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ADAPTIVE KEY COMPONENT CONTROL FOR NONLINEAR EVOLVING SYSTEMS

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ABSTRACT

The inheritance of subsystem traits in Evolving Systems is an important area of study. Evolving Systems are autonomously controlled subsystems which self-assemble into a new Evolved System with a higher purpose. Evolving Systems of aerospace structures often require additional control when assembling to maintain stability during the entire evolution process. If certain passivity traits of the subsystem components are inherited in the Evolving System, then it is possible to use an adaptive controller to restore stability in the Evolving System. This paper develops the theory for Nonlinear Evolving Systems and illustrates it with a simple example.

INTRODUCTION

The inheritance of subsystem traits in Evolving Systems is an important area of research. Evolving Systems [1, 2] are autonomously controlled subsystems which self-assemble into a new Evolved System with a higher purpose. Evolving Systems of aerospace structures often require additional control when assembling to maintain stability during the entire evolution process [3-5]. An adaptive key component controller has been shown to restore stability in Evolving Systems that would otherwise lose stability during evolution [6]. The adaptive key component controller uses a direct adaptation control law to restore stability to the Evolving System through a subset of the input and output ports on one key component of the Evolving System.

The control laws used by the adaptive key component controller to restore stability in an Evolving System are guaranteed to have bounded gains and asymptotic tracking if the Evolved System is almost strictly dissipative. (This concept is similar to almost strict passivity [6].) Hence, it is desirable to know when the dissipativity traits of the subsystem components, including the key component, are inherited in an Evolving System. We present results showing when an Evolving System will inherit the almost strict dissipativity traits of its subsystem components. We also show results to guarantee stable adaptation when an adaptive key component controller is used to restore stability. An illustrative nonlinear example is given of successful restoration of stability using an adaptive key component controller.

MATHEMATICAL FORMULATION OF EVOLVING SYSTEMS

A mathematical formulation of a nonlinear time-invariant Evolving System is given here. Consider a system of L components of individually, actively controlled subsystems which can be described by the following nonlinear equations for the ith component:

$$\begin{cases} \dot{x}_i = A_i(x_i) + B_i(x_i)u_i \\ y_i = C_i(x_i) \end{cases} \tag{1}$$

where $i = 1,2,\ldots L$. The ith component has a Lyapunov or Energy Storage Function V_i. These are the building blocks of the Evolving System. When these individual components are joined to form an Evolved System, the new entity becomes:

$$\begin{cases} \dot{x} = A(x,\varepsilon) + B(x)u \\ y = C(x) \end{cases} \tag{2}$$

with $x \equiv [x_1 \ldots x_L]^T$, $y \equiv [y_1 \ldots y_L]^T$, $u \equiv [u_1 \ldots u_L]^T$, and Lyapunov or Energy Storage Function $V \equiv \sum_{i=1}^{L} V_i$. The ith component in the above Evolved System is given by:

$$\begin{cases} \dot{x}_i = A_i(x_i) + B(x_i)u_i + \sum_{j=1}^{L} \varepsilon_{ij} A_{ij}(x_j, u_j) \\ y_i = C_i(x_i) \end{cases} \tag{3}$$

with $0 \le \varepsilon_{ij} \le 1$ and $\varepsilon_{ji} = \varepsilon_{ij}$ and where $A_{ij}(x_j, u_j)$ represents the interconnections between the ith and j^{th} components. Note that when $\varepsilon_{ij} = 0$ the system is in component form and when $\varepsilon_{ij} = 1$, the system is fully evolved. As the system evolves, or joins together, the ε_{ij}'s evolve from 0 to 1.

The components of the Evolving System are actively controlled by means of local control. Local control means dependence only on local state or local output information, i.e.,

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$u_i = h_i(x_i)$ or $h_i(y_i)$. In general, the local controller on the ith component would have the form:

$$\begin{cases} u_i = h_i(y_i, z_i) \\ \dot{z}_i = l_i(y_i, z_i) \end{cases} \tag{4}$$

where z_i is the dynamical part of the control law. Local control will be used to keep the components stable and meet the individual component performance requirements.

Once the system is fully evolved, the ith component in the fully evolved system becomes:

$$\begin{cases} \dot{x}_i = A_i(x_i) + B(x_i)u_i + \sum_{j=1}^{L} A_{ij}(x_j, u_j) \\ y_i = C_i(x_i) \end{cases} \tag{5}$$

The state space representation of the Evolved System then becomes:

$$\begin{cases} \dot{x} = A(x) + B(x)u \\ y = C(x) \end{cases} \tag{6}$$

which can also be denoted $(A(x), B(x), C(x))$ or (u, y).

INHERITANCE OF SUBSYSTEM TRAITS IN EVOLVING SYSTEMS

We say a subsystem trait, such as stability, is inherited when the Evolved System retains the characteristic of the trait from the subsystem. Previous papers have examined the inheritance of stability and shown that stability is not a generally inherited trait [1-3]. Inheritance of almost strict passivity of subsystems is desirable in Evolving Systems that use an adaptive key component controller to restore stability.

In previous papers [5, 6], a key component controller has been proposed to restore stability to Evolving Systems which would otherwise lose stability during evolution. The design approach used in the key component controller is for the control and sensing of the components to remain local and unaltered except in the case of one key component which has additional local control added to stabilize the system during evolution. The key component controller operates solely through a single set of input-output ports on the key component, see Fig. 1.

Only the key component of the Evolving System needs modification to restore the inheritance of stability. A clear advantage of the key component design is that components can be reused in many different configurations of Evolving Systems without the need for component redesign. The reuse of components which are space-qualified, or at least previously designed and unit tested, could reduce the overall system development and testing time and should result in a higher quality system with potentially significant cost savings and risk mitigation.

In many aerospace environments and applications, the parameters of a system are poorly known and difficult to obtain. Adaptive key component controllers, which make use of a direct adaptation control law, are a good design choice for restoring stability in Evolving Systems where access to precisely known parametric values is limited. The necessary condition for an Evolving System with an adaptive key component controller to be guaranteed to have bounded gains and to have asymptotic output tracking is that the system be almost strictly passive [6]. Hence, we are interested in the conditions under which the inheritance of almost strict passivity can be guaranteed in Evolving Systems.

INHERITANCE OF ALMOST STRICT DISSIPATIVITY IN EVOLVING SYSTEMS

Consider a ***Nonlinear System*** of the form:

$$\begin{cases} \dot{x} = A(x) + B(x)u \\ y = C(x) \end{cases} \tag{7}$$

We say this system is ***Strictly Dissipative*** when

$$\begin{aligned} &\exists V(x) > 0 \; \forall x \neq 0 \text{ such that} \\ &\begin{cases} \nabla V A(x) \leq -S(x) \; \forall x \\ \nabla V B(x) = C^T(x); \nabla V \equiv \text{gradient } V \end{cases} \end{aligned} \tag{8}$$

The function $V(x(t))$ is called the ***Storage Function*** for (7), and the above says that the storage rate is always less than the external power. This can be seen from

$$\begin{aligned} \dot{V} &\equiv \nabla V[A(x) + B(x)u] \\ &\leq -S(x) + C^T(x)u \\ &= -S(x) + \langle y, u \rangle \end{aligned} \tag{9}$$

Taking $u \equiv 0$, it is easy to see that (9) implies (8a) but not necessarily (8b); so (8) implies (9) but not conversely. They are only equivalent if (8a) is an equality. When equality holds in (8) and (9), the property is called Strict Passivity.

We will say a system (u, y) is ***Almost Strictly Dissipative (ASD)*** when there is some output feedback, $u = G_* y + u_r$, so that the following is strictly dissipative:

$$\begin{cases} \dot{x} = A_C(x) + B(x)u_r \\ A_C(x) \equiv A(x) + B(x)G_* C(x) \\ y = C(x) \end{cases} \tag{10}$$

Now if each component (3) is ASD, then, from (5) and (8), we have

$$\begin{cases} \nabla V_i[A_i(x_i) + B_i(x_i)G_i C_i(x_i)] \leq -S_i(x_i) + \cdots \\ \qquad\qquad + \sum_{j=1}^{L} \varepsilon_{ij} \nabla V_i A_{ij}(x_j, u_j) \\ \nabla V_i B_i(x_i) = C_i^T(x_i); \quad \nabla V_i \equiv gradient V_i \end{cases} \tag{11}$$

Due to the interconnection terms, (11) is not necessarily Strictly Dissipative. However, in some circumstances, the interconnection terms have a special form and ASD is inherited when the system evolves.

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Suppose we have a pair of subsystems of the form:

$$\begin{cases} \dot{x}_i = A_i(x_i) + B_i(x_i)u_i + B_i^A(x_i)u_i^A \\ y_i = C_i(x_i) \\ y_i^A = C_i^A(x_i) \end{cases} \tag{12}$$

where $i = 1,2$ and both subsystems $\left(\begin{bmatrix} u_1 \\ u_1^A \end{bmatrix}, \begin{bmatrix} y_1 \\ y_1^A \end{bmatrix}\right)$ and $\left(\begin{bmatrix} u_2 \\ u_2^A \end{bmatrix}, \begin{bmatrix} y_2 \\ y_2^A \end{bmatrix}\right)$ have storage functions V_i. We have the following result:

Theorem 1: If the subsystems (u_1^A, y_1^A) and (u_2^A, y_2^A) are ASD and

$$\nabla V_i B_i(x_i) = C_i^T(x_i); i = 1,2 \tag{13}$$

then the resulting feedback connection, $y_1 = u_2$ and $u_1 = -y_2$, will leave the composite system $\left(u_A \equiv \begin{bmatrix} u_1^A \\ u_2^A \end{bmatrix}, y_A \equiv \begin{bmatrix} y_1^A \\ y_2^A \end{bmatrix}\right)$ almost strictly passive.

Proof:

Let (u_i^A, y_i^A) be ASP. From (9) and (11),

$\exists G_i^*$ such that

$$\begin{cases} \nabla V_i A_i^C(x_i) \equiv \nabla V_i[A_i(x_i) + B_i^A(x_i)G_i^* C_i^A(x_i)] \\ \qquad\qquad \le -S_i(x_i) + \varepsilon_{ij}\nabla V_i A_{ij}(x_i, u_i, u_i^A) \\ \nabla V_i B_i^A(x_i) = C_i^A(x_i)^T \end{cases} \tag{14}$$

If we connect (u_1, y_1) in feedback with (u_2, y_2), then $y_1 = u_2$ and $u_1 = -y_2$ and, use (12) and (13), then we have $\nabla V_1 A_{12}(x_1, u_1, u_1^A) = \nabla V_1 B_1(x_1)u_1 = C_1^T(x_1)[-y_2] = -y_1^T y_2$ and similarly, $\nabla V_2 A_{21}(x_2, u_2, u_2^A) = y_2^T y_1$.

Let $x \equiv \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ and, from (12),

$$\begin{cases} \dot{x} = A(x) + B(x)u \\ \quad = \begin{bmatrix} A_1^C(x_1) + \varepsilon_{12}A_{12}(x_2) \\ A_2^C(x_2) + \varepsilon_{21}A_{21}(x_1) \end{bmatrix} + \begin{bmatrix} B_1^A(x_1) & 0 \\ 0 & B_2^A(x_2) \end{bmatrix}\begin{bmatrix} u_1^A \\ u_2^A \end{bmatrix} \\ y = \begin{bmatrix} y_1^A \\ y_2^A \end{bmatrix} = C(x) = \begin{bmatrix} C_1^A(x_1) \\ C_2^A(x_2) \end{bmatrix} \end{cases} \tag{15}$$

with $V = V_1 + V_2$, using (13) and $\varepsilon_{ji} = \varepsilon_{ij}$ from (3),

$$\begin{aligned} \nabla VA(x) &= \begin{bmatrix} \nabla V_1 & \nabla V_2 \end{bmatrix}\begin{bmatrix} A_1^C(x_1) + \varepsilon_{12}A_{12}(x_2) \\ A_2^C(x_2) + \varepsilon_{21}A_{21}(x_1) \end{bmatrix} \\ &= \nabla V_1 A_1(x_1) + \varepsilon_{12}(-y_1^T y_2) + \nabla V_2 A_2(x_2) + \varepsilon_{21}(y_2^T y_1) \\ &\le -[S_1(x_1) + S_2(x_2)] + \varepsilon_{21}(-y_1^T y_2) + \varepsilon_{21}(y_2^T y_1) \\ &= -S(x) \end{aligned}$$

and

$$\begin{aligned} \nabla VB(x) &= [\nabla V_1 \ \nabla V_2]\begin{bmatrix} B_1^A(x_1) & 0 \\ 0 & B_2^A(x_2) \end{bmatrix} \\ &= \begin{bmatrix} C_1^A(x_1) \\ C_2^A(x_2) \end{bmatrix}^T = C^T(x) \end{aligned}$$

Therefore $\left(u_A \equiv \begin{bmatrix} u_1^A \\ u_2^A \end{bmatrix}, y_A \equiv \begin{bmatrix} y_1^A \\ y_2^A \end{bmatrix}\right)$ is ASD with output feedback $\begin{bmatrix} u_1^A \\ u_2^A \end{bmatrix} \equiv \begin{bmatrix} G_1^* & 0 \\ 0 & G_2^* \end{bmatrix}\begin{bmatrix} y_1^A \\ y_2^A \end{bmatrix} + \begin{bmatrix} u_1^{Ar} \\ u_2^{Ar} \end{bmatrix}$ as desired. #

In [3], it was shown that the physical connection of components is equivalent to the feedback connection of the admittance of one to the impedance of the other. Consequently, if (u_1, y_1) and (u_2, y_2) are in Admittance/Impedance form, then Theorem 1 shows that ASD is an inherited property for Nonlinear Evolving Systems. Theorem 1 was also proved for the stronger Almost Strict Passivity in [8].

This is one case where the interconnections have a form that allows ASD to be inherited. Other possibilities will be investigated elsewhere.

MATHEMATICAL FORMULATION OF ADAPTIVE KEY COMPONENT CONTROLLER

We will consider an Evolving System, $(A(x), B(x), C(x))$, consisting of two components:

$$\begin{cases} \dot{x}_1 = A_1(x_1) + B_1(x_1)u_1 + B_1^A(x_1)u_1^A \\ y_1 = C_1(x_1) \\ y_1^A = C_1^A(x_1) \end{cases} \tag{16}$$

and

$$\begin{cases} \dot{x}_2 = A_2(x_2) + B_2(x_2)u_2 \\ y_2 = C_2(x_2) \end{cases} \tag{17}$$

Without loss of generality, we can let component 1 be the key component since the system can be rewritten to switch component 1 with component 2. Also, we may think of Component 2 as all the rest of the Evolving System to which the Key Component and its Adaptive Controller will be connected.

The Adaptive Key Component Controller on Component 1 will be given by:

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$$\begin{cases} u_1^A = G_1 y_1^A \\ \dot{G}_1 = -y_1^A (y_1^A)^T h_1; h_1 > 0 \end{cases} \tag{18}$$

This controller uses only the input and output ports (u_1^A, y_1^A) on Component 1.

Theorem 2: Assume that V_1 and V_2 are positive $\forall x \neq 0$ and radially unbounded, and $\left(A(x), B(x), C(x)\right)$ are continuous functions of x and $S(x)$, above, is positive $\forall x \neq 0$ and has continuous partial derivatives in x. Furthermore, assume:

a) Component 2 (u_2, y_2) is strictly dissipative and in impedance form;
b) Component 1 (u_1^A, y_1^A) is almost strictly dissipative;
c) Component 1 (u_1, y_1) is in admittance form

Then the Adaptive Key Component Controller (18) produces global asymptotic state stability $x \equiv \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \xrightarrow[t\to\infty]{} 0$ with bounded adaptive gains when Component 1 is joined with Component 2 into an Evolved System and the outputs $y_i = C_i(x_i) \xrightarrow[t\to\infty]{} 0$.

Proof: Since the physical connection of Component 1 to Component 2 is equivalent to the feedback connection $u_1 = -y_2$ and $u_2 = y_2$, by Theo.1 we have that the closed-loop system (u_1^A, y_1^A) below is ASD:

$$\begin{cases} \dot{x}_1 = A_1(x_1) - \varepsilon B_1(x_1) C_2(x_2) + B_1^A(x_1) u_1^A \\ \dot{x}_2 = A_2(x_2) + \varepsilon B_2(x_2) C_1(x_1); 0 \le \varepsilon \le 1 \\ y_1^A = C_1^A(x_1) \end{cases} \tag{19}$$

Rewrite (18), using G_1^* constant,

$$\begin{cases} u_1^A = G_1 y_1^A = G_1^* y_1^A + \Delta G_1 y_1^A; \Delta G_1 \equiv G_1 - G_1^* \\ \Delta \dot{G}_1 = \dot{G}_1 = -y_1^A (y_1^A)^T h_1; h_1 > 0 \end{cases} \tag{20}$$

Combining (19) and (20) yields:

$$\begin{cases} \dot{x}_1 = A_1^C(x_1) - \varepsilon B_1(x_1) C_2(x_2) + B_1^A(x_1) w_1^A \\ \qquad \text{with } w_1^A \equiv \Delta G_1 y_1^A \\ \qquad \text{and } A_1^C(x_1) \equiv A_i(x_i) + B_i^A(x_i) G_i^* C_i^A(x_i) \\ \dot{x}_2 = A_2(x_2) + \varepsilon B_2(x_2) C_1(x_1); 0 \le \varepsilon \le 1 \\ y_1^A = C_1^A(x_1) \end{cases} \tag{21}$$

Let $V = V_1 + V_2$ and we have:

$$\dot{V} = -S(x) + \left\langle y_1^A, w_1^A \right\rangle \tag{22}$$

Form $V_G \equiv \frac{1}{2} tr(\Delta G_1 h_1^{-1} \Delta G_1^T)$ and obtain from (20):

$$\begin{aligned} \dot{V}_G &\equiv tr(\Delta \dot{G}_1 h_1^{-1} \Delta G_1^T) \\ &= -tr(y_1^A (y_1^A)^T h_1 h_1^{-1} \Delta G_1^T) \\ &= -tr(y_1^A (w_1^A)^T) \\ &= -\left\langle y_1^A, w_1^A \right\rangle \end{aligned} \tag{23}$$

Define: $V(x, \Delta G) \equiv V(x) + V_G(\Delta G)$ and, from (22) and (23), we have:

$$\begin{aligned} \dot{V}(x, \Delta G) &\equiv \dot{V}(x) + \dot{V}_G(\Delta G) \\ &= -S(x) + \left\langle y_1^A, w_1^A \right\rangle - \left\langle y_1^A, w_1^A \right\rangle \\ &= -S(x) \le 0 \end{aligned} \tag{24}$$

This guarantees that all trajectories $(x, \Delta G)$ are bounded. If $\dot{V}(x, \Delta G)$ is uniformly continuous or $\ddot{V}(x, \Delta G)$ is bounded, then Barbalat's Lemma [9] yields: $S(x) \xrightarrow[t\to\infty]{} 0$, and the positivity and continuity of $S(x)$ imply that $x \equiv \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \xrightarrow[t\to\infty]{} 0$.

Consider

$$\begin{aligned} \ddot{V}(x, \Delta G) &= -\dot{S}(x) \\ &\le \left| \dot{S}(x) \right| \\ &= \left| \frac{\partial S(x)}{\partial x} \dot{x} \right| \\ &\le \left\| \frac{\partial S(x)}{\partial x} \right\| \left\| \dot{x} \right\| \\ &\le \left\| \frac{\partial S(x)}{\partial x} \right\| \left[\left\| A(x) \right\| + \left\| B(x) \right\| \left\| w_1^A \right\| \right] \\ &\le \left\| \frac{\partial S(x)}{\partial x} \right\| \left[\left\| A(x) \right\| + \left\| B(x) \right\| \left\| \Delta G_1 \right\| \left\| C_1^A(x_1) \right\| \right] \end{aligned}$$

which is bounded because $(x, \Delta G)$ is bounded, $S(x)$ has continuous partial derivatives and $\left(A(x), B(x), C(x)\right)$ are continuous, and a continuous function of bounded $x(t)$ is also bounded in t.

So, $y_i = C_i(x_i) \xrightarrow[t\to\infty]{} 0$ because $C_i(x_i)$ is continuous. #

It should be noted that the above results might only hold on a neighborhood $N_i(0, r_i) \equiv \left\{ x_i / \left\| x_i \right\| < r_i \right\}$. However, then the stability in Theo. 2 is only locally asymptotic to the origin.

SPECIAL CASE: INTERNAL STATE NONLINEARITY

Here we look at a special case of the above theory when the only nonlinearity is in the internal state structure of each component. This means (12) becomes:

$$\begin{cases} \dot{x}_i = A_i x_i + h(x_i) + B_i u_i + B_i^A u_i^A \\ y_i = C_i x_i \\ y_i^A = C_i^A x_i \end{cases} \tag{25}$$

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The nonlinearities are assumed to satisfy $h_i(0)=0$ and a Lipschitz continuity condition:

$$\begin{aligned}&\|h_i(x)-h_i(y)\| \le \mu_i\|x-y\| \\ &\forall x,y \text{ with } \mu_i>0\end{aligned} \tag{26}$$

We can choose quadratic Storage Functions: $V(x_i)\equiv\frac{1}{2}x_i^T P_i x_i$. We will assume the linear part of each component (A_i,B_i^A,C_i^A) is Almost Strict Positive Real (ASPR), i.e. $\exists G_i^* \ni A_i^C \equiv A_i + B_i G_i^* C_i$ satisfies:

$$\begin{cases}(A_i^C)^T P_i + P_i A_i^C = -Q_i \\ P_i B_i^A = (C_i^A)^T\end{cases} \tag{27}$$

with $P_i>0,\ Q_i>0$. In addition, we will assume:

$$P_i B_i = C_i^T \tag{28}$$

with the same $P_i>0$. Then, from (27)-(28),

$$\begin{cases}\nabla V_i A_i^C(x_i) = x_i^T P_i[(A_i + B_i^A G_i^* C_i^A)x_i + h_i(x_i) + B_i u_i] \\ \qquad = -\frac{1}{2}x_i^T Q_i x_i + x_i^T P_i h_i(x_i) + y_i^T u_i \\ \nabla V_i B_i^A(x_i) = x_i^T P_i B_i^A = (C_i^A x_i)^T\end{cases} \tag{29}$$

Now, from (26) and the Cauchy-Schwarz inequality, we have

$$\begin{aligned}-\frac{1}{2}x_i^T Q_i x_i + x_i^T P_i h_i(x_i) &\le -\left(\frac{\lambda_{\min}(Q_i)}{2} - \lambda_{\max}(P_i)\mu_i\right)x_i^T x_i \\ &= -\gamma_i\|x_i\|^2 \equiv -S_i(x_i)\end{aligned}$$

Therefore (29) becomes

$$\begin{cases}\nabla V_i A_i^C(x_i) = x_i^T P_i[(A_i + B_i^A G_i^* C_i^A)x_i + \cdots \\ \qquad + h_i(x_i) + B_i u_i] \\ \qquad \le -S_i(x_i) + \langle y_i, u_i\rangle \\ \nabla V_i B_i^A(x_i) = x_i^T P_i B_i^A = (C_i^A x_i)^T \\ \text{with } S_i(x_i) \equiv \gamma_i\|x_i\|^2 > 0\ \forall x_i \ne 0 \\ \text{when } \gamma_i \equiv \frac{\lambda_{\min}(Q_i)}{2} - \lambda_{\max}(P_i)\mu_i > 0\end{cases} \tag{30}$$

From this analysis, we have the following result:

Theorem 3: If the linear part, (A_i,B_i^A,C_i^A), of (25) is ASPR, then (28) holds, and the Lipschitz constant in (26) satisfies:

$$0<\mu_i<\frac{\lambda_{\min}(Q_i)}{2\lambda_{\max}(P_i)} \tag{31}$$

Then, when Component 1 is joined with Component 2, the Adaptive Key Component controller (18) will maintain closed-loop stability.

Proof: From (31), we have (30) and can apply Theo. 2 to achieve the result. #

It is well known that (27) is equivalent to $C_i B_i > 0$ and the open-loop transfer functions $T_i(\mathrm{s}) \equiv C_i^A(sI - A_i)^{-1}B_i$ are minimum phase.

Also, the above results need only hold on a neighborhood $N_i(0,r)$ and (31) will hold when the size of the neighborhood r is small enough. However, then the stability is only locally asymptotic.

NONLINEAR ILLUSTRATIVE EXAMPLE

Example 1, which is shown in Fig. 2, is a two component nonlinear flexible structure Evolving System. The components of Example 1 are stable when they are unconnected components, but the Evolving System fails to inherit the stability of the components. This example will be used to demonstrate the successful use of an adaptive key component controller to restore stability.

The dynamical equations for the components of Example 1 are:

$$\text{comp. 1}: \begin{cases}m_1\ddot{q}_1 = u_1 - \varepsilon_{12}k_{12}(q_1-q_2) \\ y_1 = [q_1,\dot{q}_1]^{\mathrm{T}}\end{cases}$$

$$\text{comp. 2}: \begin{cases}m_2\ddot{q}_2 = u_2 - \varepsilon_{12}k_{12}(q_2-q_1) - \cdots \\ \qquad -[k_{22}(q_2-q_3) + \mu_{22}\sin(q_2-q_3)] \\ m_3\ddot{q}_3 = u_3 - [k_{22}(q_3-q_2) + \mu_{22}\sin(q_3-q_2)] \\ y_2 = [q_2,\dot{q}_2]^{\mathrm{T}} \\ y_3 = [q_3,\dot{q}_3]^{\mathrm{T}}\end{cases} \tag{17}$$

with $m_1=30$, $m_2=1$, $m_3=1$, $k_{12}=4$, $k_{22}=1$ and $\mu_{22}=0.5$. Example 1 has the following controllers:

$$\begin{cases}u_1 = -(0.9s+0.1)q_1 \\ u_2 = -\left(\frac{0.1}{s}+0.2s+0.5\right)q_2 \\ u_3 = -(0.6s+1)q_3\end{cases} \tag{18}$$

The subsystem components are stable in closed-loop form when they are unconnected, i.e., $\varepsilon_{12}=0$. When $\varepsilon_{12}=1$, the system is fully evolved and it is unstable as seen in Fig. 3.

A Simulink model was created to implement an adaptive key component controller for Example 1 as described in the previous section. Simulations were run in which the connection parameter, ε_{12}, ranged from 0 to 1, allowing the system to go from unconnected components to a fully Evolved System. The key component controller was able to maintain system stability during the entire evolution process when it used the input-output ports on mass 1 of component 1, see Fig. 3. When component 1 was the key component, $(\overline{A},\overline{B},\overline{C})$ is ASD.

CONCLUSION

We have presented a result (Theo 1) describing when a Nonlinear Evolving System will inherit the almost strict

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dissipativity trait of its subsystem components. In Theo.2 we show an adaptive key component controller that will guarantee that stability is inherited by the Evolved System, and a special case is considered in Theo.3 where only internal state nonlinearity is present. A simple nonlinear example was given of successful inheritance of almost strict dissipativity .

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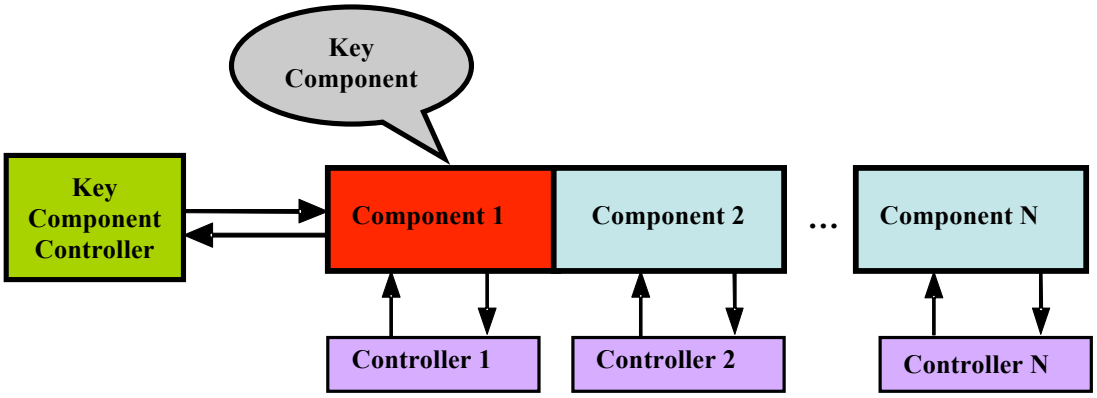

Figure 1. Key component controller architecture.

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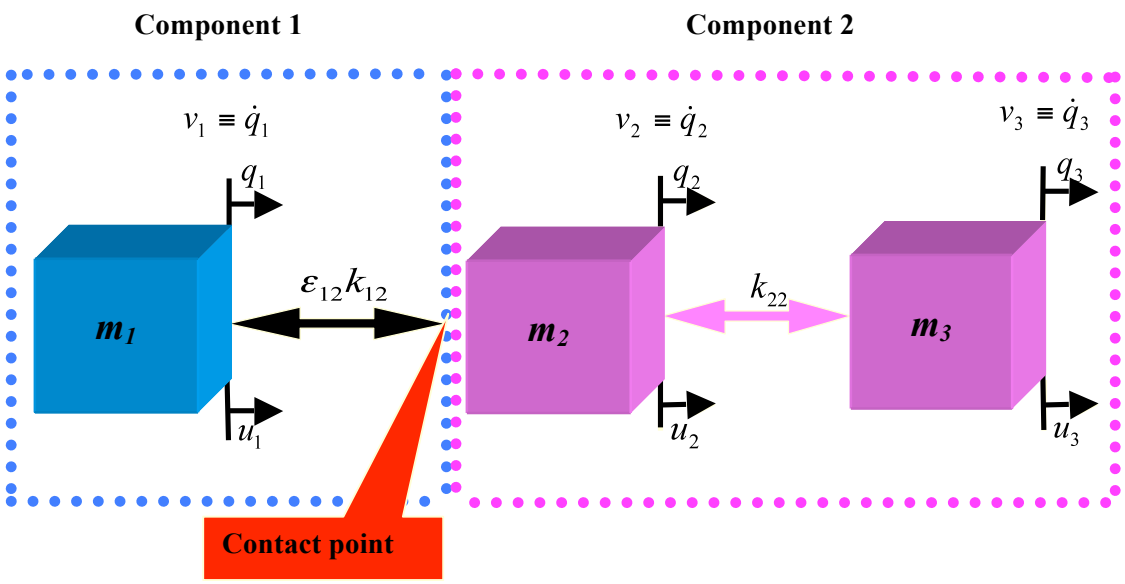

Figure 2. Example 1: A two component flexible structure Evolving System.

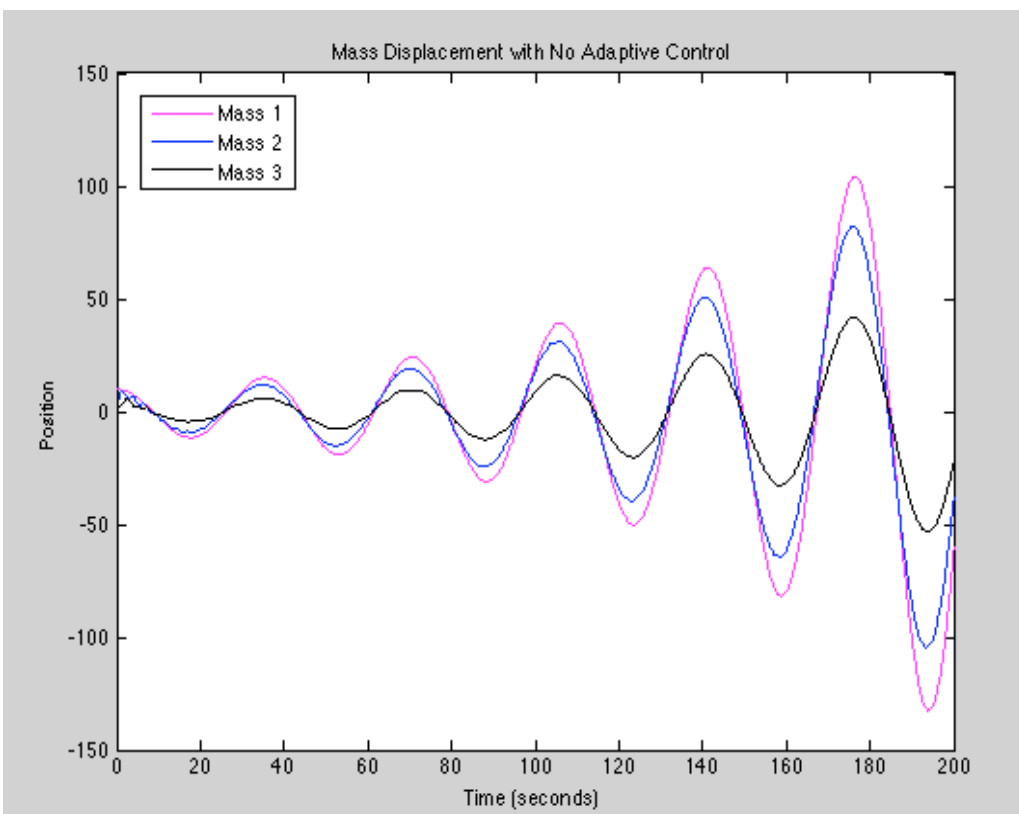

Figure 3. Ex. 1 after evolution with no adaptive key component controller.

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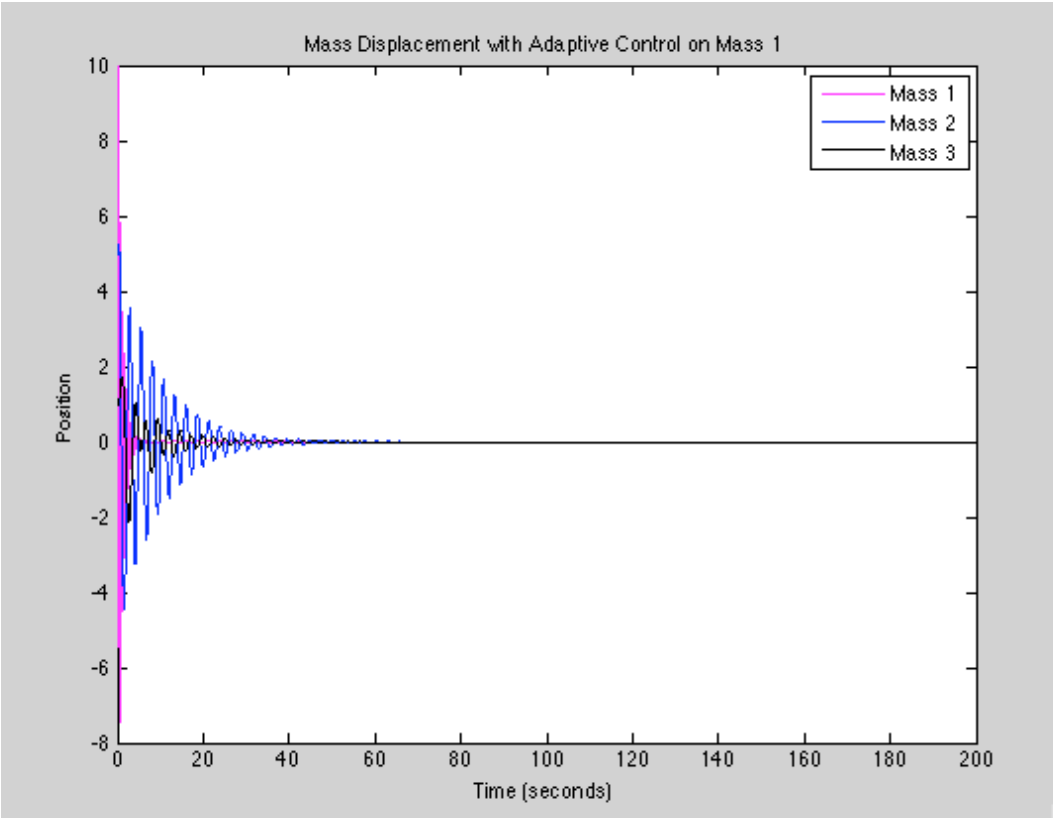

Figure 4. Ex. 1 with adaptive key component controller on mass 1.

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