Inferring Spectra from Interferograms:
Application to the Multiplexing Fourier Transform Spectrometer

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It is well-known that interferometers are sensitive devices for inferring spectra when the incoming radiation is narrowband. Conversely, interferometers lack sensitivity for detecting broadband radiation. In order to boost the sensitivity of Fourier Spectrometers, USNO has constructed a multiplexed Fourier Transform Spectrometer that takes incoming starlight, passes it through an interferometer with a variable delay line, and disperses the recombined white light fringe with a diffraction grating onto a CCD detector. By varying the delay line over time, the data provided by each pixel on the detector is a series of juxtaposed narrowband interferograms. The result of acquiring numerous narrowband interferograms is that the spectrum inferred from these data is a factor of \(R_g\) (i.e. the resolving power of the grating) more sensitive than an equivalent interferometer without a grating. In effect, the sensitivity boost imparted by our innovation is equivalent to the sensitivity increase enjoyed by conventional grating spectrometers when they were upgraded with multipixel CCD detectors (i.e. the entire integration time is spent integrating on all colors). One way to consider the effect of adding a grating spectrometer to a Fourier Transform Spectrometer is to note that the interferogram consists of a central fringe packet with a width of \(R_g\) wavelengths, and that increasing \(R_g\) simply increases the portion of the interferogram that contains significant and meaningful signal. From the standpoint of the spectrum, the noise in any Fourier spectrum is a constant proportional to the total number of photons in the interferogram. By partitioning the entire optical spectrum into smaller wavelength portions and by properly combining the narrowband spectra, the out-of-band noise can be effectively filtered out of the final spectrum.

In order for this procedure to succeed, it is critical that the narrowband spectra be properly estimated from the interferogram data, and combined in an optimal way so as to minimize additional noise contributions. The basic forward problem is straightforward since the interferogram is related to the spectrum via the inverse cosine transform. Since spectra are continuous, we can obtain results that more accurate than those obtained with the discrete Fourier transform (DFT) by modeling the spectrum as a linear-piecewise function, and using Bayesian methods to derive the posterior probability of the narrowband spectral model given the narrowband interferogram. A straightforward algorithm, based on Newton’s method is developed, which finds the maximum a posteriori narrowband spectrum in addition to the position of the central fringe. By juxtaposing the narrowband results, the majority of the spectrum is readily reconstructed. This methodology is extremely useful since we can use the accuracy of the analysis results at any time to decide which future measurements to make. In addition, the explicit description of the forward problem will allow us to better model the instrument and simultaneously calibrate the instrument and acquire data.

We will show data interferograms and inferred spectra acquired from stellar targets acquired with the 25” Clay Center Observatory telescope at the Dexter and Southfield Schools in Brookline Massachusetts, USA.