Elliptic Grid Generation of Spiral-Bevel Pinion Gear Typical of OH-58 Helicopter Transmission

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Abstract

This paper discusses the source term treatment in the numerical solution of elliptic partial differential equations for an interior grid generation problem in generalized curvilinear coordinates. The geometry considered is that of a planar cross-section of a generic spiral-bevel gear tooth typical of a pinion in the OH-58 helicopter transmission. The source terms used are appropriate for an interior grid domain where all the boundaries are prescribed via a combination of Dirichlet and Neumann boundary conditions.

New constraints based on the Green's Theorem are derived which uniquely determine the coefficients in the source terms[1]. These constraints are designed for boundary clustered grids where gradients in physical quantities need to be resolved adequately. However, it is seen that the present formulation works satisfactorily for mild clustering also. Thus, a fully automated elliptic grid generation technique is made possible where there is no need for a parametric study of these parameters since the new relations fix these free parameters uniquely.

Keywords:

Elliptic Grid Generation; Spiral-bevel pinion gear.

1 Introduction

There has been a large amount of effort devoted to developing and enhancing the grid generation capability [1,2,3,4] through the solution of elliptic partial differential equations (pdes). The elliptic pdes used in the grid generation problems near boundaries are similar to the equations used in nuclear physics, diffusion-reaction problems, vortex problems, electric space charge problems, steady state heat transfer (conduction and convection) through long thin fins, etc. In the grid generation problems, these pdes contain appropriate source terms that control the distribution of grid points especially near the boundaries. In the literature, the elliptic pdes used for grid generation are erroneously referred to as Poisson equations which contain source terms that are functions of only the independent variables, whereas, in the pdes for grid generation,

[1] Invention under review for NASA Patent

these inhomogeneous terms also contain terms proportional to the dependent variables. Actually, in grid generation problems, close to a curvilinear boundary, the governing equations reduce to the long thin fin heat transfer equations with a finite heat transfer coefficient in the transverse direction (normal to the plane of paper) and a large heat transfer coefficient in the lateral direction.

The focus in the studies referred to above has been on developing body conforming grids around bodies for external fluid flow simulations. The grids thus generated are smooth with at least first two derivatives continuous, appropriately stretched or clustered normal to any given coordinate direction and orthogonal over most of the grid domain. The inhomogeneous terms afford a grid control to satisfy clustering and orthogonality around specific surfaces (in three dimensions) and lines (in two dimensions).

In external flows, these inhomogeneous terms, i.e., the source terms and the dependent variable proportional terms are designed to vanish away from the body so the problem reduces to solving a Laplacian away from the body.

In the present study, the inhomogeneous terms used are appropriate for an interior grid generation problem where all the boundaries enveloping the grid will affect the solution through these terms. These terms are designed by interpreting their meaning physically through the principle of conservation of thermal energy close to the grid boundaries.

The geometry treated here is that of a planar cross-section of a spiral-bevel pinion gear tooth typical of the OH-58 helicopter transmission pinion. This study is driven by the need to generate time-series vibration signatures from the OH-58 helicopter transmission by finite difference simulation of the appropriate structural dynamic equations. The choice of elliptic pdes for grid generation is entailed by the need to generate time series data as accurately as possible (see relative comparison with other representative grid generation methods in Ref. 5).

2 Problem Definition

The two-dimensional governing equations for an elliptic grid generation problem in an appropriately defined planar domain are [1,2]

$$\xi_{xx} + \xi_{yy} = P(\xi,\eta)$$

$$\eta_{xx} + \eta_{yy} = Q(\xi,\eta)$$

where ξ and η are the generalized curvilinear coordinates, x and y are the Cartesian coordinates, and the $P(\xi,\eta)$ and $Q(\xi,\eta)$ are the inhomogeneous terms.

The form of the inhomogeneous terms, P and Q, is, e.g., exponential [2] and is given by

$$P(\xi,\eta) = -a_i(\eta)sgn(\xi - \xi_i)exp(-b_i|\xi - \xi_i|) \tag{1a}$$

$$Q(\xi,\eta) = -c_i(\xi)sgn(\eta - \eta_i)exp(-d_i|\eta - \eta_i|) \tag{1b}$$

where i refers to the grid boundary in question.

For the sake of argument, without loss of generality, if we take the case where $\xi > \xi_i$ and $\eta > \eta_i$, then we have the inhomogeneous terms as

$$P(\xi,\eta) = -a_i(\eta)exp(-b_i(\xi - \xi_i)) \tag{1c}$$

$$Q(\xi,\eta) = -c_i(\xi)exp(-d_i(\eta - \eta_i)) \tag{1d}$$

At the boundaries, where $\xi = \xi_i$ and $\eta = \eta_i$, Equations (1c) and (1d) respectively become

$$P(\xi_i,\eta) = -a_i(\eta)$$

and

$$Q(\xi, \eta_i) = -c_i(\xi)$$

When $b_i|\xi - \xi_i|$ or $d_i|\eta - \eta_i|$ is small, the inhomogeneous terms take the form given by

$$P(\xi, \eta) = -a_i(\eta)(1 - b_i(\xi - \xi_i))$$

and

$$Q(\xi, \eta) = -c_i(\xi)(1 - d_i(\eta - \eta_i))$$

Therefore, the governing equation for, e.g., ξ, in the vicinity of the boundary ξ_i, becomes

$$\xi_{xx} + \xi_{yy} = -a_i(\eta)(1 - b_i(\xi - \xi_i))$$

or

$$\xi_{xx} + \xi_{yy} - a_i(\eta)b_i\xi = -a_i(\eta) - a_i(\eta)b_i\xi_i \tag{2}$$

If the term, $a_ib_i\xi$, were absent, the resulting equation would turn out to be a Poisson equation. The equation given above arises, e.g., in the steady state heat conduction problems in long thin fins, where ξ is the temperature and where the heat transfer coefficient in the transverse thin direction is moderate but is large in the lateral direction, and there is a balance amongst the heat conducted through the fin, heat carried away from or to it through convection in proportion to this moderate heat transfer coefficient and the heat sources/sinks distributed over the domain.

If we define a new variable

$$\theta = \xi - \xi_i$$

then Equation(2) becomes

$$\theta_{xx} + \theta_{yy} - a_ib_i\theta = -a_i \tag{3}$$

The term, $-a_i$, can be thought of as a heat source/sink term.

Equation(3) tells us that when $\xi > \xi_i$, there is a balance between the heat convected from a control volume in the interior to the boundary ξ_i, heat conducted out of this control volume and the heat lost from the control volume due to the heat sink, $a_i(\eta)$. Conversely, when $\xi < \xi_i$, there is a balance between the heat convected from the boundary ξ_i to a control volume in the interior, heat conducted out of the control volume and the heat generated in the control volume due to the source, $a_i(\eta)$.

From Equation(3), it can be seen that for a given convective heat flux (given number of grid lines), as the product, a_ib_i decreases, the heat transfer coefficient decreases proportionally in magnitude which means that the temperature gradient at the boundary ξ_i has increased so that ξ approaches ξ_i rapidly. This means that there is a large gradient in ξ from the grid boundary i to the interior, thereby resulting in a highly clustered grid near the boundary.

Similarly, if we consider the case when $\xi_i > \xi$, then we have

$$\xi_{xx} + \xi_{yy} - a_i(\eta)b_i\xi = -a_i(\eta) - a_i(\eta)b_i\xi_i$$

or,

$$\theta_{xx} + \theta_{yy} - a_i(\eta)b_i\theta = a_i(\eta)$$

where $\theta = \xi_i - \xi$

Away from this grid boundary, $b_i|\xi - \xi_i|$ or $b_i|\eta - \eta_i|$ is large, and we are left with the Laplace equation, $\Delta\xi = 0$ or $\Delta\eta = 0$. Extremum principle is unconditionally maintained there, since the solution is harmonic in this case.

Referring to Equation(3), the Green's Theorem gives us

$$\int\int_S (-a_i + a_ib_i\theta)d\sigma = \int_C \partial_n\theta ds \tag{4}$$

where S is the surface area of a closed domain, C is the boundary enclosing this domain, n is the normal to the surface, $d\sigma$ is the elemental area and ds is an elemental arc.

The integrands on the left hand side, $-a_i$ and $a_i b_i \theta$ represent the heat source/sink term and the convection term respectively, and the integrand on the right hand side represents the heat flux through the boundary C.

Equation(4) is used as a constraint to fix b_i uniquely for a solution consistent with the specification of the boundary data. The extremum principle will be satisfied at the ith boundary, which is the requirement in the grid generation problems, since the energy conservation principle is satisfied. The term, $-a_i(\eta)$, is calculated iteratively through the solution process, which together with b_i ensures the grid orthogonality and a given grid spacing at the ith grid boundary.

In the design of these inhomogeneous terms, there is no restriction on the nature of the source term, a_i. It can change sign which indicates the presence of sources and sinks, subject to the constraint given by Equation(4). Otherwise, improper combinations of sources and sinks will violate the extremum principle. If over the domain, there is a net rate of heat generation due to the source/sink combination, then there has to be a positive heat flux convected away and vice-versa. This requirement will automatically be satisfied by Equation(4).

If there is a point heat source present in the domain, the isothermals (temperature contour lines) will tend to cluster around it since the gradients in the vicinity of the source will be positive toward the source and high, depending upon the strength of the source, and conversely for a heat sink. Same argument applies to a line heat source and sink. By analogy, if the source term turns out to be positive over some parts of the domain, then the curvilinear coordinate lines will tend towards lines with higher coordinate values and vice-versa.

3 Solution Procedure

First, the boundary data are selected appropriate to the physics of the problem, so that the gradients in physical quantities can be resolved adequately. Since there is a symmetry plane and a rotational symmetry present in the present problem, the grid is reflected about this symmetry plane and then rotated around completely about a moving axis of periodicity, thus substantially reducing the computational effort in generating the gear tooth grid.

Then, by interchanging the independent and dependent variables, the governing equations to be solved in the computational space (ξ, η) become

$$\alpha x_{\xi\xi} - 2\beta x_{\xi\eta} + \gamma x_{\eta\eta} = -J^2(P(\xi,\eta)x_\xi + Q(\xi,\eta)x_\eta)$$

$$\alpha y_{\xi\xi} - 2\beta y_{\xi\eta} + \gamma y_{\eta\eta} = -J^2(P(\xi,\eta)y_\xi + Q(\xi,\eta)y_\eta)$$

These equations are solved in the computational space using a line SOR relaxation algorithm where each coordinate line in one curvilinear coordinate direction is solved semi-implicitly using the Thomas algorithm for tri-diagonal systems. The inhomogeneous terms referred to above are designed and incorporated so that a desired grid behavior near the boundaries is achieved [6].

The inhomogeneous problem is solved using a technique similar to that of Ref. 3 by over-relaxing the inhomogeneous terms during the iteration process. The inhomogeneous terms used in Ref. 3 are well suited for external boundary value problems where they allow for clustering in only one curvilinear coordinate direction, normal to the body. But, in internal boundary value problems, inhomogeneous terms have to take account of the influence of the boundaries in both curvilinear coordinate directions. The inhomogeneous terms used here allow for clustering in both coordinate directions.

The inhomogeneous terms, $P(\xi,\eta)$ and $Q(\xi,\eta)$, are evaluated at the boundaries in terms of the left hand side at each line relaxation sweep. Then outward from each boundary, the inhomogeneous terms are attenuated through an exponential function in each direction, as discussed above. In ξ direction, outward from a given ξ_i boundary, this exponential term is of the form, $-a_i exp(-b_i|\xi - \xi_i|)$, and, in η direction, it is of the form, $-a_i exp(-b_i|\eta - \eta_i|)$.

The boundary constraint given by Equation (4) is applied to a finite slender strip by evaluating the heat source/sink term and the convective flux term over the strip close to the boundary with the heat flux calculated around the strip.

4 Results

Figure 1 shows a finite-difference grid model of pinion and driven gears in mesh. The grids were generated automatically without any manual prescription of the free parameters.

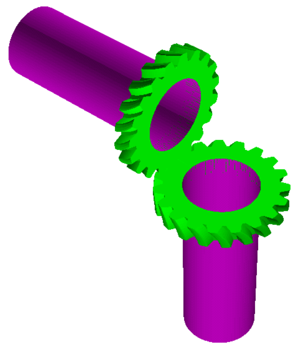


Figure 1: Finite difference grid model of pinion and driven gears in mesh

Figure 2 below shows a close-up of the pinion gear and shaft.

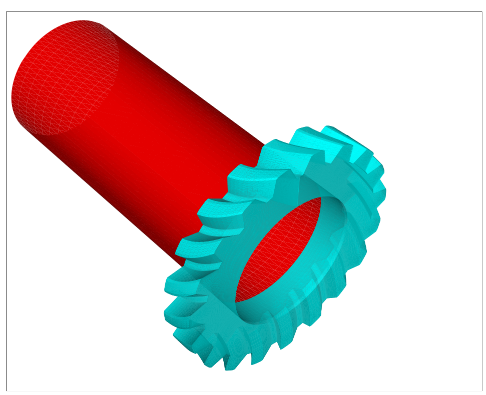


Figure 2: Close-up view of the pinion gear and shaft

The spiral-bevel nineteen tooth pinion gear typical of a OH58 helicopter is shown in Fig. 3 and the pinion shaft is shown in Fig. 4 below.

Figure 3: Finite difference grid of the 19-tooth spiral-bevel pinion gear

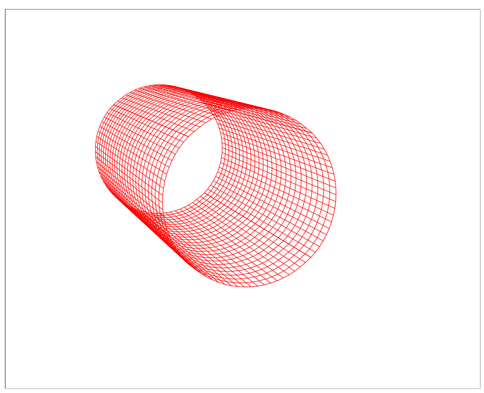


Figure 4: Finite difference grid of the pinion shaft

In Fig. 5 below, a cross-section of the pinion gear is shown. In Fig. 6, the corresponding cross-sectional grids for an individual pinion gear tooth and pinion shaft are shown. The tooth grid shows a desired clustering near and around the tooth.

As is the case with generalized curvilinear coordinates, the cross-sectional grid for the shaft does not contain any geometric singularities. The polar coordinate singularity at the origin has been removed and manifests itself through the four triangular grid cells at the boundary as is seen in Fig. 6(b) below.

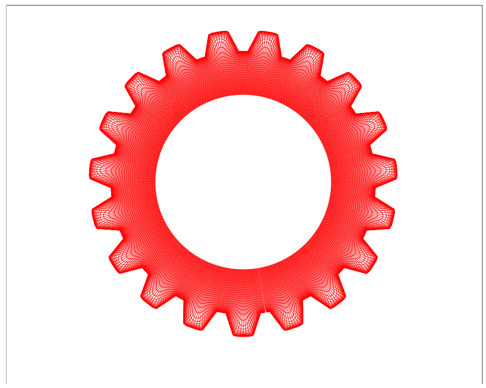
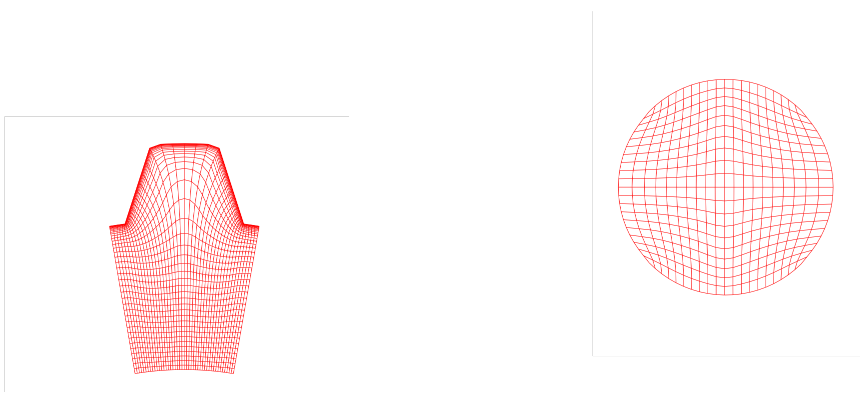


Figure 5: Cross-section of the pinion gear



(a) (b)

Figure 6: Cross-sectional grids (a) pinion gear tooth (b) pinion shaft

5 Concluding Remarks

The boundary constraints for elliptic grid generation problems developed in this study have been demonstrated to be applicable to a practical problem of gear teeth grid generation. Smooth clustered grids have been generated using these constraints without any recourse to redistribution of grid points which has been a common approach used in elliptic grid generation problems until now. With new constraints, elliptic grids can be generated in simulation time without any manual intervention thus making problems of structural dynamics and fluid dynamics over compliant boundaries straightforward.

6 REFERENCES

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