HYBRID DAMAGE ADAPTIVE FLIGHT CONTROL WITH MODEL INVERSION ADAPTATION

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Abstract: This paper presents a hybrid neural net adaptive control method for damaged aircraft. The hybrid method utilizes an adaptive scheme that adjusts a dynamic inversion control model to account for damaged plant dynamics. The model inversion adaptive scheme is based on two approaches: 1) an indirect adaptive law based on the Lyapunov theory, and 2) a recursive least squares method for parameter estimation of damaged plant dynamics. Simulations show that the hybrid adaptive control can provide a significant improvement in the tracking performance over the direct adaptive control.

Keywords: Adaptive control, Flight control, Neural networks, Recursive least squares.

1. INTRODUCTION

While air travel remains the safest mode of transportation, accidents do occur on rare occasions. American Airlines Flight 587 illustrates the reality of hazards due to structural failures of airframe components (National Transportation Safety Board, 2004). Recently, the DHL incident involving an Airbus A300-B4 cargo aircraft in 2003 illustrates the ability to maintain controlled flight in the presence of structural damage and hydraulic loss (Lemaignan, 2005). In a damage event, aircraft may experience a loss of lift, changes in aerodynamic characteristics and mass properties, and other effects that can manifest in an unstable, non-equilibrium flight. Reduced structural rigidity of a damaged airframe may cause elastic motions that can interfere with a flight control system or impose unknown load constraints on pilot commands. Under off-nominal flight conditions, a flight control system needs to be able to adapt to changes in aircraft dynamics due to damage.

Over the past several years, various adaptive control methods have been investigated. Adaptive flight control provides a possibility for enhancing aircraft stability and performance by enabling a flight control system to adapt to system uncertainties. Adaptive control laws may be divided into direct and indirect approaches. Indirect adaptive control methods enable control parameters to be computed from on-line parameter estimation of plant dynamics. Parameter estimation techniques such as recursive least-squares and neural networks have been used in indirect adaptive control methods. In recent years, model-reference direct adaptive control using neural networks has been a topic of great research interest (Rysdyk, and Calise, 1998; Johnson, *et al.*, 2000). Neural network is known to be a good universal approximator of many nonlinear functions that can be used to model a large class of uncertain plant dynamics. Nonetheless, key challenges still remain ahead in verification and certification of adaptive flight control software which have prevented it from being universally adopted in the aviation industry (Jacklin, *et al.*, 2006).

NASA has developed an intelligent flight control program (Williams-Hayes, 2005) to demonstrate a neural net adaptive flight control for an F-15 fighter aircraft. The intelligent flight control uses a dynamic inversion controller with a neural net direct adaptive law based on the work of Rysdyk and Calise (1998) to provide consistent handling qualities. This architecture uses a reference model to specify desired handling qualities. On-line learning neural networks are used to compensate for errors and adapt to changes in aircraft dynamics. While this architecture has been used with good success in simulations, the possibility of a high gain control due to aggressive learning can exist. A high gain control can potentially result in an actuator command that may saturate the control authority and or excite unmodeled dynamics of the damaged aircraft which can adversely affect the stability of the adaptive law. Therefore, we propose a hybrid adaptive flight control method to reduce the possibility of a high gain control. The hybrid adaptive control performs on-line estimation of damaged plant dynamics based on two methods: 1) an indirect adaptive law and 2) a recursive least squares method, both of which provide update laws for the model inversion control. A simulation of a damaged generic transport aircraft has shown that the hybrid adaptive flight control is effective in improving the command tracking performance.

2. HYBRID DAMAGE ADAPTIVE CONTROL

In a damage event, an aircraft becomes out of trim resulting from a sudden shift in the center of gravity (CG) and changes in aerodynamic characteristics and mass properties. The nonlinear equations of the aircraft angular motion with the translational motion in trim are

$$\begin{aligned}\bar{I}_{xx}\dot{p}-\bar{I}_{xy}\dot{q}-\bar{I}_{xz}\dot{r}+\bar{I}_{xy}pr-\bar{I}_{xz}pq+\left(\bar{I}_{zz}-\bar{I}_{yy}\right)qr \\ +\bar{I}_{yz}\left(r^2-q^2\right)+m\left(qv+rw\right)\Delta x-mpv\Delta y \\ -mpw\Delta z=\left(C_l^*+\Delta\bar{C}_l\right)QSb\end{aligned} \tag{1}$$

$$\begin{aligned}\bar{I}_{yy}\dot{q}-\bar{I}_{xy}\dot{p}-\bar{I}_{yz}\dot{r}+\bar{I}_{yz}pq-\bar{I}_{xy}qr+\left(\bar{I}_{xx}-\bar{I}_{zz}\right)pr \\ +\bar{I}_{xz}\left(p^2-r^2\right)-mqu\Delta x+m\left(pu+rw\right)\Delta y \\ -mqw\Delta z=\left(C_m^*+\Delta\bar{C}_m\right)QS\bar{c}+\delta_T T_{max}z_e\end{aligned} \tag{2}$$

$$\begin{aligned}\bar{I}_{zz}\dot{r}-\bar{I}_{xz}\dot{p}-\bar{I}_{yz}\dot{q}+\bar{I}_{xz}qr-\bar{I}_{yz}pr+\left(\bar{I}_{yy}-\bar{I}_{xx}\right)pq \\ +\bar{I}_{xy}\left(q^2-p^2\right)-mru\Delta x-mrv\Delta y \\ +m\left(pu+qv\right)\Delta z=\left(C_n^*+\Delta\bar{C}_n\right)QSb\end{aligned} \tag{3}$$

where the mass and inertia values are post-damage values, $(\Delta x, \Delta y, \Delta z)$ is the CG shift position relative to the original CG, and $\Delta\bar{C}_{l,m,n}$ are the changes in the aerodynamic moment coefficients. Generally, these parameters are unknown and can have a drastic effect on trim stability of an aircraft.

When a damage occurs, the aircraft must be retrimmed in order to maintain desired airspeed, altitude, and heading. Concurrently, pilot rate commands should be reasonably achieved by the rate-command-attitude-hold (RCAH) control. If the damage is asymmetric, the aircraft motion is generally coupled in both longitudinal and lateral directions. Therefore, a non-zero bank angle will be required to trim the aircraft (Nguyen, *et al.*, 2006)

$$\Delta\phi=\frac{-\left(\Delta C_Y+C_{Y,\alpha}\Delta\alpha+C_{Y,\boldsymbol{\delta}}\boldsymbol{\delta}\right)QS}{mg\left[\cos\left(\gamma^*+\alpha^*\right)-\sin\left(\gamma^*+\alpha^*\right)\Delta\alpha\right]} \tag{4}$$

where $\boldsymbol{\delta}=\left(\delta_a,\delta_e,\delta_r\right)^T$ is the flight control surface deflection vector, $\Delta\alpha$ is the incremental trim angle of attack, and the asterisk symbol denotes the nominal undamaged trim states. Trimming the damaged aircraft with the bank angle will result in a reduced bank angle limit and a non-level flight. For a level flight, the damaged aircraft can be trimmed with the sideslip angle, but the rudder control authority will be reduced.

Based on feedback linearization, the linearized plant dynamics of the damaged aircraft can be expressed as

$$\dot{\boldsymbol{\omega}}=\dot{\boldsymbol{\omega}}^*+\Delta\dot{\boldsymbol{\omega}}=\mathbf{F}_1\boldsymbol{\omega}+\mathbf{F}_2\boldsymbol{\sigma}+\mathbf{G}\boldsymbol{\delta} \tag{5}$$

where $\boldsymbol{\omega}=\left(p,q,r\right)^T$, $\boldsymbol{\sigma}=\left(\Delta\alpha,\Delta\beta,\Delta\phi,\Delta\delta_T\right)^T$ is the incremental trim state vector, $\Delta\dot{\boldsymbol{\omega}}$ is the unknown damaged plant dynamics, and $\dot{\boldsymbol{\omega}}^*$ is the nominal plant dynamics

$$\dot{\boldsymbol{\omega}}^*=\mathbf{F}_1^*\boldsymbol{\omega}+\mathbf{F}_2^*\boldsymbol{\sigma}+\mathbf{G}^*\boldsymbol{\delta} \tag{6}$$

$\mathbf{F}_1^*$, $\mathbf{F}_2^*$, $\mathbf{G}^*$ are nominal state and control transition matrices which are assumed to be known.

Figure 1 illustrates the proposed hybrid adaptive flight control. The control architecture comprises: 1) a reference model that translates rate commands into desired acceleration commands, 2) a proportional-integral (PI) feedback control for rate stabilization and tracking, 3) a dynamic inversion controller that computes actuator commands using desired acceleration commands, 4) a neural net direct adaptation due to Rysdyk and Calise (1998), and 5) a model inversion adaptation that adjusts the model to match the damaged aircraft dynamics. The neural net direct adaptation is designed to reduce the dynamic inversion error by estimating an augmented acceleration command. The possibility of a high gain control can exist with this direct adaptation if the dynamic inversion error is large that would require a large learning rate. This may be undesirable since unmodeled dynamics such as structural modes can potentially be excited by the adaptive signal. The proposed hybrid adaptive control can improve the current direct adaptive control.

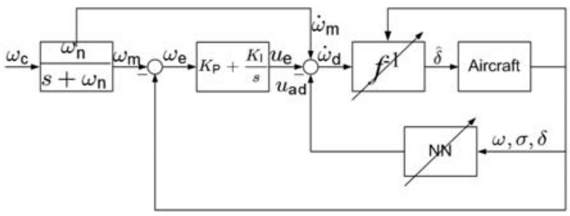

Fig. 1 - Hybrid Adaptive Flight Control

A first-order reference model is used to filter a rate command $\boldsymbol{\omega}_c$ into a reference model rate $\boldsymbol{\omega}_m$ and a reference model acceleration $\dot{\boldsymbol{\omega}}_m$

$$\dot{\boldsymbol{\omega}}_m+\boldsymbol{\omega}_n\boldsymbol{\omega}_m=\boldsymbol{\omega}_n\boldsymbol{\omega}_c \tag{7}$$

where $\boldsymbol{\omega}_n = \text{diag}(\omega_p, \omega_q, \omega_r)$ is the reference model frequency matrix.

The reference frequency parameters must be chosen appropriately in order to obtain a good transient response that satisfies actuator position and rate limits. The reference model parameters can be tuned using an adaptive critic approach to ensure that the flight control can track the reference model in order to achieve desired handling qualities (Krishnakumar, *et al.*, 2003).

The reference model rate $\boldsymbol{\omega}_m$ is compared with the actual rate $\boldsymbol{\omega}$ to form a tracking error signal $\boldsymbol{\omega}_e = \boldsymbol{\omega}_m - \boldsymbol{\omega}$. A pseudo-control vector $\mathbf{u}_e$ incorporates a PI feedback scheme to better handle rate errors detected from the rate feedback. The error dynamics, defined by proportional and integral gains, must be fast enough to track the reference model, yet slow enough to not interfere with actuator dynamics. A windup protection is designed to limit the integrator at its current value when a control surface is commanded beyond its limit. The pseudo-control vector $\mathbf{u}_e$ is computed as

$$\mathbf{u}_e = \mathbf{K}_P \boldsymbol{\omega}_e + \mathbf{K}_I \int_0^t \boldsymbol{\omega}_e d\tau \tag{8}$$

The PI control incorporates frequencies that match the reference model frequencies and damping ratios

$$\mathbf{K}_P = 2\boldsymbol{\zeta}\boldsymbol{\omega}_n \quad \mathbf{K}_I = \boldsymbol{\omega}_n^2 \tag{9}$$

where $\boldsymbol{\zeta} = \text{diag}(\zeta_p, \zeta_q, \zeta_r)$ is a damping ratio matrix.

In order for the controller to track the reference model acceleration $\dot{\boldsymbol{\omega}}_m$, the desired acceleration $\dot{\boldsymbol{\omega}}_d$ is set to

$$\dot{\boldsymbol{\omega}}_d = \dot{\boldsymbol{\omega}}_m + \mathbf{u}_e - \mathbf{u}_{ad} \tag{10}$$

where $\mathbf{u}_{ad}$ is the direct adaptive signal that cancels out the dynamic inversion error, so that the desired acceleration $\dot{\boldsymbol{\omega}}_d$ is equal to the reference model acceleration $\dot{\boldsymbol{\omega}}_m$ in an ideal setting when the tracking error goes to zero asymptotically.

A dynamic inversion controller is computed to obtain an estimated control surface deflection command $\hat{\boldsymbol{\delta}}$ to achieve the desired acceleration $\dot{\boldsymbol{\omega}}_d$

$$\hat{\boldsymbol{\delta}} = \hat{\mathbf{G}}^{-1}\left(\dot{\boldsymbol{\omega}}_d - \hat{\mathbf{F}}_1\boldsymbol{\omega} - \hat{\mathbf{F}}_2\boldsymbol{\sigma}\right) \tag{11}$$

where $\hat{\mathbf{F}}_1 = \mathbf{F}_1^* + \Delta\hat{\mathbf{F}}_1$, $\hat{\mathbf{F}}_2 = \mathbf{F}_2^* + \Delta\hat{\mathbf{F}}_2$, $\hat{\mathbf{G}} = \mathbf{G}^* + \Delta\hat{\mathbf{G}}$ are the the estimated damaged plant matrices and $\hat{\mathbf{G}}$ is assumed to be invertible.

Because the true damaged plant dynamics are unknown, the dynamic inversion controller incurs an error equal to

$$\boldsymbol{\varepsilon} = \dot{\boldsymbol{\omega}} - \dot{\boldsymbol{\omega}}_d = \Delta\boldsymbol{\varepsilon} - \Delta\hat{\mathbf{F}}_1\boldsymbol{\omega} - \Delta\hat{\mathbf{F}}_2\boldsymbol{\sigma} - \Delta\hat{\mathbf{G}}\hat{\boldsymbol{\delta}} \tag{12}$$

where $\Delta\boldsymbol{\varepsilon} = \Delta\dot{\boldsymbol{\omega}}$.

The tracking error dynamics then becomes

$$\dot{\mathbf{e}} = \mathbf{A}\mathbf{e} + \mathbf{B}\mathbf{u}_{ad} + \mathbf{B}\Delta\hat{\mathbf{F}}_1\boldsymbol{\omega} + \mathbf{B}\Delta\hat{\mathbf{F}}_2\boldsymbol{\sigma} + \mathbf{B}\Delta\hat{\mathbf{G}}\hat{\boldsymbol{\delta}} - \mathbf{B}\Delta\boldsymbol{\varepsilon} \tag{13}$$

where $\mathbf{e} = \left(\int_0^t \boldsymbol{\omega}_e d\tau, \boldsymbol{\omega}_e\right)^T$ is the tracking error and

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{K}_I & -\mathbf{K}_P \end{bmatrix}, \ \mathbf{B} = \begin{bmatrix} \mathbf{0} \\ \mathbf{I} \end{bmatrix} \tag{14}$$

The direct adaptive signal $\mathbf{u}_{ad}$ is computed from a single-layer sigma-pi neural network

$$\mathbf{u}_{ad} = \mathbf{W}^T\boldsymbol{\beta}(\mathbf{C}_1, \mathbf{C}_2, \mathbf{C}_3, \mathbf{C}_4, \mathbf{C}_5, \mathbf{C}_6) \tag{15}$$

where $\boldsymbol{\beta} = (\mathbf{C}_1, \mathbf{C}_2, \mathbf{C}_3, \mathbf{C}_4, \mathbf{C}_5, \mathbf{C}_6)^T$ is a basis function with $\mathbf{C}_i$, $i = 1, \ldots, 6$, as inputs into the neural network consisting of control commands, sensor feedback, and bias terms

$$\mathbf{C}_1 = V^2 \begin{bmatrix} \boldsymbol{\omega}^T & \alpha\boldsymbol{\omega}^T & \beta\boldsymbol{\omega}^T \end{bmatrix} \tag{16}$$

$$\mathbf{C}_2 = V^2 \begin{bmatrix} 1 & \alpha & \beta & \alpha^2 & \beta^2 & \alpha\beta & \alpha\beta^2 \end{bmatrix} \tag{17}$$

$$\mathbf{C}_3 = V^2 \begin{bmatrix} \boldsymbol{\delta}^T & \alpha\boldsymbol{\delta}^T & \beta\boldsymbol{\delta}^T \end{bmatrix} \tag{18}$$

$$\mathbf{C}_4 = \begin{bmatrix} p\boldsymbol{\omega}^T & q\boldsymbol{\omega}^T & r\boldsymbol{\omega}^T \end{bmatrix} \tag{19}$$

$$\mathbf{C}_5 = \begin{bmatrix} u\boldsymbol{\omega}^T & v\boldsymbol{\omega}^T & w\boldsymbol{\omega}^T \end{bmatrix} \tag{20}$$

$$\mathbf{C}_6 = \begin{bmatrix} 1 & \theta & \phi & C_T \end{bmatrix} \tag{21}$$

The update law for the neural net weights $\mathbf{W}$ is

$$\dot{\mathbf{W}} = -\Gamma\left(\boldsymbol{\beta}\mathbf{e}^T\mathbf{P}\mathbf{B} + \mu\left\|\mathbf{e}^T\mathbf{P}\mathbf{B}\right\|\mathbf{W}\right) \tag{22}$$

where $\Gamma > 0$ is a learning rate, $\mu > 0$ is an e-modification parameter, $\mathbf{P}$ solves the Lyapunov equation $\mathbf{A}^T\mathbf{P} + \mathbf{P}^T\mathbf{A} = -\mathbf{Q}$ for some positive-definite matrix $\mathbf{Q}$, and $\|.\|$ is a Frobenius norm.

The goal is to compute $\hat{\mathbf{F}}_1, \hat{\mathbf{F}}_2, \hat{\mathbf{G}}$ by a model inversion adaptive law. Two approaches are considered: 1) an indirect adaptive law using the Lyapunov theory, and 2) a recursive least squares method for optimal estimation of these damaged plant matrices.

2.1 Indirect Adaptive Law: The model inversion adaptation can be computed by the following update law

$$\dot{\boldsymbol{\Phi}} = -\boldsymbol{\Lambda}\left(\boldsymbol{\theta}\mathbf{e}^T\mathbf{P}\mathbf{B} + \boldsymbol{\eta}\left\|\mathbf{e}^T\mathbf{P}\mathbf{B}\right\|\boldsymbol{\Phi}\right) \tag{23}$$

where $\boldsymbol{\Phi}^T = \left(\mathbf{W}_{\boldsymbol{\omega}}^T, \mathbf{W}_{\boldsymbol{\sigma}}^T, \mathbf{W}_{\boldsymbol{\delta}}^T\right)$ is a weight matrix, $\boldsymbol{\theta}^T = \left(\boldsymbol{\omega}^T\boldsymbol{\beta}_{\boldsymbol{\omega}}^T, \boldsymbol{\sigma}^T\boldsymbol{\beta}_{\boldsymbol{\sigma}}^T, \hat{\boldsymbol{\delta}}^T\boldsymbol{\beta}_{\boldsymbol{\delta}}^T\right)$ is an input matrix of state and control vectors and corresponding basis functions, $\boldsymbol{\Lambda} = \text{diag}(\Gamma_{\boldsymbol{\omega}}, \Gamma_{\boldsymbol{\sigma}}, \Gamma_{\boldsymbol{\delta}}) > 0$ is a learning rate matrix, and $\boldsymbol{\eta} = \text{diag}(\mu_{\boldsymbol{\omega}}, \mu_{\boldsymbol{\sigma}}, \mu_{\boldsymbol{\delta}})$ is an e-modification parameter matrix. The e-modification term provides robustness to unmodeled dynamics (Nadrendra and Annaswamy, 1987).

The damaged plant matrices are then computed as

$$\Delta\hat{\mathbf{F}}_1 = \mathbf{W}_{\boldsymbol{\omega}}^T\boldsymbol{\beta}_{\boldsymbol{\omega}}, \ \Delta\hat{\mathbf{F}}_2 = \mathbf{W}_{\boldsymbol{\sigma}}^T\boldsymbol{\beta}_{\boldsymbol{\sigma}}, \ \Delta\hat{\mathbf{G}} = \mathbf{W}_{\boldsymbol{\delta}}^T\boldsymbol{\beta}_{\boldsymbol{\delta}} \tag{24}$$

The proof is in the appendix.

2.2 Recursive Least Squares Method: Suppose the dynamic inversion error can be written as

$$\boldsymbol{\varepsilon} = \boldsymbol{\Phi}^T\boldsymbol{\theta} + \Delta\boldsymbol{\varepsilon} \tag{25}$$

where $\Delta\varepsilon$ is the estimation error of $\Delta\dot{\omega}$. Then, the estimated dynamic inversion error is

$$\hat{\varepsilon} = \dot{\hat{\boldsymbol{\omega}}} - \mathbf{F}_1^* \boldsymbol{\omega} - \mathbf{F}_2^* \boldsymbol{\sigma} - \mathbf{G}^* \hat{\boldsymbol{\delta}} \tag{26}$$

where $\dot{\hat{\boldsymbol{\omega}}}$ is the estimated acceleration.

If the error is unbiased, then the recursive least squares method can be applied to estimate the plant dynamics. The model inversion adaptation using the recursive least squares method is given by

$$\dot{\boldsymbol{\Phi}} = (1+\xi)^{-1} \mathbf{R}\boldsymbol{\theta} \left(\hat{\varepsilon}^T - \boldsymbol{\theta}^T \boldsymbol{\Phi} \right) \tag{27}$$

where

$$\dot{\mathbf{R}} = -(1+\xi)^{-1} \mathbf{R}\boldsymbol{\theta}\boldsymbol{\theta}^T \mathbf{R} \tag{28}$$

$$\xi = \boldsymbol{\theta}^T \mathbf{R} \boldsymbol{\theta} \tag{29}$$

The weight matrix $\mathbf{R}$ has a very similar form to the Kalman filter with Eq. (**??**) as the differential Riccati equation for a zero-order plant dynamics. In practice, we implement the recursive least squares method in a discrete time with a directional forgetting factor (Bobal, 2005) according to

$$\boldsymbol{\Phi}_{i+1} = \boldsymbol{\Phi}_i + (1+\xi_{i+1})^{-1} \mathbf{R}_{i+1} \boldsymbol{\theta}_i \left[\hat{\varepsilon}_{i+1}^T - \boldsymbol{\theta}_i^T \boldsymbol{\Phi}_i \right] \tag{30}$$

$$\mathbf{R}_{i+1} = \mathbf{R}_i - \left(\psi_{i+1}^{-1} + \xi_{i+1} \right)^{-1} \mathbf{R}_i \boldsymbol{\theta}_i \boldsymbol{\theta}_i^T \mathbf{R}_i \tag{31}$$

where ψ is defined as

$$\psi_{i+1} = \varphi_{i+1} - \xi_i^{-1} (1 - \varphi_{i+1}) \tag{32}$$

The directional forgetting factor φ is calculated as

$$\varphi_{i+1}^{-1} = 1 + (1+\rho) \ln (1+\xi_{i+1}) + \left[\frac{\eta_{i+1} (1+\vartheta_{i+1})}{1+\xi_{i+1}+\eta_{i+1}} - 1 \right] \frac{\xi_{i+1}}{1+\xi_{i+1}} \tag{33}$$

where ρ is a constant, and η and ϑ are parameters with the following update laws

$$\eta_{i+1} = \lambda_{i+1}^{-1} \left\| \hat{\varepsilon}_{i+1} - \boldsymbol{\Phi}_i^T \boldsymbol{\theta}_i \right\|^2 \tag{34}$$

$$\vartheta_{i+1} = \varphi_{i+1} (1 + \vartheta_i) \tag{35}$$

$$\lambda_{k+1} = \varphi_{i+1} \left[\lambda_k + (1+\xi_{i+1}) \left\| \hat{\varepsilon}_{i+1} - \boldsymbol{\Phi}_i^T \boldsymbol{\theta}_i \right\|^2 \right] \tag{36}$$

3. NUMERICAL SIMULATIONS

To evaluate the hybrid adaptive flight control, a simulation was performed for a twin-engine generic transport model (GTM) defined by NASA Langley Research Center (Bailey, *et al.*, 2005), as shown in Fig. 2. An aerodynamic modeling of the damaged aircraft is performed using a CFD code to estimate aerodynamic coefficients and stability and control derivatives. In this simulation, we choose a damage configuration corresponding to a 30% loss of the left wing.

Fig. 2 - Generic Transport Model

The pilot pitch command is simulated with a series of step input pitch doublets. The tracking performance of the three control laws is compared in Fig. 3. The hybrid indirect adaptive scheme shows an improvement in the tracking performance over the direct adaptive scheme after one doublet, but it is the hybrid recursive least squares method that provides the best tracking performance of all.

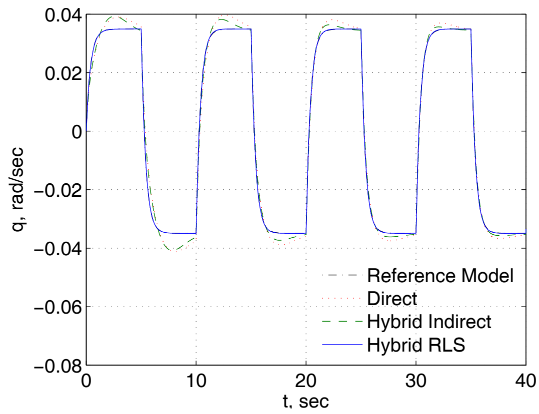

Fig. 3 - Pitch Doublet Tracking Performance

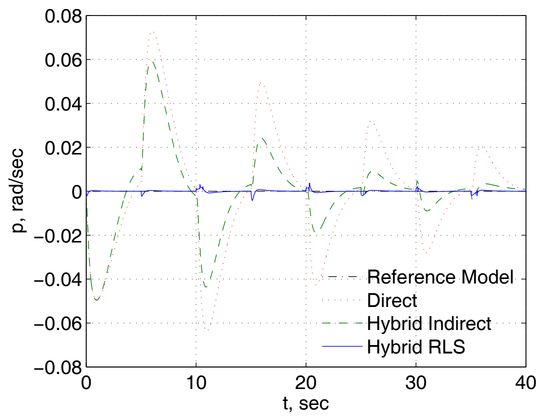

Fig. 4 - Roll Response

Figures 4 and 5 are the the roll and yaw responses during the pitch doublet maneuver. The hybrid recursive least squares method controls the roll and yaw rates much better than both the direct and hybrid indirect adaptive schemes. Figure 6 shows the Frobenius norms of the tracking error for the three adaptive control schemes. The hybrid recursive least squares method achieves the smallest tracking error norm, while the tracking error norm for the hybrid indirect adaptive scheme decreases over time faster than that for the direct adaptive scheme.

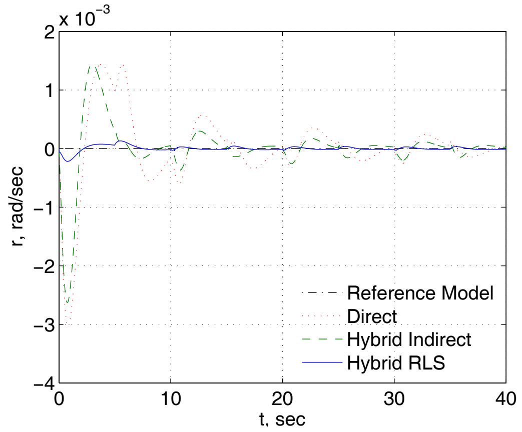

Fig. 5 - Yaw Response

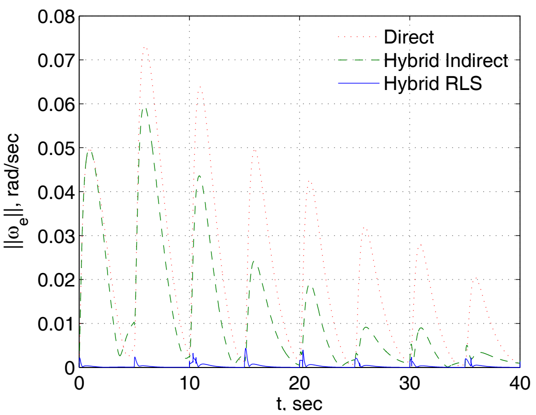

Fig. 6 - Tracking Error Norm

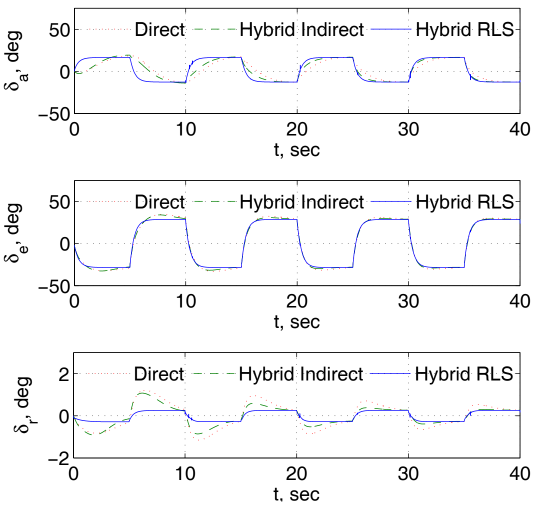

Fig. 7 - Allocation of Control Surface Deflections

The allocation of control surface deflections to achieve the pitch doublet maneuver is as shown in Fig. 7. Due to the cross coupling between the longitudinal and lateral motions, the pitch maneuver requires all three control surface deflections. The remaining right aileron control input is needed to compensate for the left rolling moment due to the asymmetric wing damage. A small rudder control input is also required to compensate for an adverse yaw moment. The aileron and rudder deflection commands for both the direct and hybrid indirect adaptive schemes do not track well initially with the expected commands which would resemble those for the hybrid recursive least squares method. Over time, the adaptation brings the control surface deflection commands for the direct and hybrid indirect adaptive schemes closer to the expected values.

4. CONCLUSIONS

This paper presents a hybrid neural net adaptive flight control method that performs on-line estimation of damaged aircraft plant dynamics by a model inversion adaptation. This proposed method can potentially reduce the possibility of a high gain control in the current direct adaptation strategy. The on-line estimation of the damaged plant dynamics is provided by two model inversion adaptive laws based on an indirect adaptive control method and a recursive least squares method. A control simulation of a wing-damaged generic transport model has shown that the hybrid indirect adaptive law improves the tracking performance over the direct adaptive scheme alone. Moreover, the hybrid recursive least squares method outperforms both the direct and hybrid indirect adaptive schemes.

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APPENDIX

$\mathbf{A}$ is assumed to be Hurwitz. Let $\mathbf{W} = \mathbf{W}^* + \tilde{\mathbf{W}}$ and $\mathbf{\Phi} = \mathbf{\Phi}^* + \tilde{\mathbf{\Phi}}$ where the asterisk symbol denotes the ideal weight matrices that cancel out the residual error $\Delta\varepsilon$ and the tilde symbol denotes the weight deviations. The ideal weight matrices are unknown but they may be assumed constant and bounded to stay within a $\mathbf{\Delta}$-neighborhood of the error ε such that

$$\mathbf{\Delta} = \sup_{\boldsymbol{\omega},\boldsymbol{\sigma},\boldsymbol{\delta}} \left\| \mathbf{W}^{*T}\boldsymbol{\beta} + \mathbf{\Phi}^T\boldsymbol{\theta} - \Delta\varepsilon \right\| \tag{37}$$

We define the following Lyapunov candidate function

$$V = \mathbf{e}^T\mathbf{P}\mathbf{e} + \mathrm{tr}\left(\frac{\tilde{\mathbf{W}}^T\tilde{\mathbf{W}}}{\Gamma} + \tilde{\mathbf{\Phi}}^T\mathbf{\Lambda}^{-1}\tilde{\mathbf{\Phi}}\right) \tag{38}$$

where $\mathbf{P} \geq \mathbf{0}$ and tr $(.)$ denotes the trace operation.

The time derivative of the Lyapunov candidate function is computed as

$$\dot{V} = \dot{\mathbf{e}}^T\mathbf{P}\mathbf{e} + \mathbf{e}^T\mathbf{P}\dot{\mathbf{e}} + 2\mathrm{tr}\left(\frac{\tilde{\mathbf{W}}^T\dot{\tilde{\mathbf{W}}}}{\Gamma} + \tilde{\mathbf{\Phi}}^T\mathbf{\Lambda}^{-1}\dot{\tilde{\mathbf{\Phi}}}\right) \tag{39}$$

Defining $\mathbf{A}^T\mathbf{P} + \mathbf{P}^T\mathbf{A} = -\mathbf{Q}$, we get

$$\begin{aligned}\dot{V} \leq& -\mathbf{e}^T\mathbf{Q}\mathbf{e} + 2\mathbf{e}^T\mathbf{P}\mathbf{B}\left(\tilde{\mathbf{W}}^T\boldsymbol{\beta} + \tilde{\mathbf{\Phi}}^T\boldsymbol{\theta} + \mathbf{\Delta}\right)\\ &+2\mathrm{tr}\left[-\tilde{\mathbf{W}}^T\boldsymbol{\beta}\mathbf{e}^T\mathbf{P}\mathbf{B} - \mu\tilde{\mathbf{W}}^T\left\|\mathbf{e}^T\mathbf{P}\mathbf{B}\right\|\left(\mathbf{W}^* + \tilde{\mathbf{W}}\right)\right.\\ &\left. + \tilde{\mathbf{\Phi}}^T\mathbf{\Lambda}^{-1}\dot{\tilde{\mathbf{\Phi}}}\right]\end{aligned} \tag{40}$$

Completing the square yields

$$\begin{aligned}&2\mathrm{tr}\left[-\mu\tilde{\mathbf{W}}^T\left\|\mathbf{e}^T\mathbf{P}\mathbf{B}\right\|\left(\mathbf{W}^* + \tilde{\mathbf{W}}\right)\right] = \\ &-2\mu\left\|\mathbf{e}^T\mathbf{P}\mathbf{B}\right\|\left(\left\|\frac{\mathbf{W}^*}{2} + \tilde{\mathbf{W}}\right\|^2 - \left\|\frac{\mathbf{W}^*}{2}\right\|^2\right)\end{aligned} \tag{41}$$

Let $\rho(\mathbf{Q})$ and $\rho(\mathbf{P})$ be the spectral radii of $\mathbf{Q}$ and $\mathbf{P}$. Since $\|\mathbf{B}\| = 1$ and $\dot{\tilde{\mathbf{\Phi}}} = \dot{\mathbf{\Phi}}$, we establish that

$$\begin{aligned}\dot{V} \leq& -\rho(\mathbf{Q})\|\mathbf{e}\|^2 + \frac{\rho(\mathbf{P})\|\mathbf{e}\|}{2}\left(4\|\mathbf{\Delta}\| + \mu\|\mathbf{W}^*\|^2\right)\\ &-2\mu\rho(\mathbf{P})\|\mathbf{e}\|\left\|\frac{\mathbf{W}^*}{2} + \tilde{\mathbf{W}}\right\|^2\\ &+2\mathrm{tr}\left[\tilde{\mathbf{\Phi}}^T\left(\mathbf{\Lambda}^{-1}\dot{\mathbf{\Phi}} + \boldsymbol{\theta}\mathbf{e}^T\mathbf{P}\mathbf{B}\right)\right]\end{aligned} \tag{42}$$

In order to guarantee that $\dot{V} \leq 0$, we require that the trace operator be equal to zero, thus resulting in the indirect adaptive law in Eq. (**??**) without the e-modification term. In addition, we also require that

$$\|\mathbf{e}\| > \frac{\rho(\mathbf{P})}{2\rho(\mathbf{Q})}\left(4\|\mathbf{\Delta}\| + \mu\|\mathbf{W}^*\|^2\right) \tag{43}$$

The time rate of change of the Lyapunov candidate function is then strictly negative and therefore it would guarantee that the signals are bounded since

$$V(\infty) \leq V(0) - 2\mu\rho(\mathbf{P})\int_0^\infty \|\mathbf{e}\|\left\|\frac{\mathbf{W}^*}{2} + \tilde{\mathbf{W}}\right\|^2 dt \tag{44}$$

Thus, $V < \infty$ is bounded as $t \to \infty$. Therefore, we establish that $\|\mathbf{e}\| \to 0$ so that $\left\|\dot{\mathbf{\Phi}}\right\| \to 0$ as $t \to \infty$. This means that the adaptive laws will result in a convergence of the estimated $\Delta\hat{\mathbf{F}}_1$, $\Delta\hat{\mathbf{F}}_2$, and $\Delta\hat{\mathbf{G}}$ to their steady state values.

It can easily be shown that with the e-modification term in the indirect adaptive law, the time rate of change of the Lyapunov candidate function becomes

$$\begin{aligned}\dot{V} \leq& -2\rho(\mathbf{P})\|\mathbf{e}\|\left(\mu\left\|\frac{\mathbf{W}^*}{2} + \tilde{\mathbf{W}}\right\|^2\right.\\ &\left. + \left\|\boldsymbol{\eta}\left(\frac{\mathbf{\Phi}^*}{2} + \tilde{\mathbf{\Phi}}\right)\right\|^2\right)\end{aligned} \tag{45}$$

Thus, the effect of the e-modification terms is to increase the negative time rate of change of the Lyapunov candidate function so that as long as the effects of unmodeled dynamics and or disturbances do not exceed the value of $\dot{V}$, the adaptive signals should remain bounded.