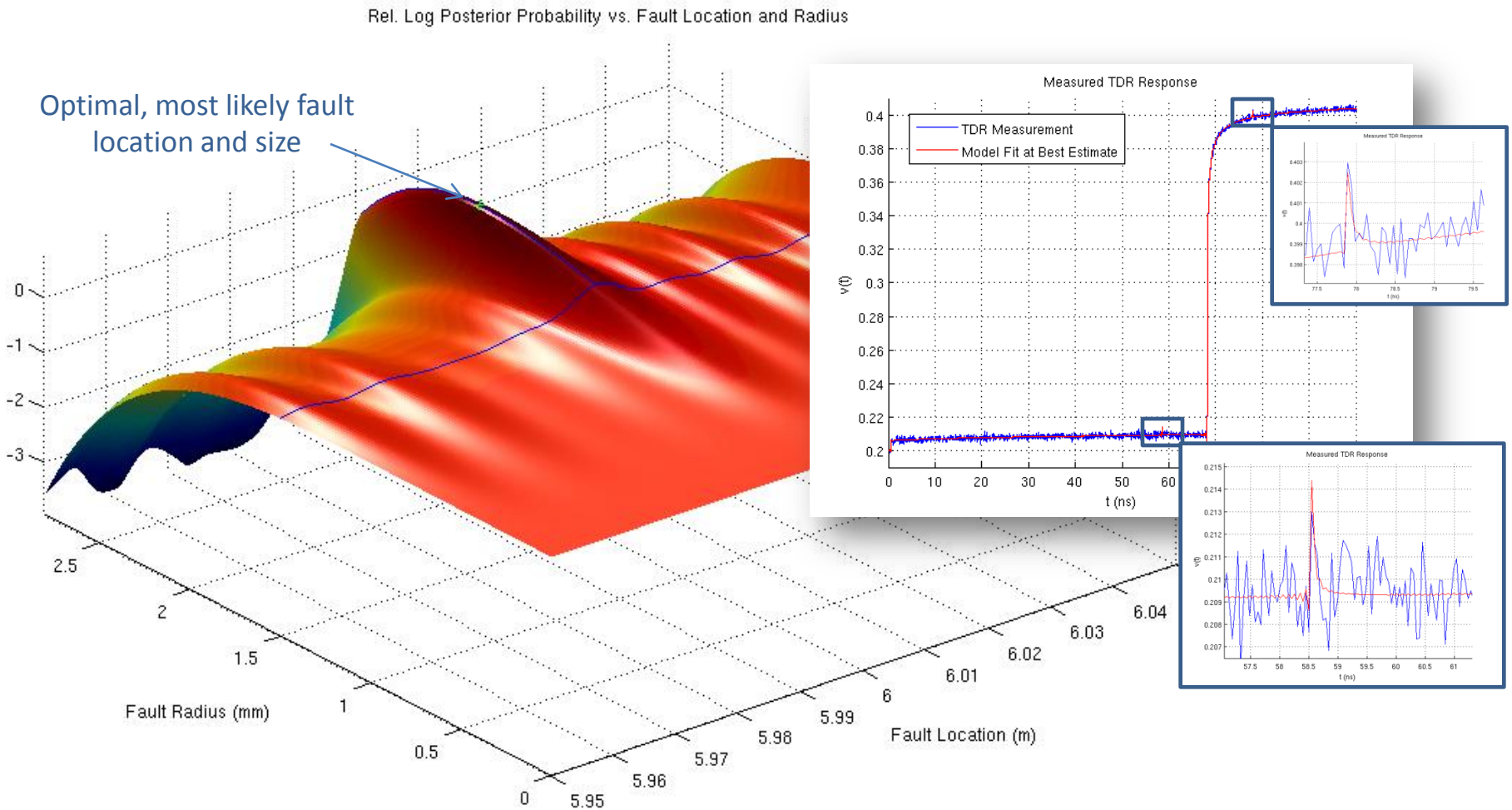
Find the most probable parameters given the observed data and model

Simulated Example Posterior PDF



Fault signature is not visually detectable on the TDR measurement, but very detectable on the Posterior PDF

Simulated Example Posterior PDF



Fault signature is not visually detectable on the TDR measurement, but very detectable on the Posterior PDF

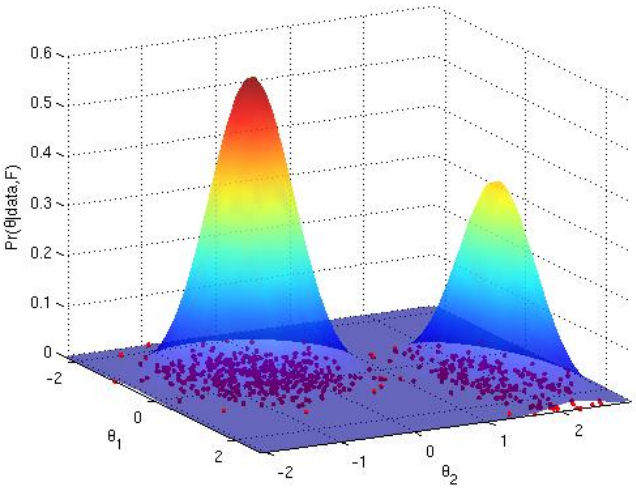
Markov Chain Monte-Carlo

For k=1,2,…,N sample $\theta_k \sim \Pr(\theta|data,F)$

The optimal estimate is then:

$\Theta^* = \text{argmax}\ \Pr(\theta_k|data,F)$

The spread of the N samples provide uncertainty information

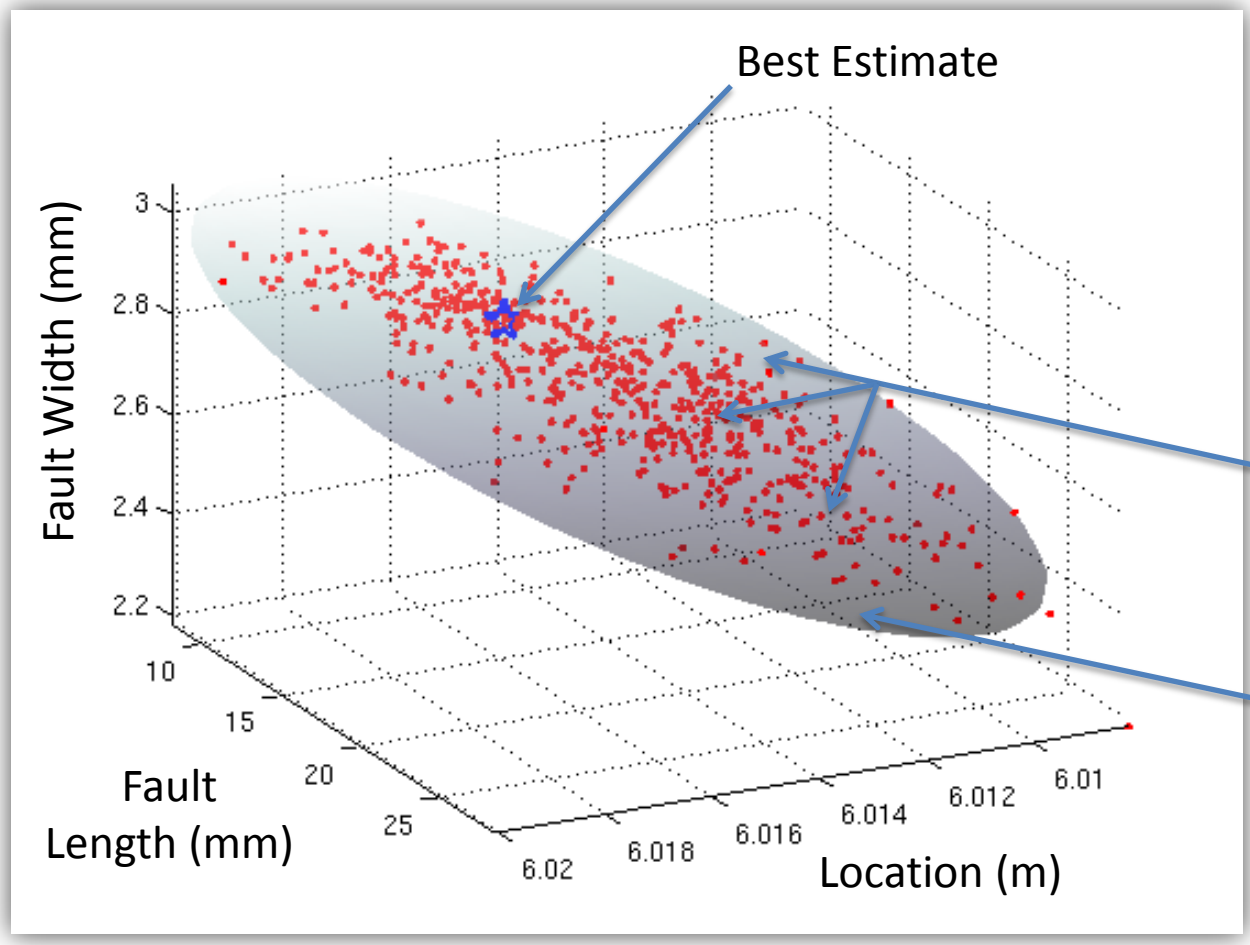


The samples are obtained using the *Nested Sampling* algorithm, a variant of the famous Metropolis Hastings Algorithm

- The main drawback is this approach is slow, it takes around 8 hours to solve our fault inversion problem
- The main advantage is it solves a general nonlinear inversion problem **without** the need for derivative information, a good initial guess, or additional Gaussian approximations

Optimal Estimate and Uncertainty

Chafing Fault Parameter Estimates



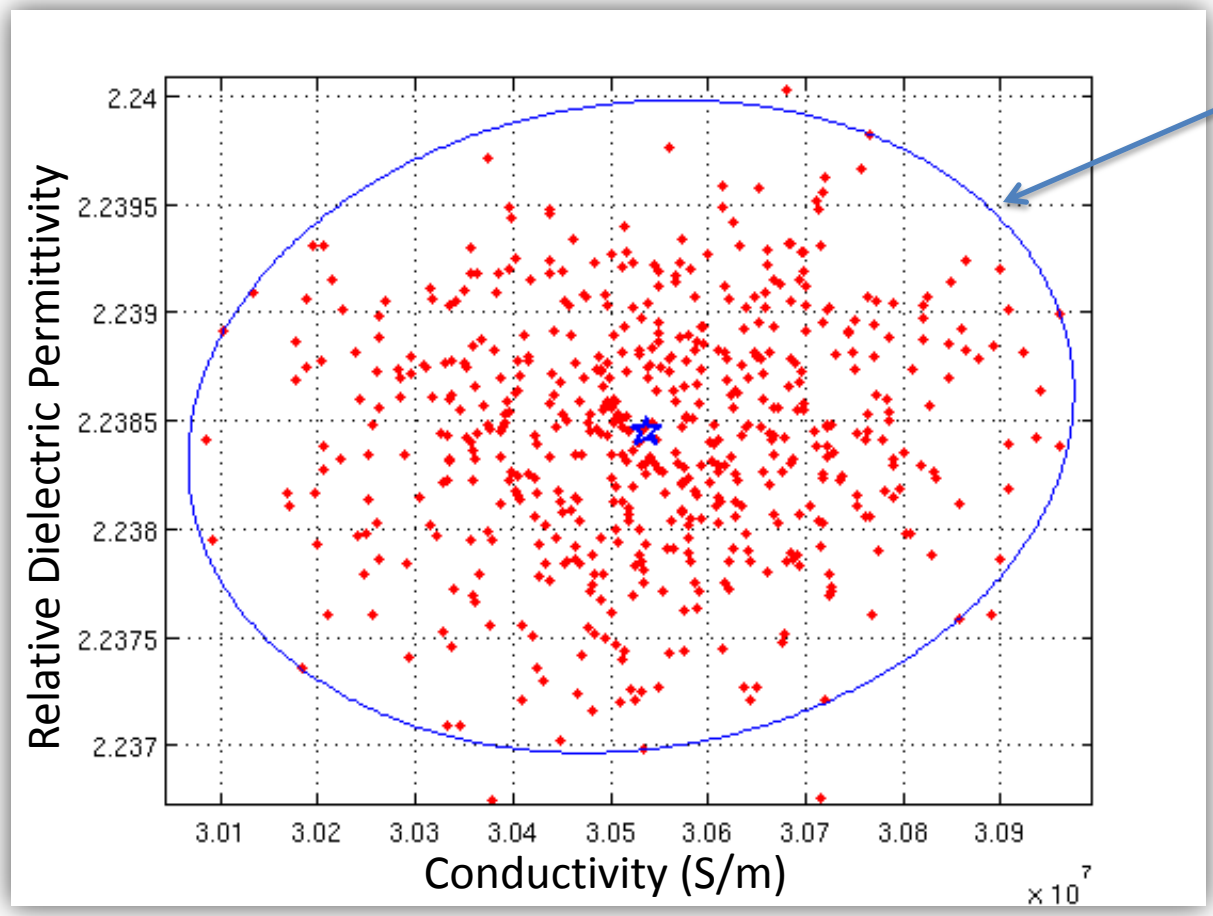
Lab Measurements:
Location: 6.007 ± 0.012 (m)
Width: 3 ± 1 mm
Length: 11 ± 1 mm
(actual fault is ellipsoidal)

Posterior Samples
$\theta_k \sim \Pr(\theta | data, F)$

95% Confidence Ellipsoid

Optimal Estimate and Uncertainty

RG58 Coaxial Cable Parameter Estimates



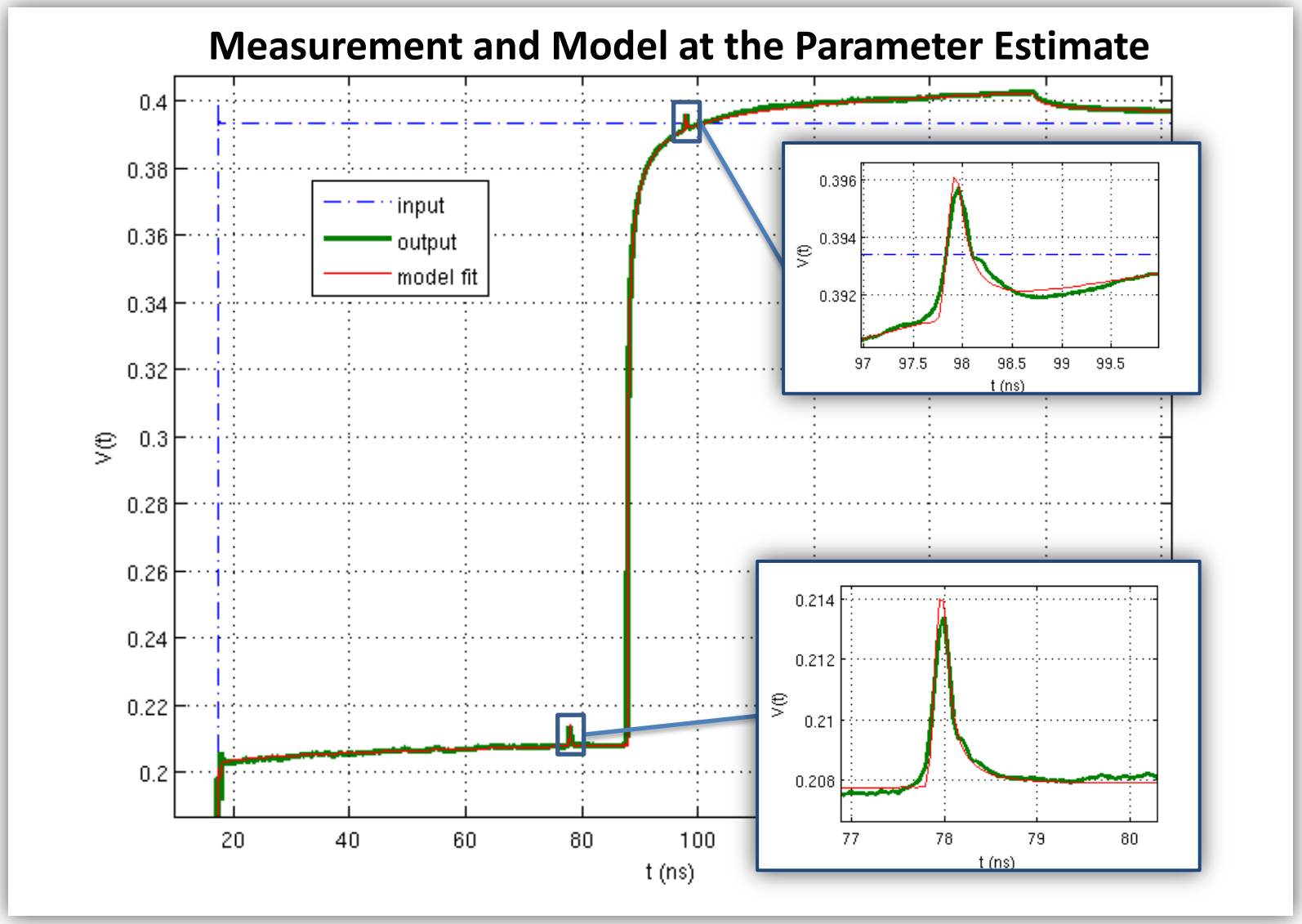
95% Confidence Ellipse

Text Book Values:
Permittivity (PE): 2.25
Conductivity (Cu) : 5.8 x 10^7

Text book values are typical and vary due to temperature, humidity, impurities, etc …

Our Estimate reflects the most likely values for this particular cable sample and test conditions

Model Fit Verification



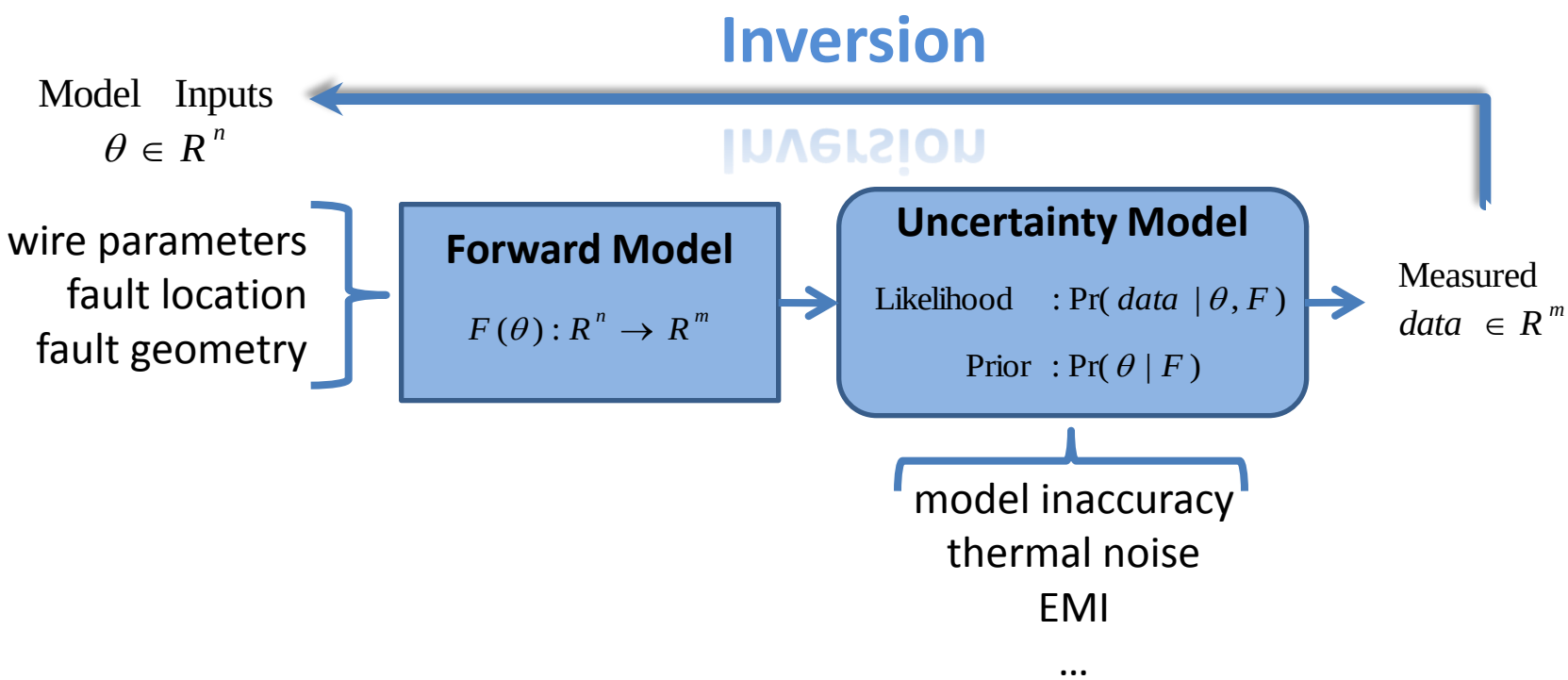
Conclusion

- Main purpose of this work was to establish a general framework for TDR based fault detection in a wiring system
  - Presented one example for chafing fault detection in RG58 coax
  - Plan to extend the work to other common wiring geometries such as twisted shielded pair
- The probabilistic inversion approach is the way to go:
  - Provides a clear definition of what an optimal estimate is
  - Enables quantification of fault detectability (as estimation uncertainty)
- **Ultimate Goals:**
  - **Address trade space research questions**
  - **Develop improved fault detection software that works in practice**

Questions ?

Supporting Material

Model Inversion



Inversion Approach

- Direct Approach: $F^{-1}(data) = \theta$
  - Almost always works **poorly** in practice because *there is uncertainty between the model output and the measured data*
  - Finding the inverse of the forward model, if it exists, is often intractable in practice. The most general forward model is an *algorithm*.
- Probabilistic Approach
  - Requires specifying a ***prior pdf***, which encodes known information about the inputs given **no** measured data
  - Also requires a ***likelihood pdf***, which encodes the probability of measuring the data given the inputs

Admitting uncertainty in the modeling and measurement process leads to robust inversion and much more …

MCMC Inversion Example

Measured Data: $y = F(\theta) + \text{noise} \in R^{8192}, \ \theta \in R^4$

Likelihood: $\Pr(y \mid \theta, F) = \left(2\pi\sigma^2\right)^{-m/2} e^{-\frac{1}{2\sigma^2}\left\|F(\theta)-y\right\|^2}$

Prior: $\theta \sim \text{Uniform} \ (\theta_{\min}, \theta_{\max})$

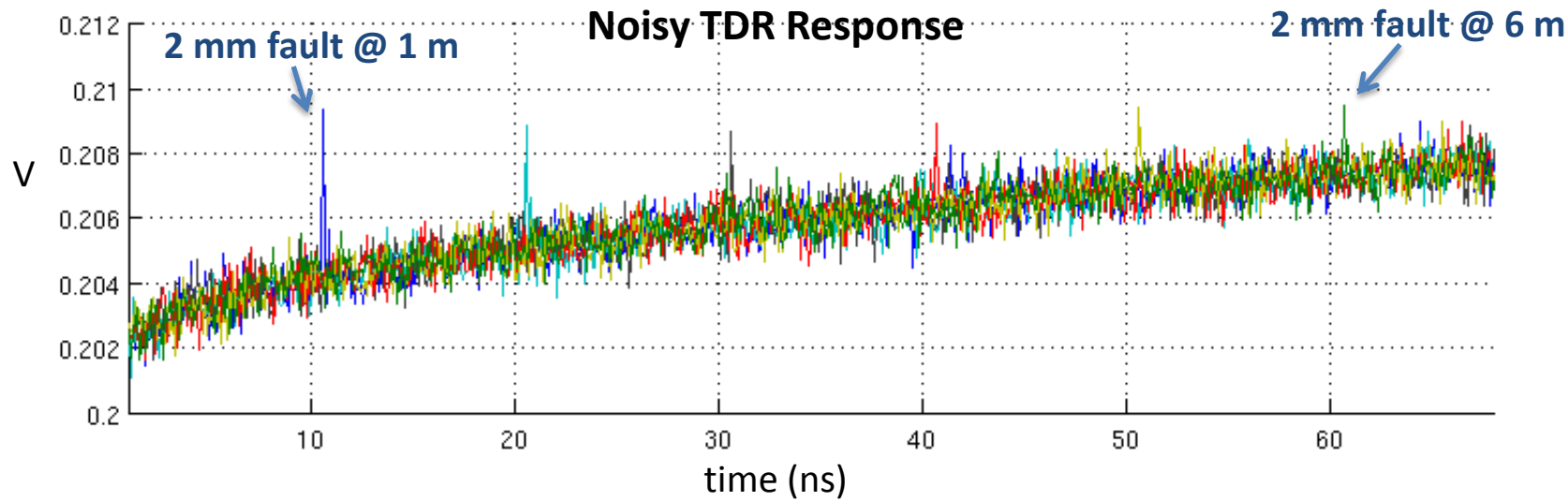
Solve: $\text{maximize} \quad \Pr(\theta \mid y, F) \propto \Pr(y \mid \theta, F)\Pr(\theta \mid F)$

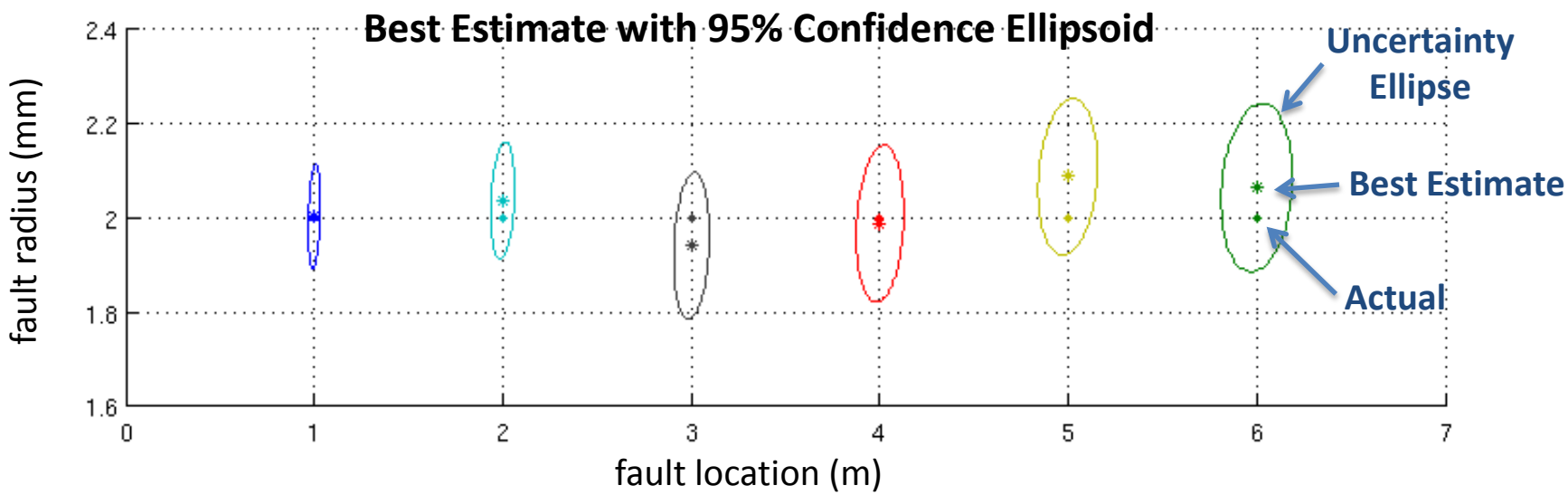
Approach: $\text{For } k = 1, 2, \ldots, N \text{ sample } \theta_k \sim \Pr(\theta \mid y, F) \text{ using MCMC.}$

Optimal estimate: $\theta^* \sim \arg\max_{k=1,\ldots,N} \Pr(\theta_k \mid y, F)$

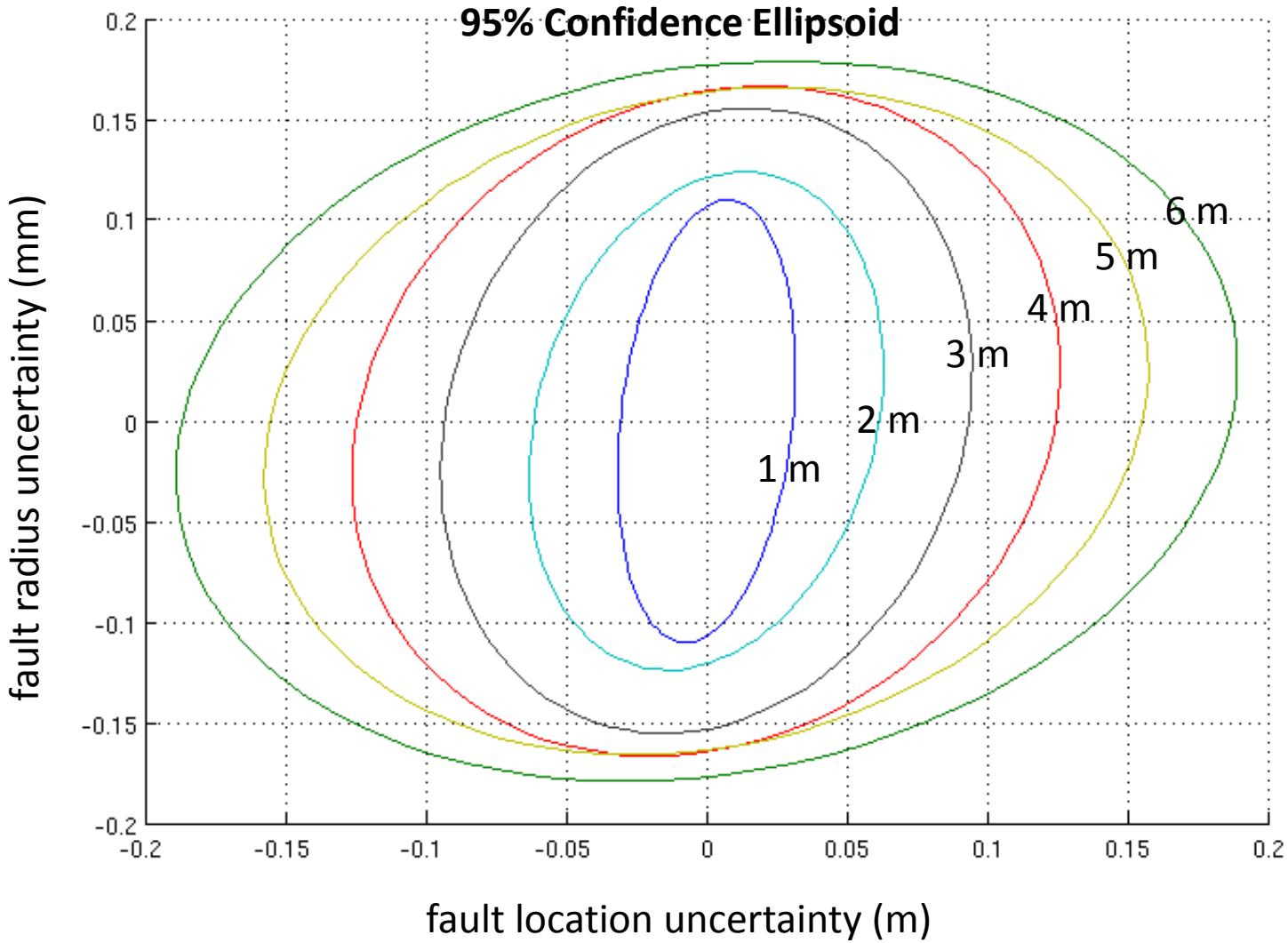
Uncertainty: Spread of the sampled θ_k's

Estimation with Uncertainty



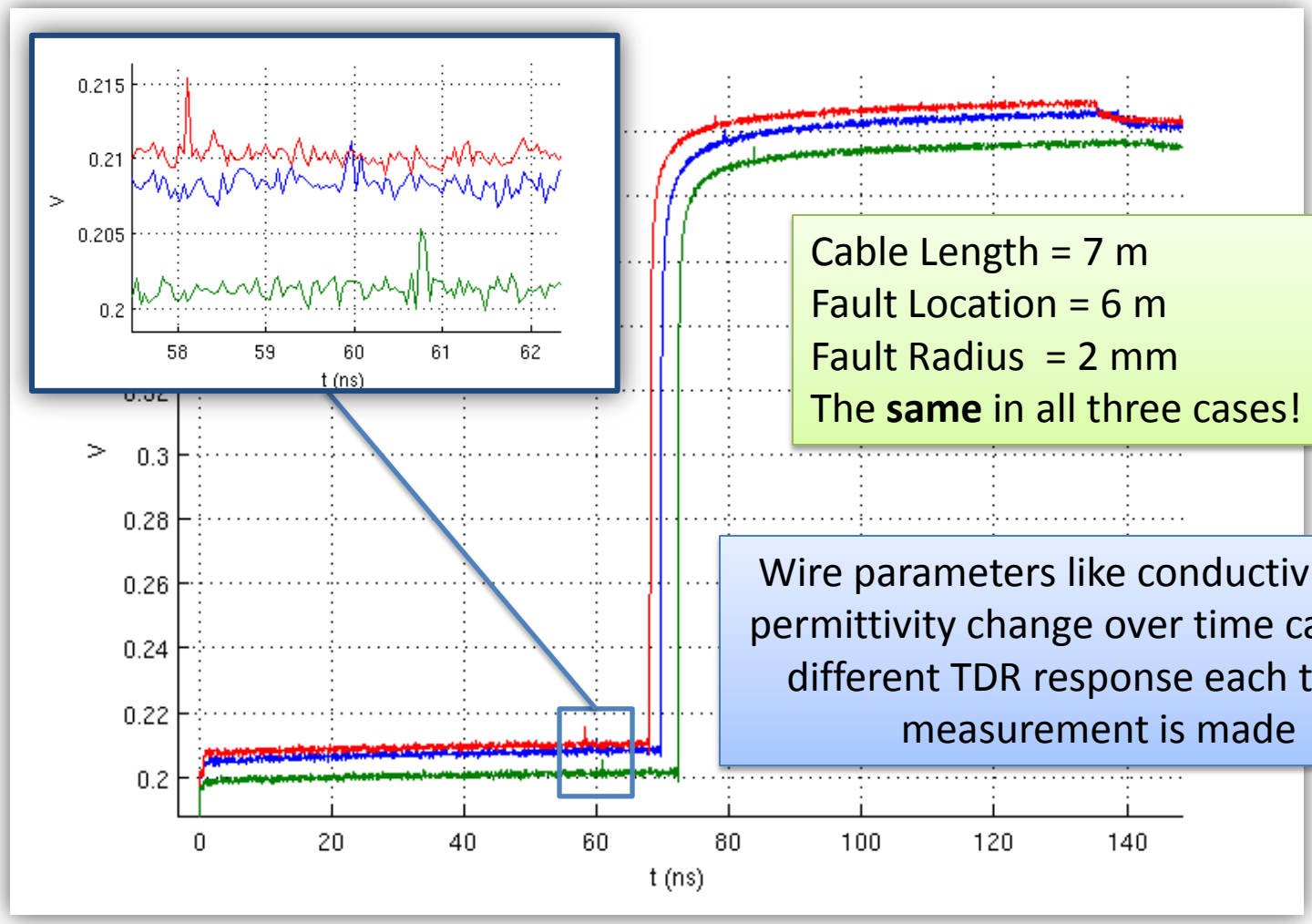


Uncertainty Growth with Distance for a 2 mm Fault in RG58 Coax



The Baseline Problem

3 Simulated Measurements of the Same Cable and Fault

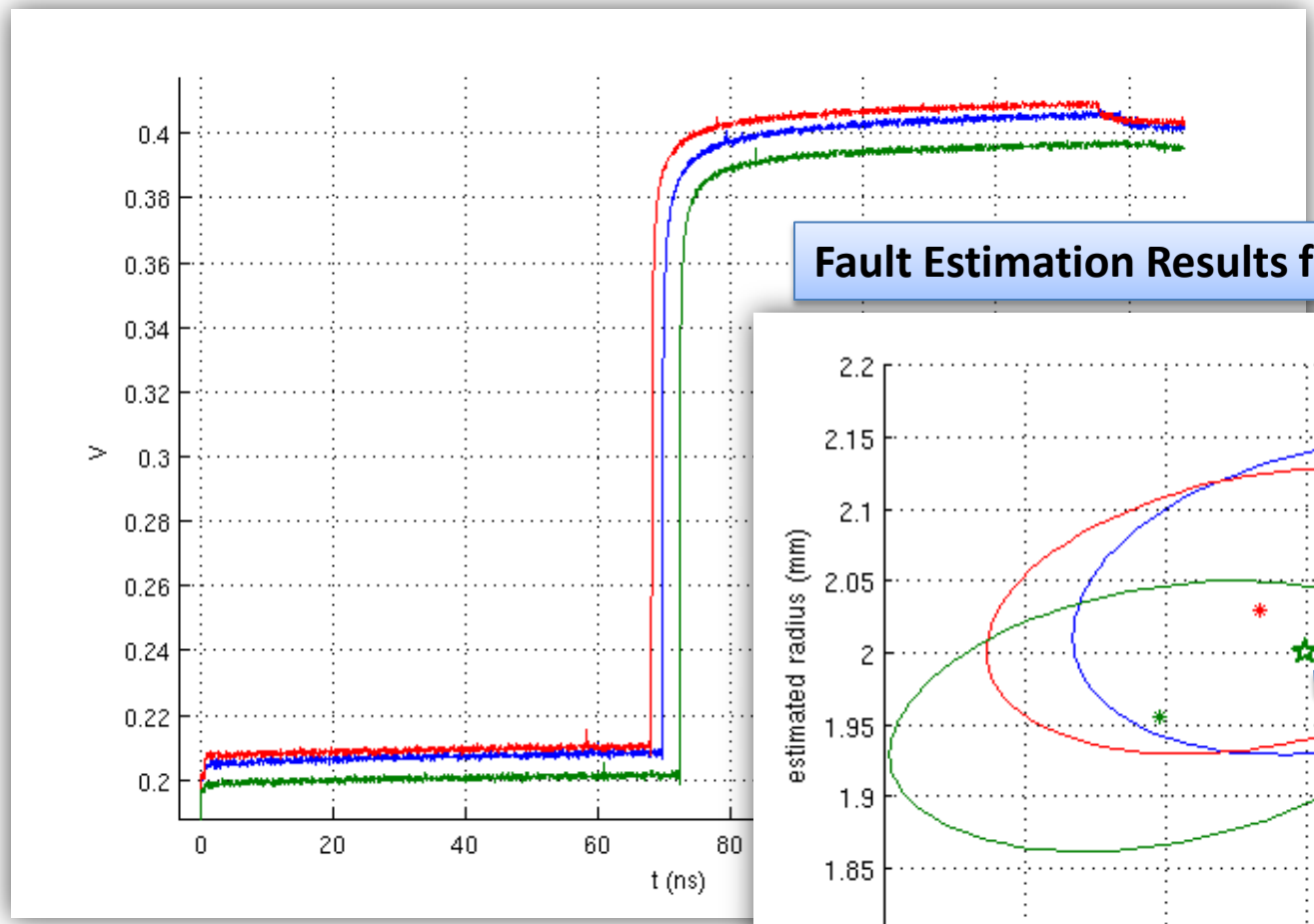
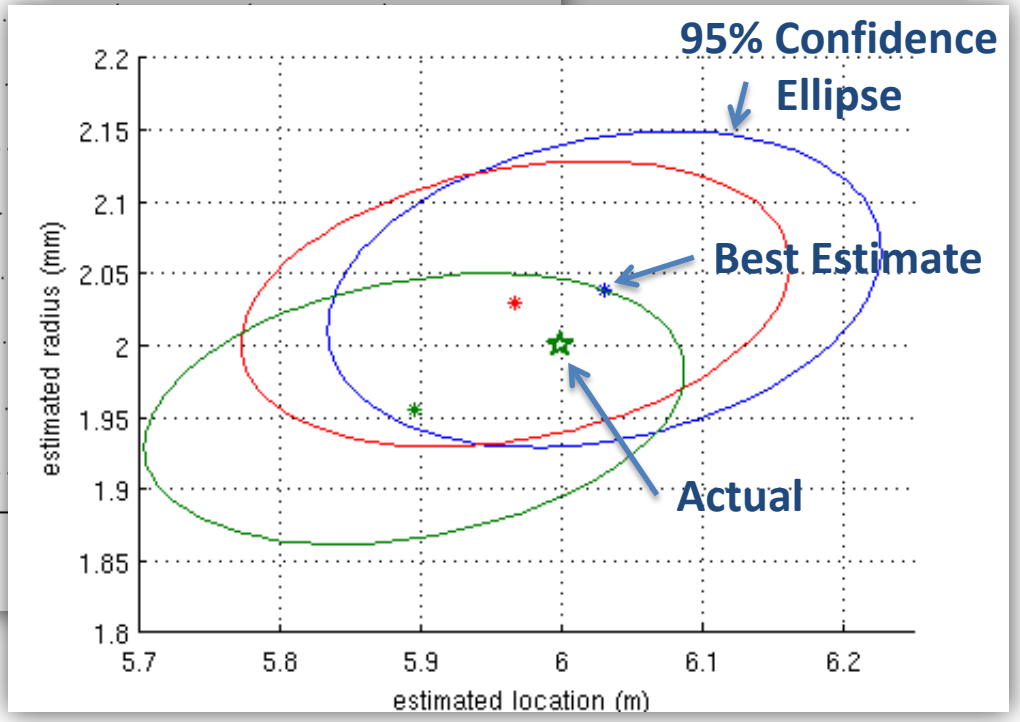


Cable Length = 7 m
Fault Location = 6 m
Fault Radius = 2 mm
The **same** in all three cases!

Wire parameters like conductivity, and permittivity change over time causing a different TDR response each time a measurement is made

No Baseline Needed!

3 Simulated Measurements of the Same Cable and Fault



Fault Estimation Results for Each Simulation