Inferring Automata with State-local Alphabet Abstractions

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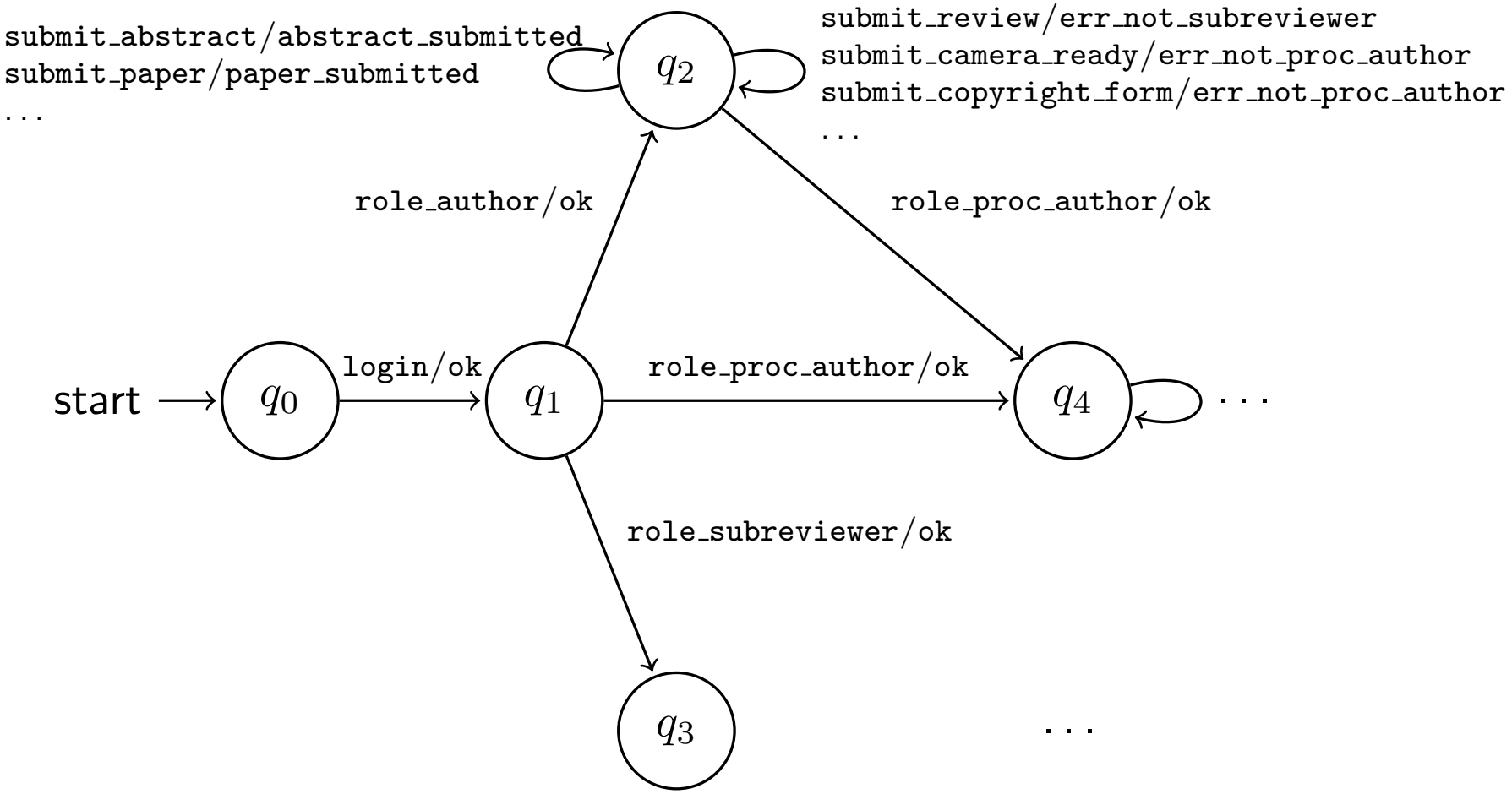
Example: A Conference Service



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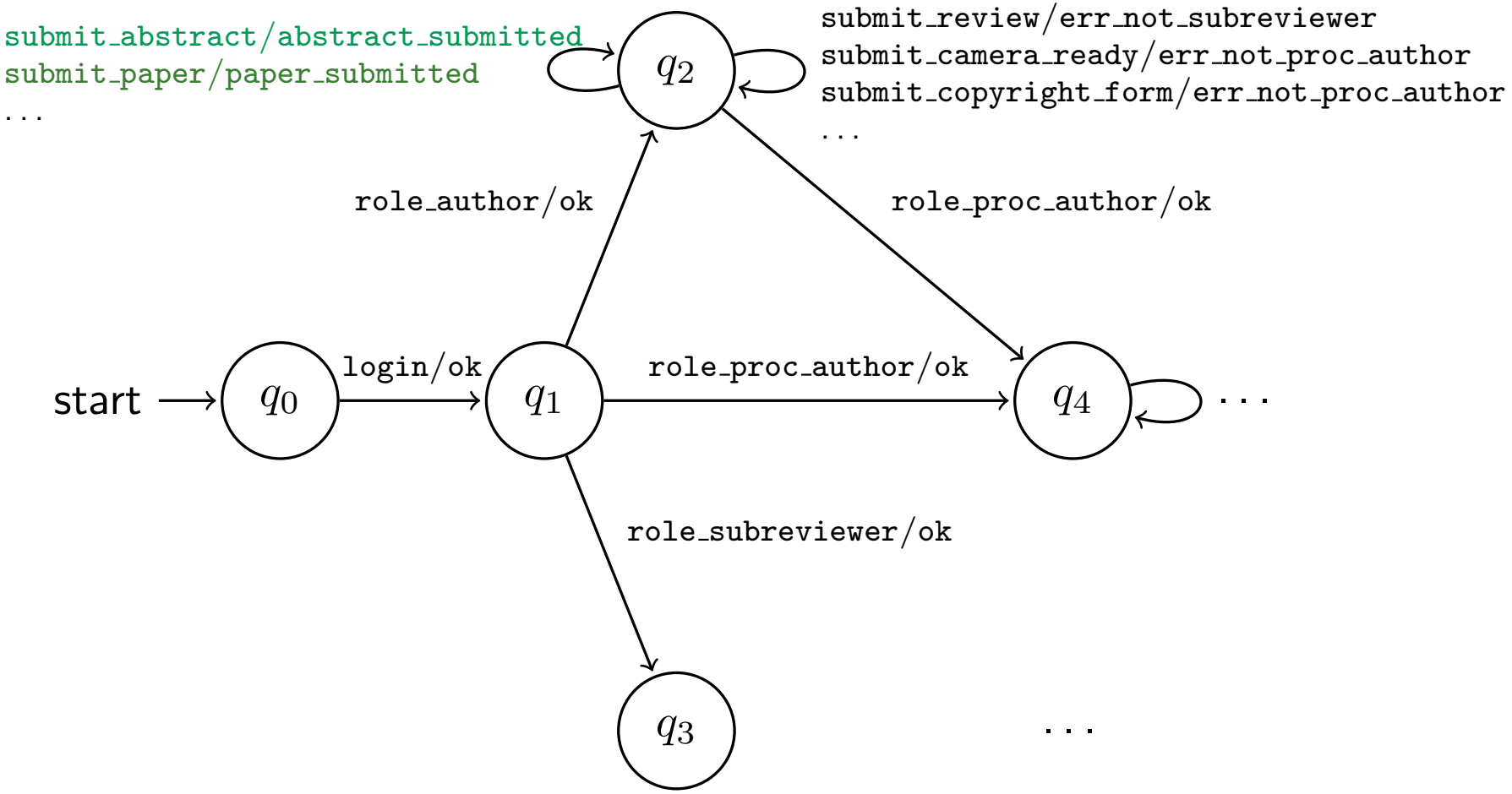

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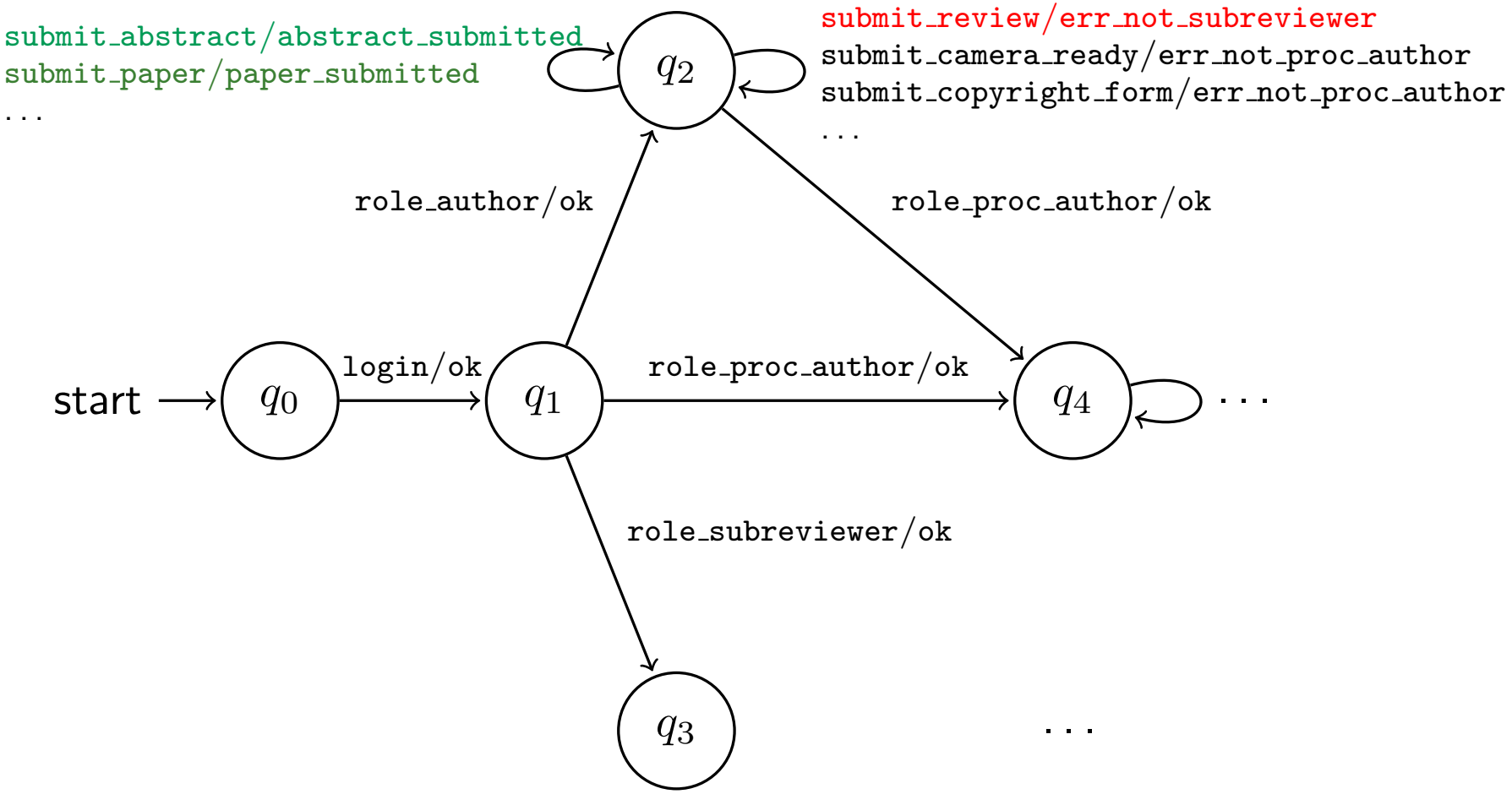

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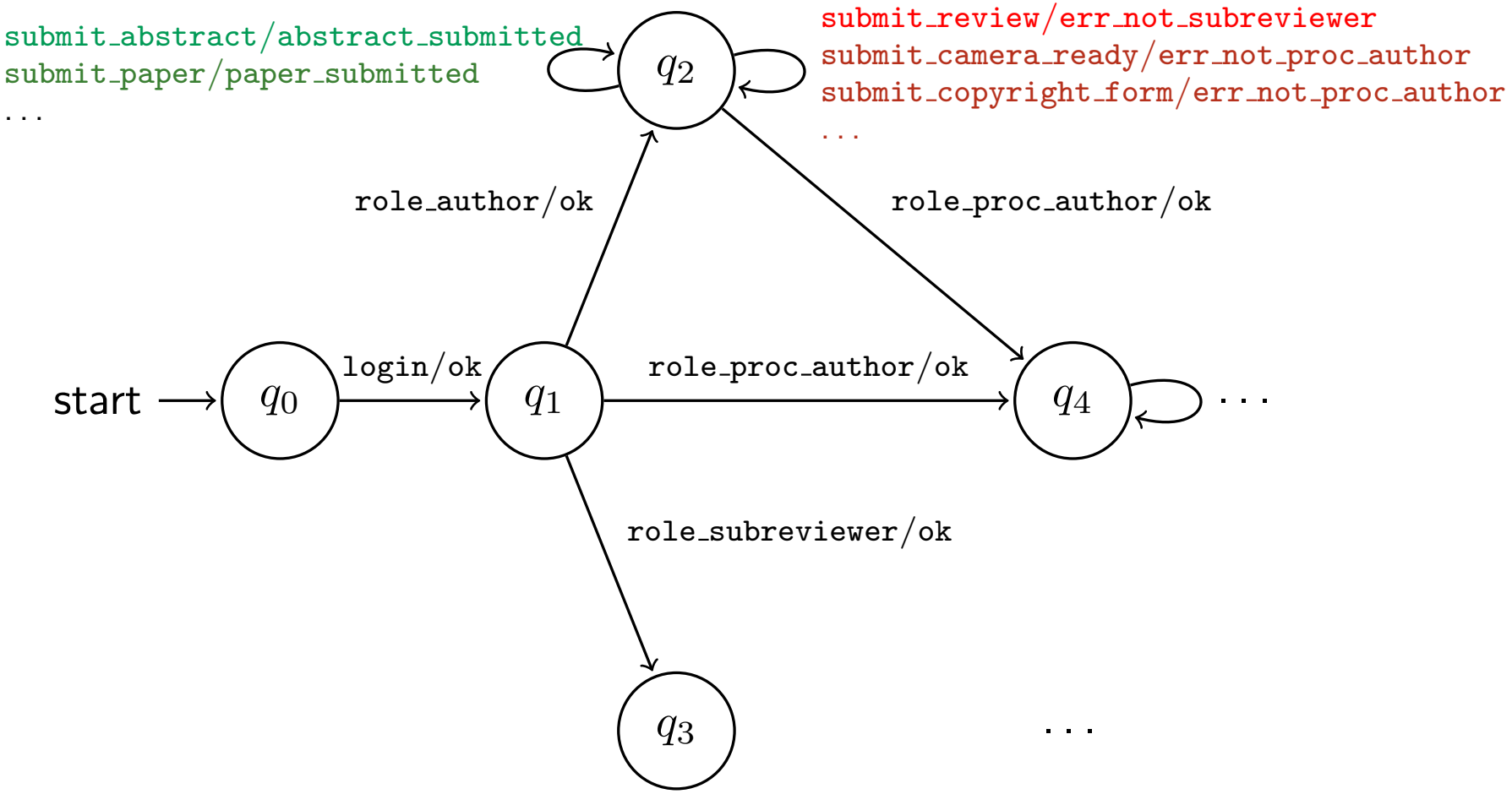

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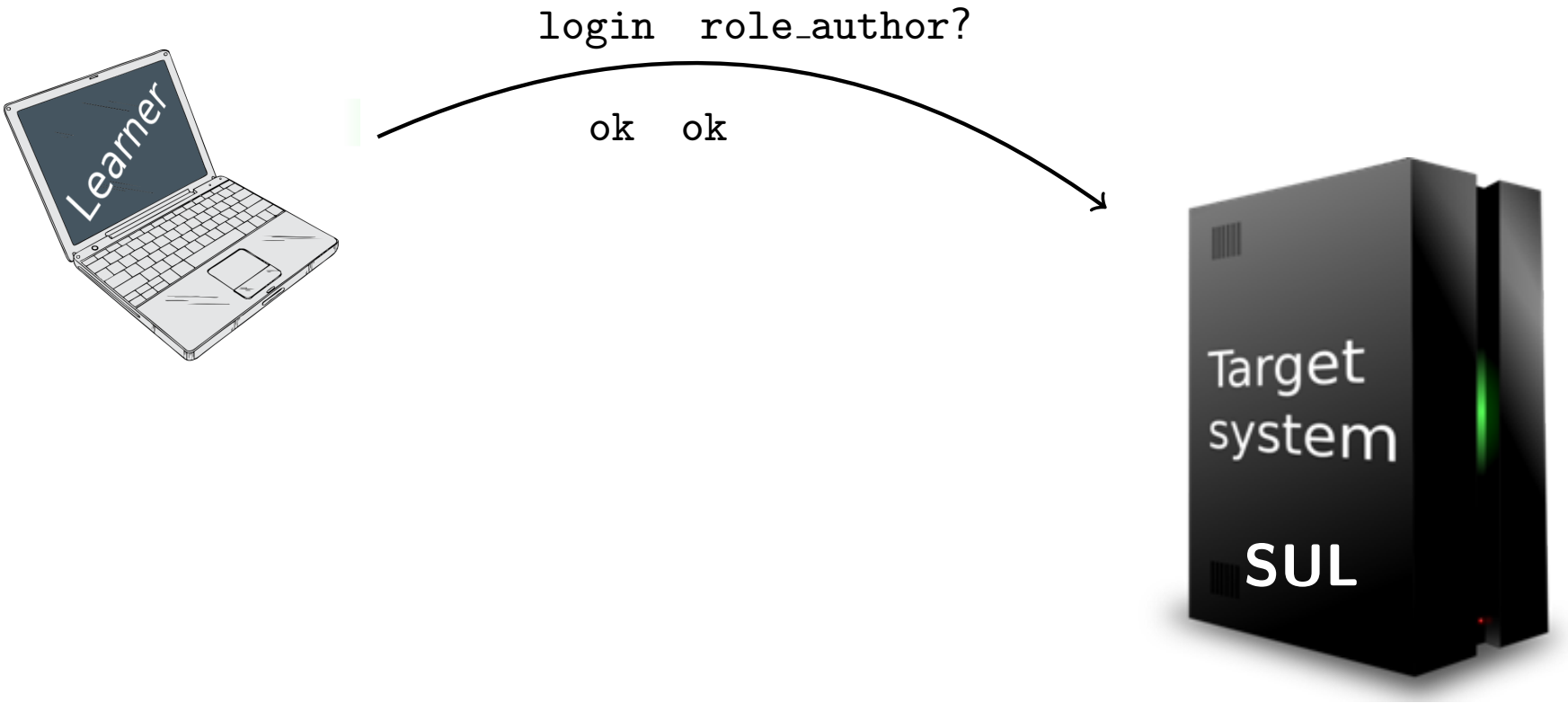

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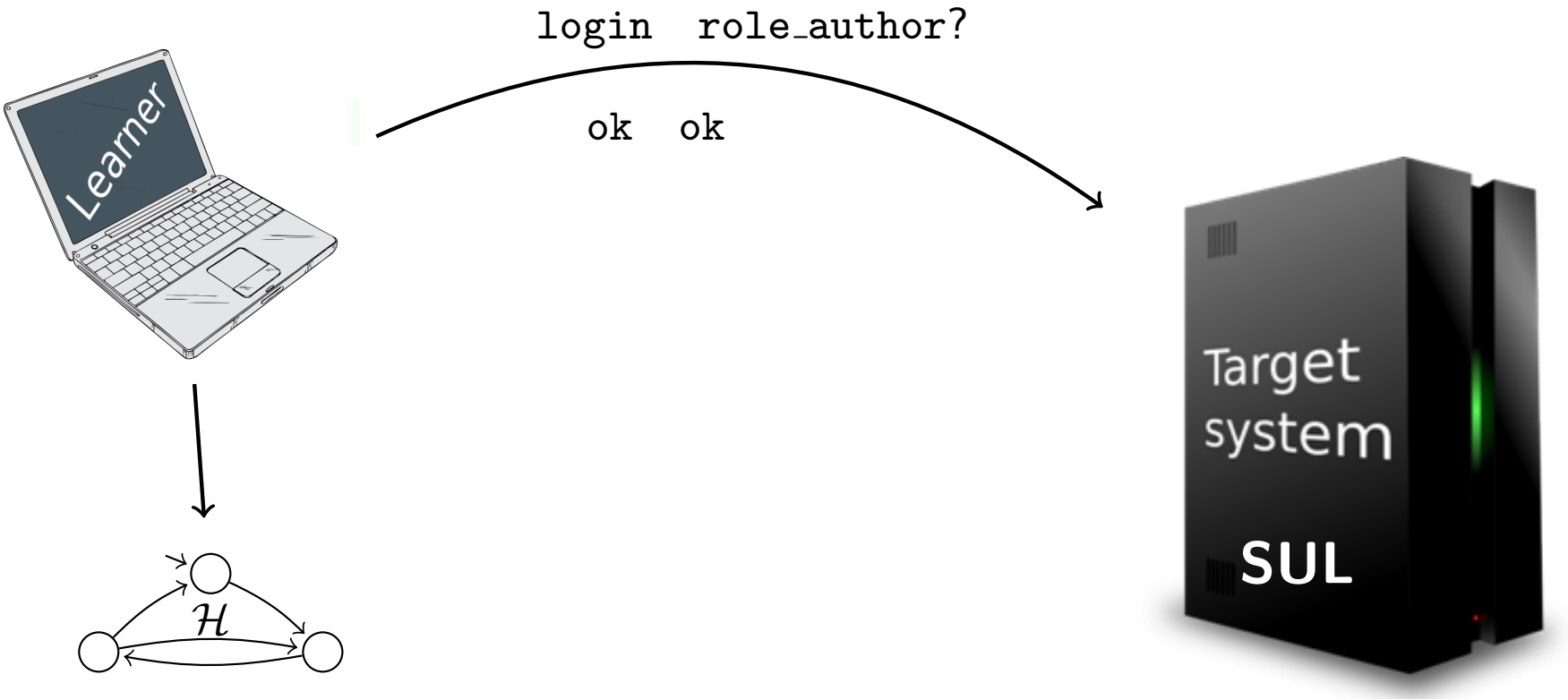

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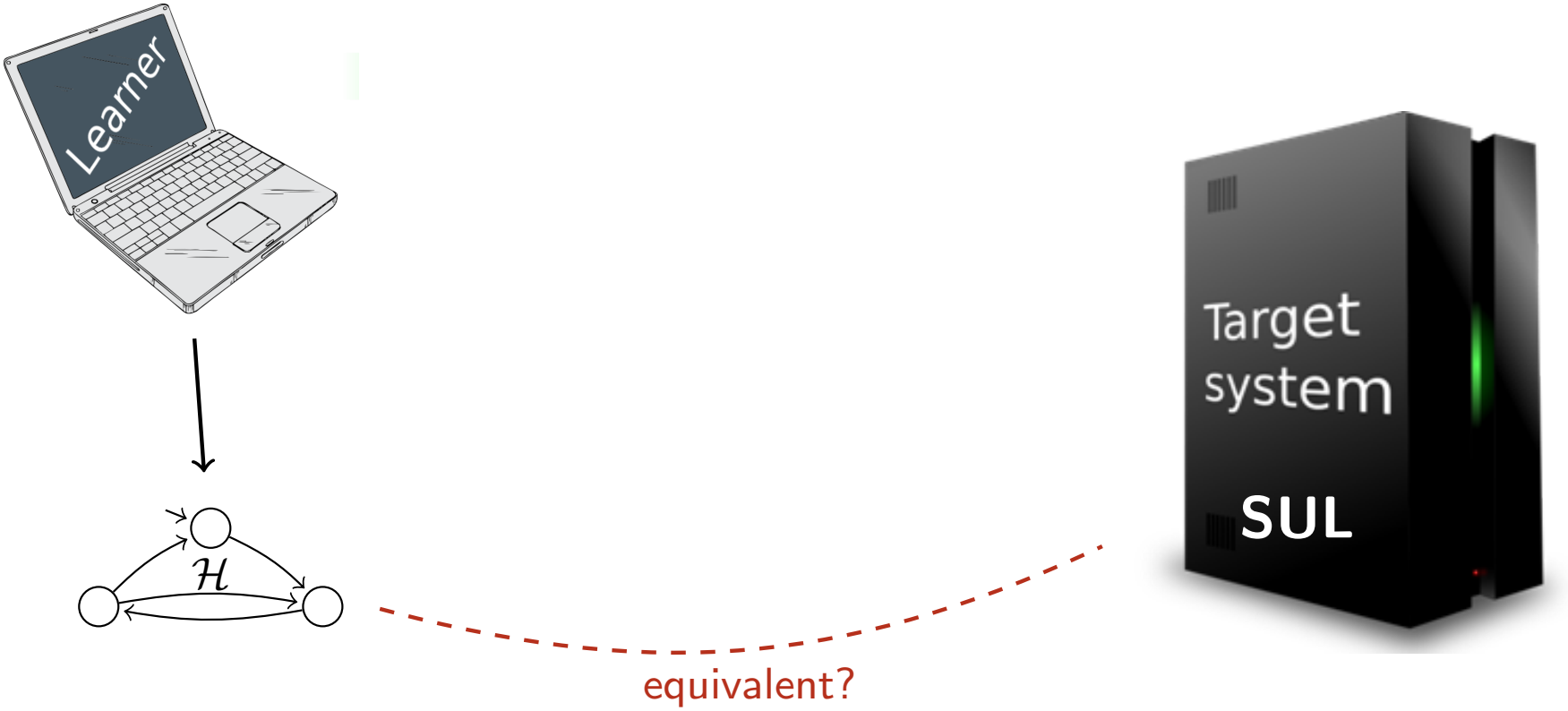

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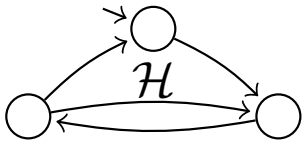

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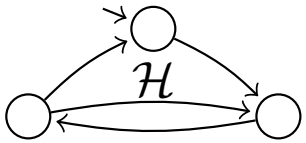
Yes: done

No: counterexample
$w \in \Sigma^* : [\![SUL]\!](w) \neq [\![\mathcal{H}]\!](w)$



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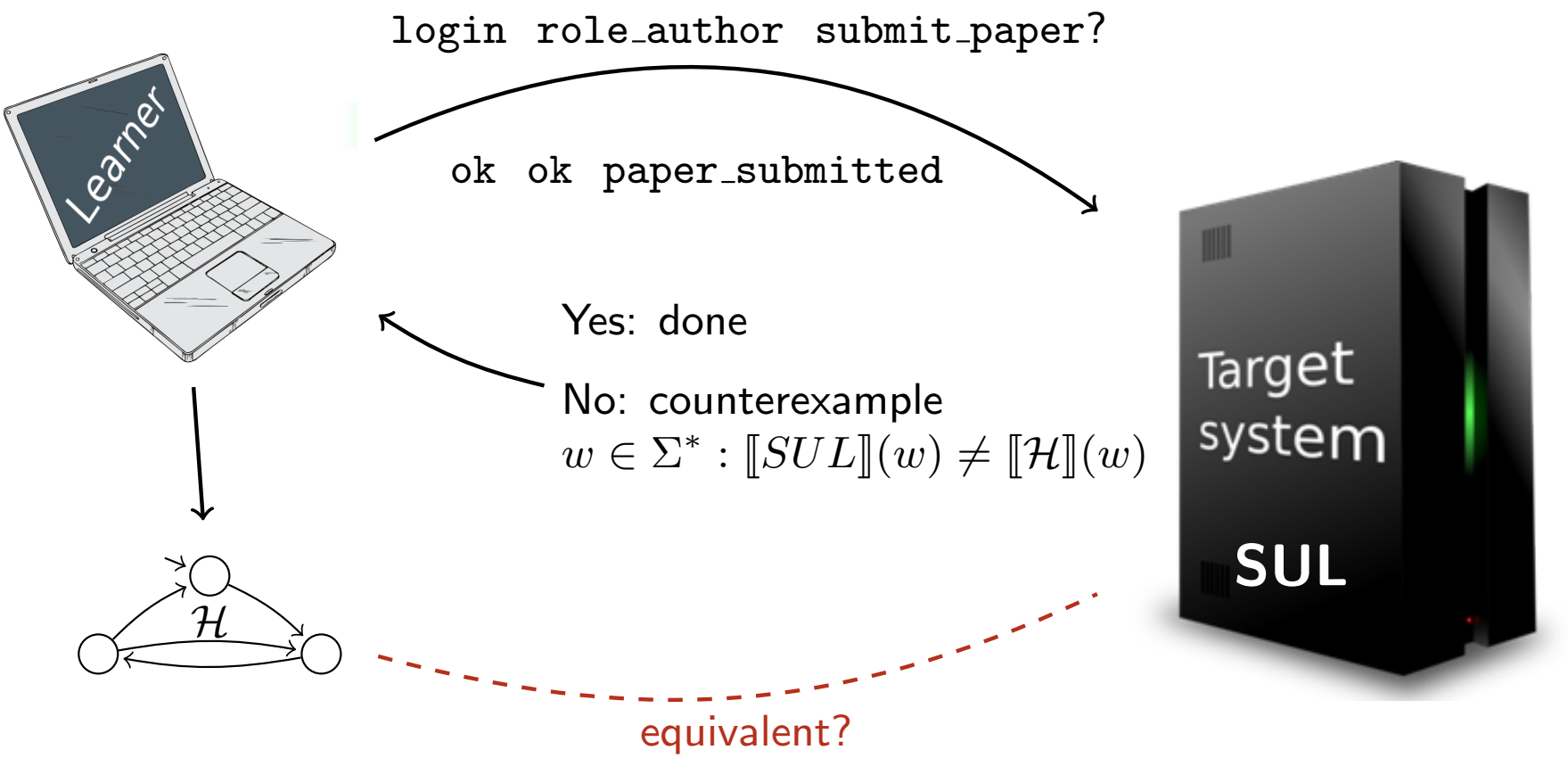

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Active Automata Learning Formalized

- Originally for learning regular sets (DFAs) [Angluin, 1987]; adapted to reactive systems (Mealy machines) [Niese, 2003]
- Input alphabet Σ, output alphabet Ω
- Interaction with "teacher" via
  - Membership Queries (MQs): What is the response to some input $w \in \Sigma^*$?
  - Equivalence Queries (EQs): Is the conjectured hypothesis correct?
- Formally: $MQ\colon \Sigma^* \to \Omega$, $EQ\colon S \to \Sigma^* \cup \{\text{yes}\}$

The L^* Algorithm

- Central data structure: Observation Table

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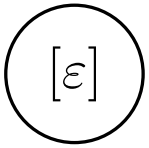

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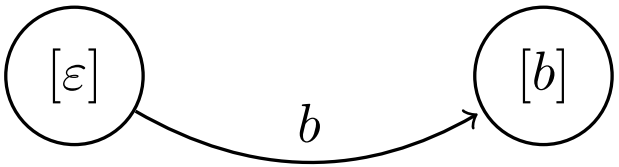

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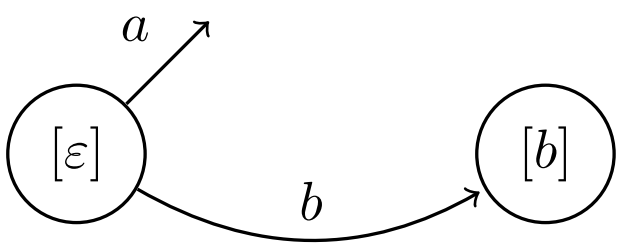

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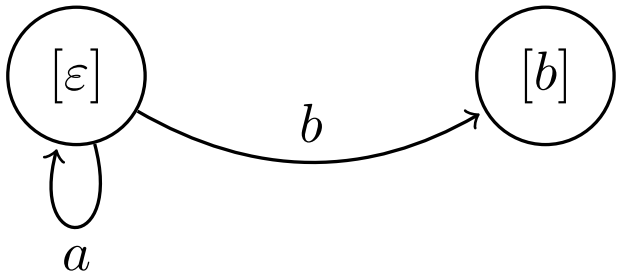

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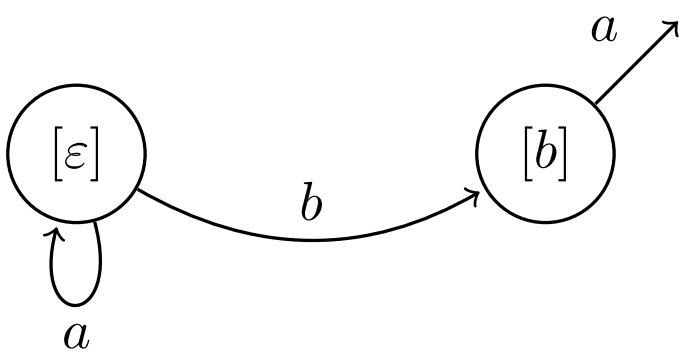

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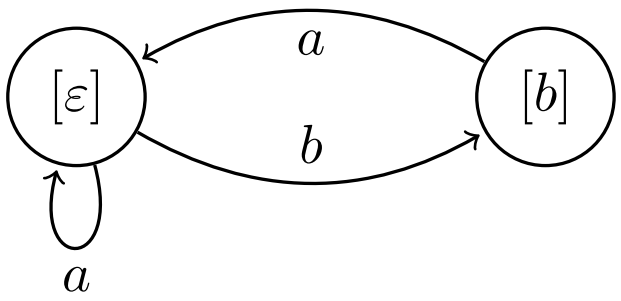

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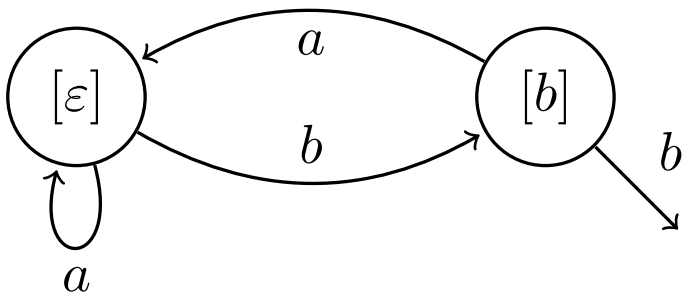

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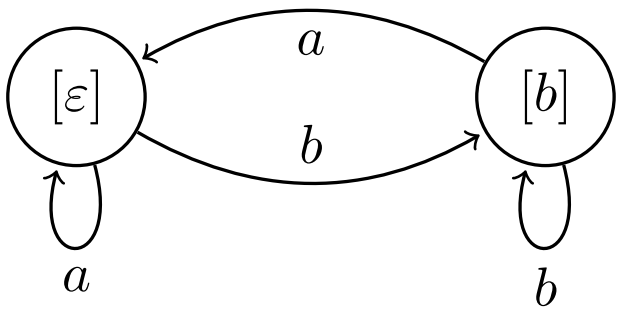

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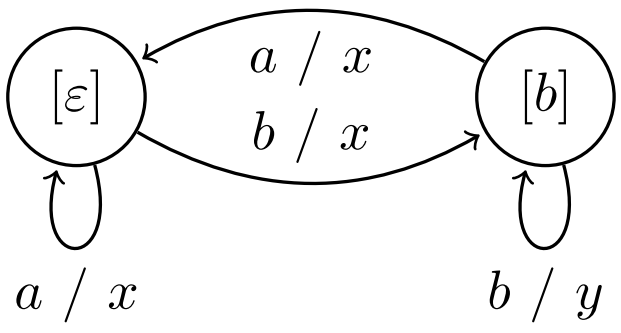

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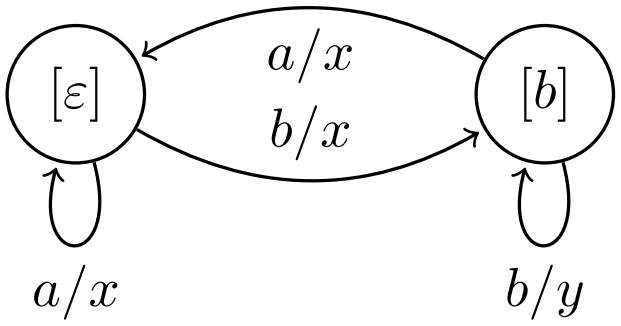

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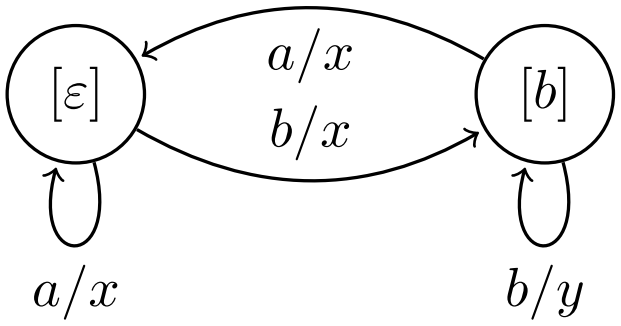

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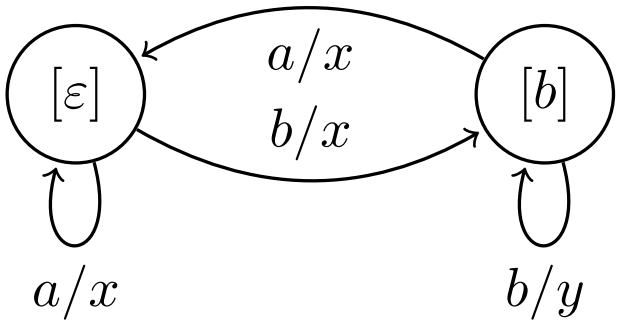

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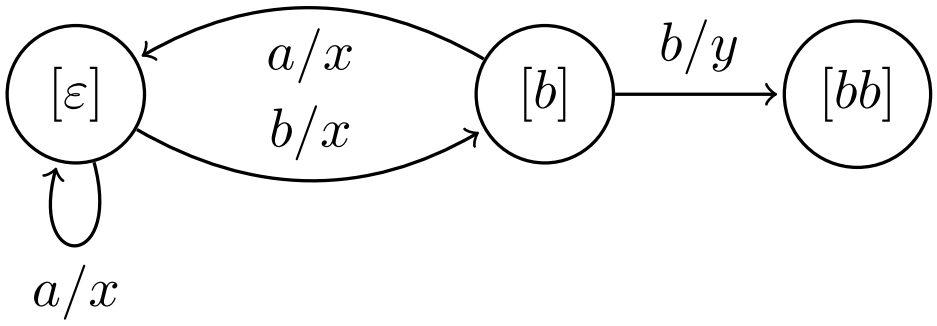

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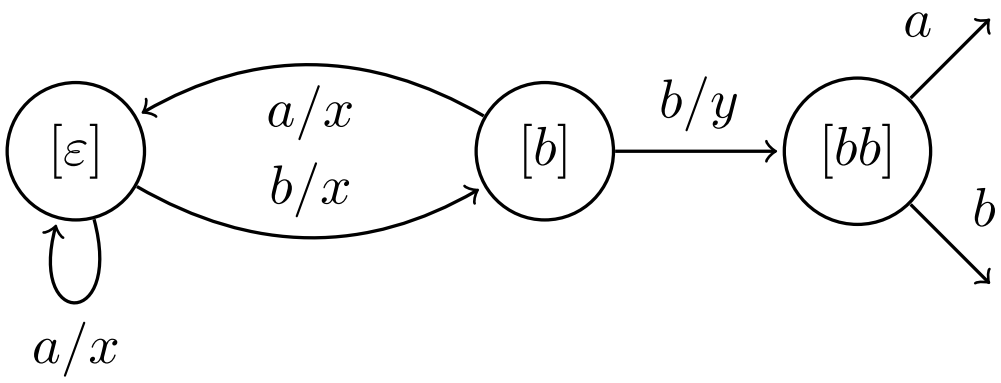

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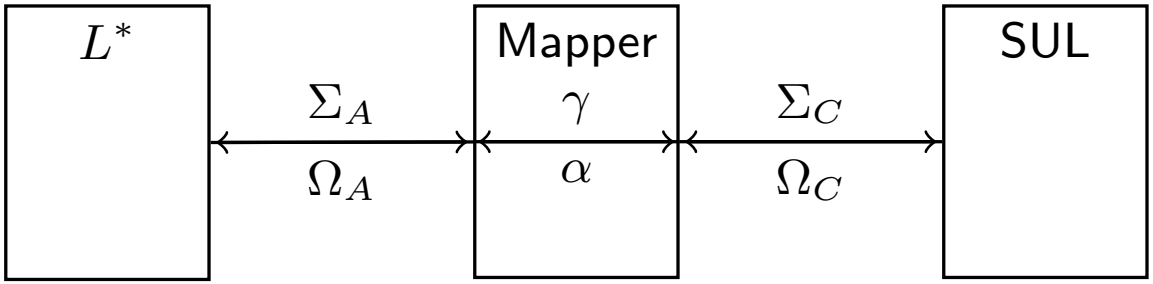

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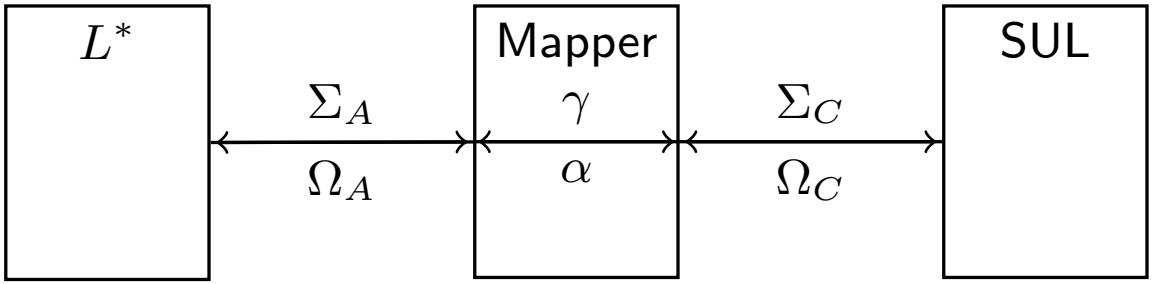

- Case-study: Inferring model of the EU biometric passport [Aarts et al., 2010]
  - Requires a-priori knowledge $\Rightarrow$ conflicts with black-box setting

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- Goal: Find coarsest determinism-preserving abstraction (DPA)

Automated Alphabet Abstraction Refinement

- Main idea: Augment black-box learning to also learn alphabet abstraction [Howar et al., 2011]

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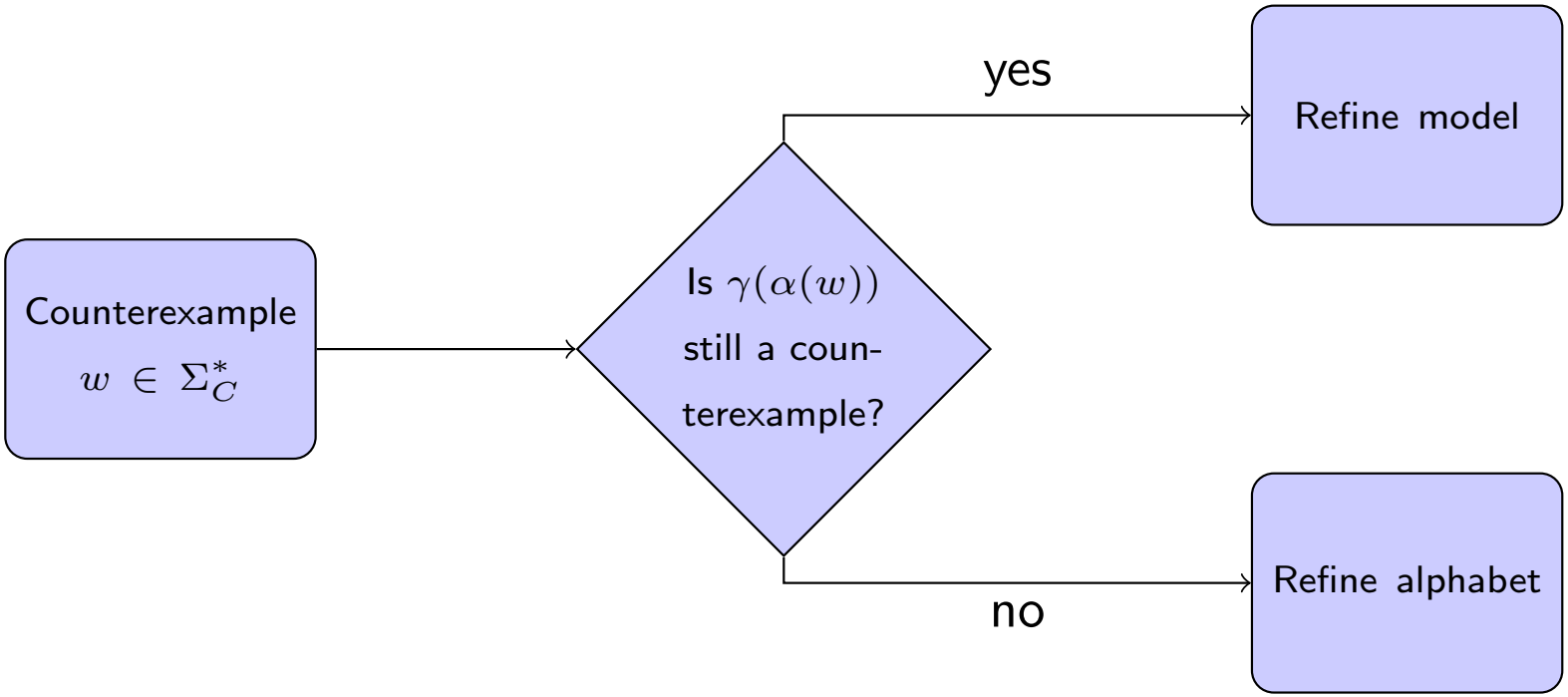

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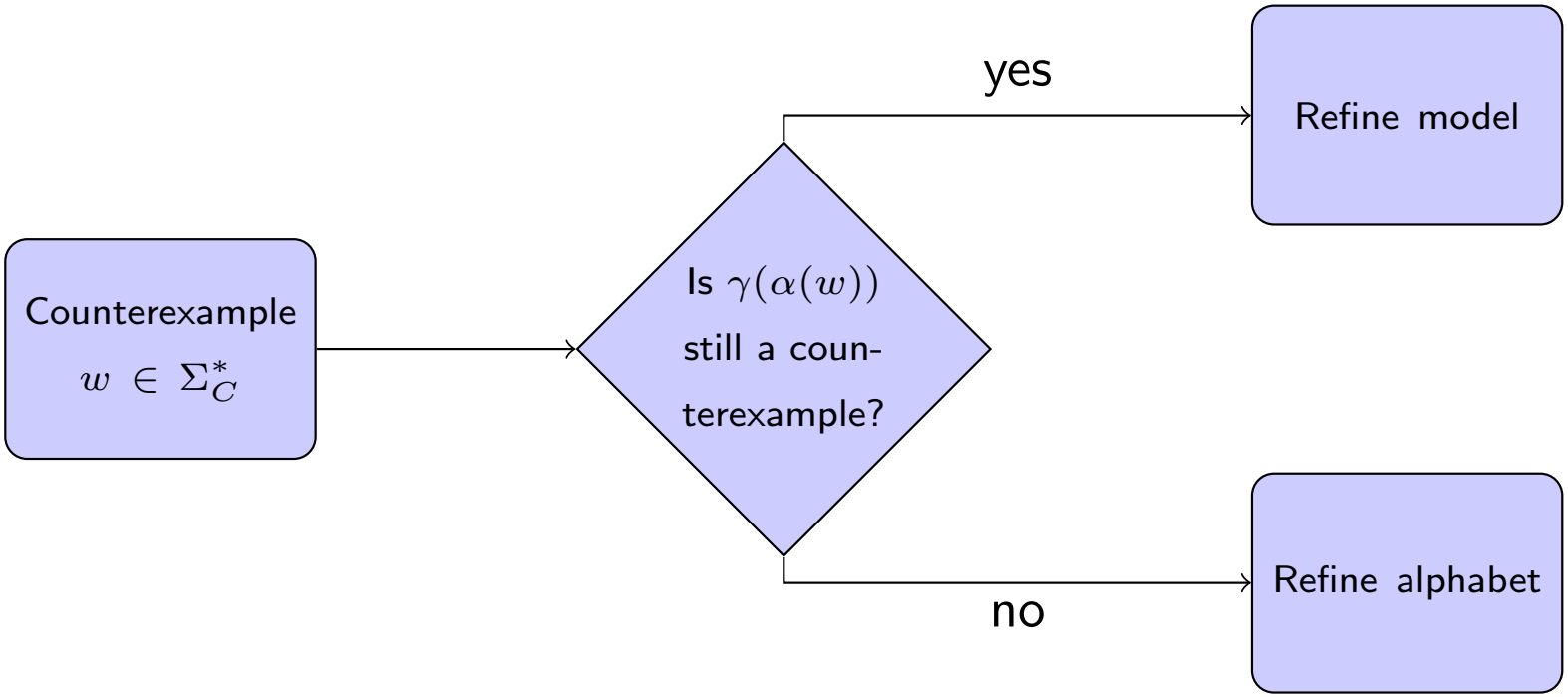

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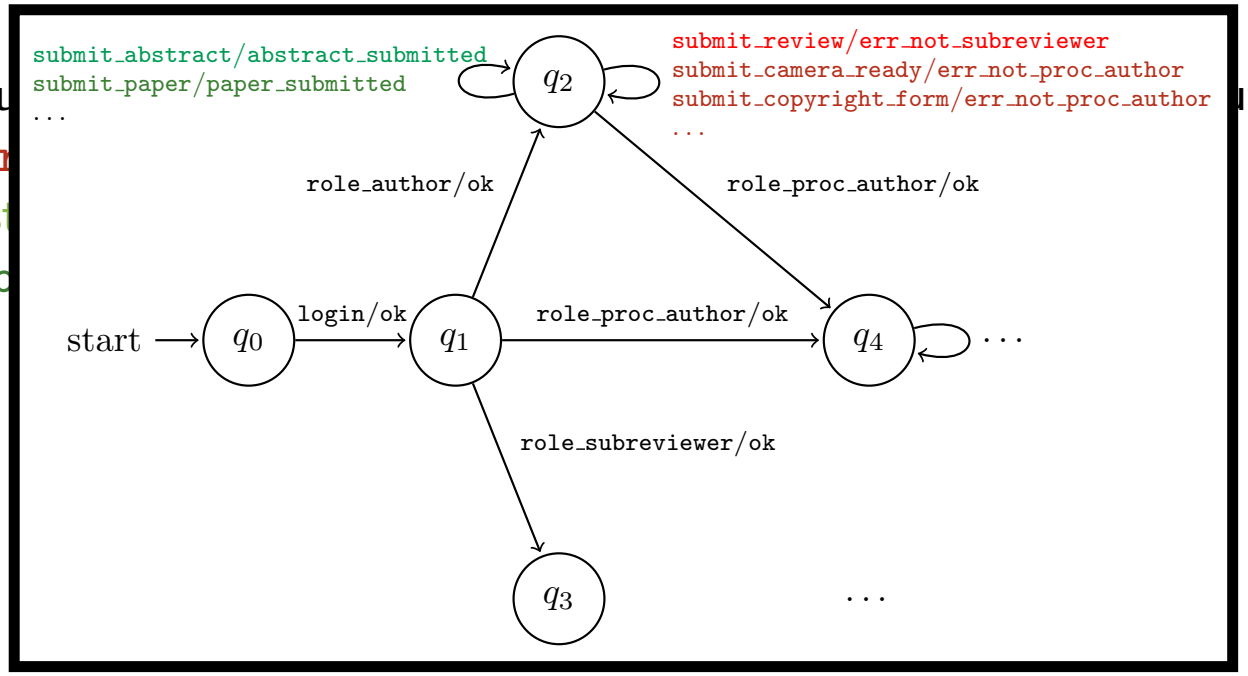

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- Idea: Abstraction at a state-local level (this paper)
- Challenges:
  - (How to distinguish states w/o a global (abstract) alphabet?)
  - How to organize per-state black-box alphabet abstractions?
  - How to refine models and/or abstractions from counterexamples?

Organizing Abstractions – Abstraction Trees

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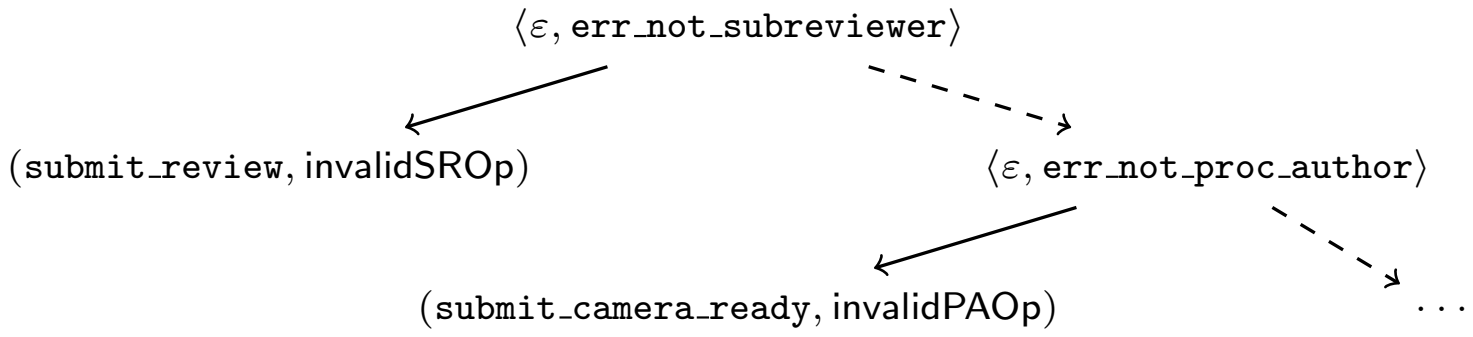

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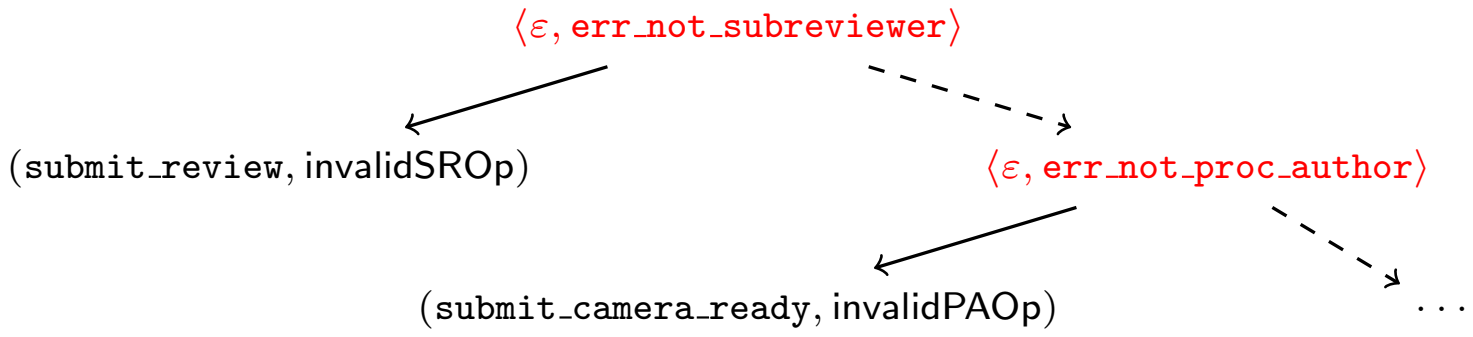

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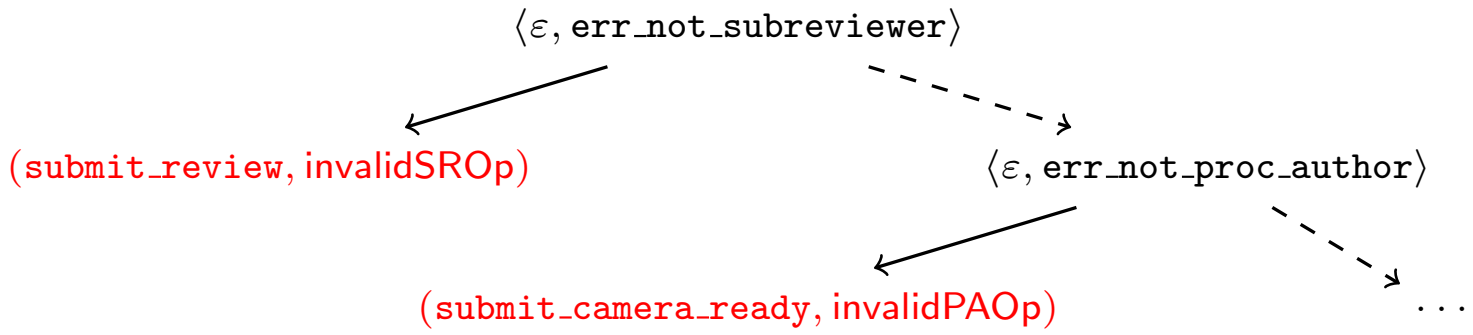

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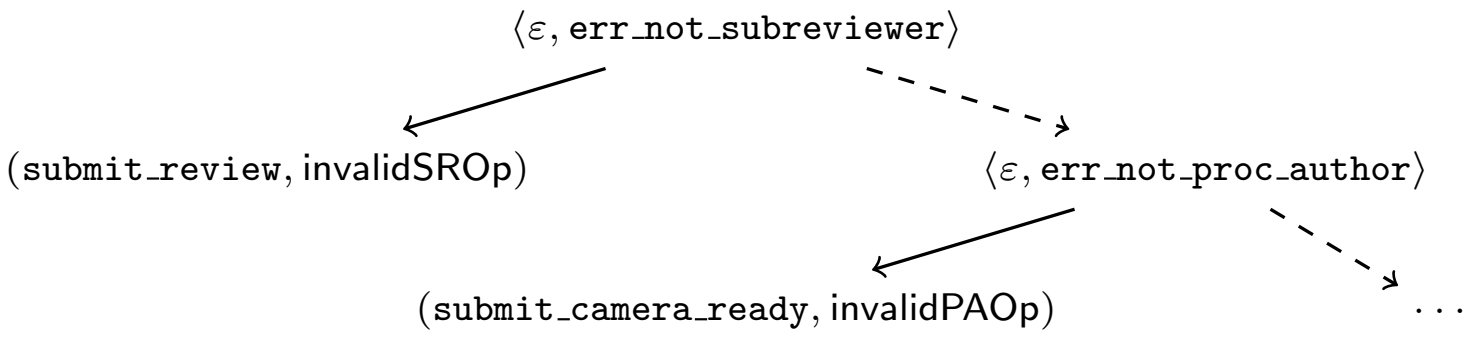
Abstraction Trees



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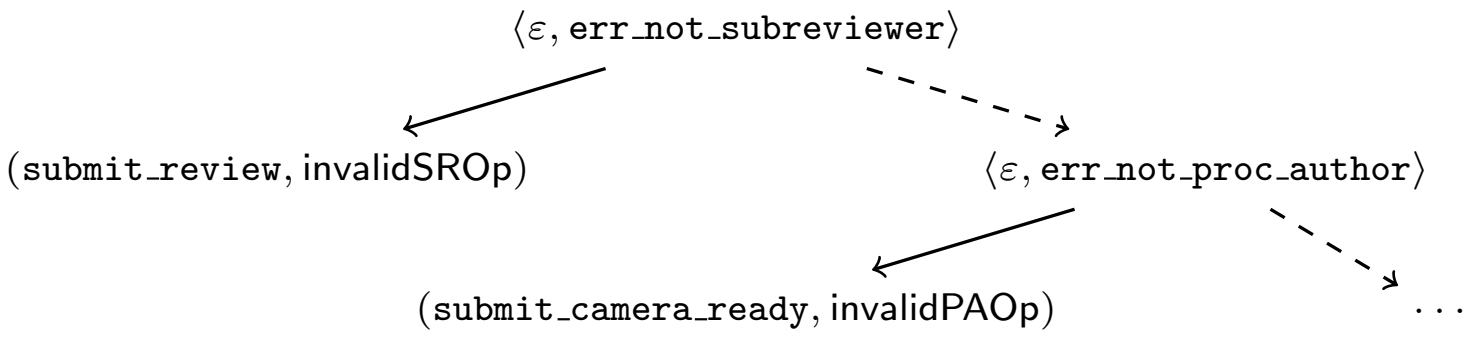

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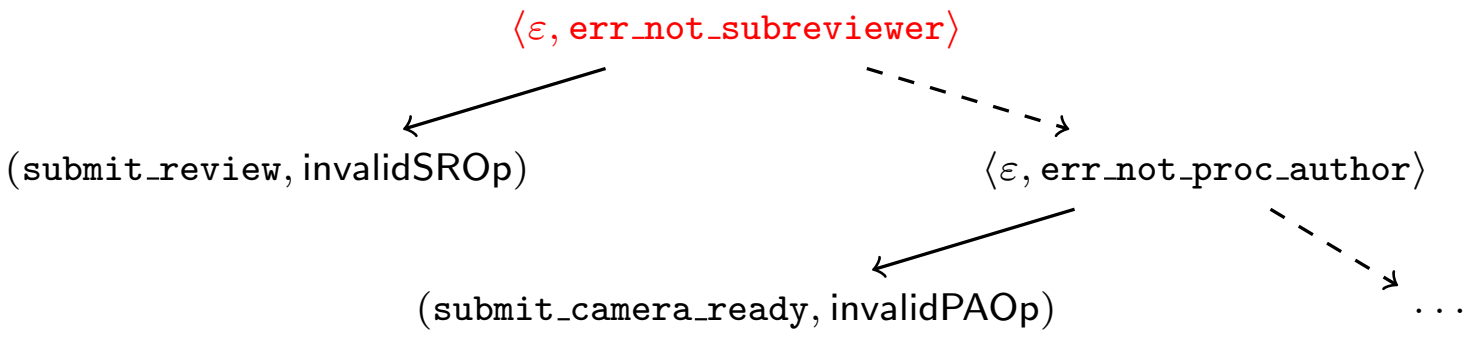

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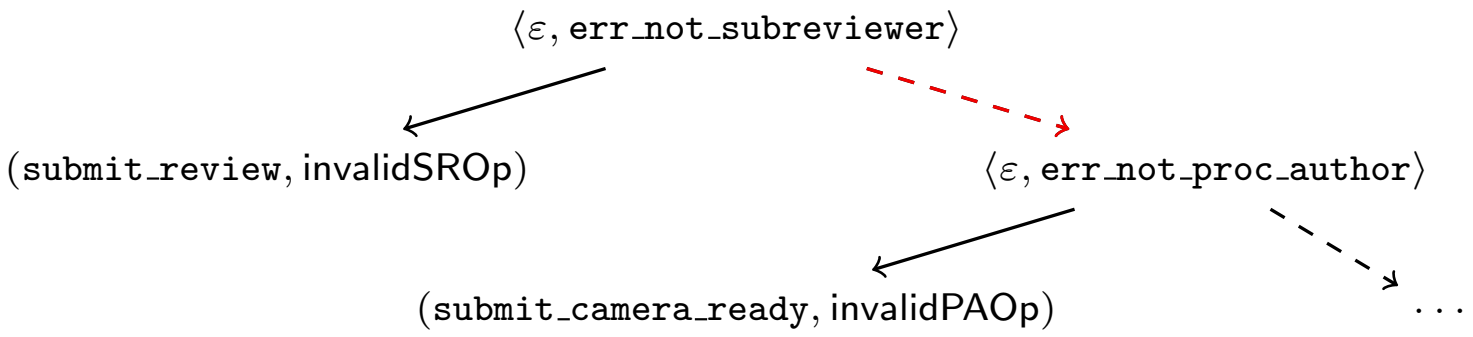

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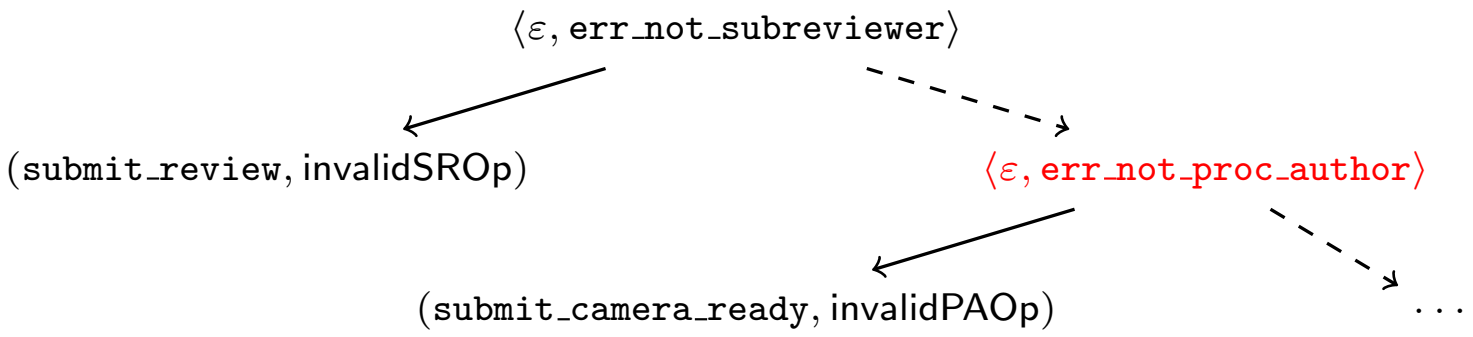

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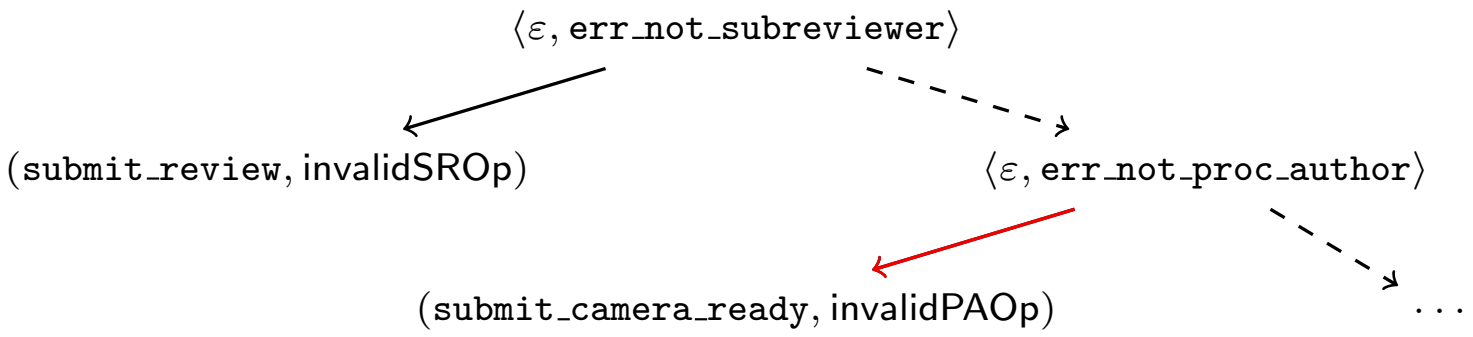

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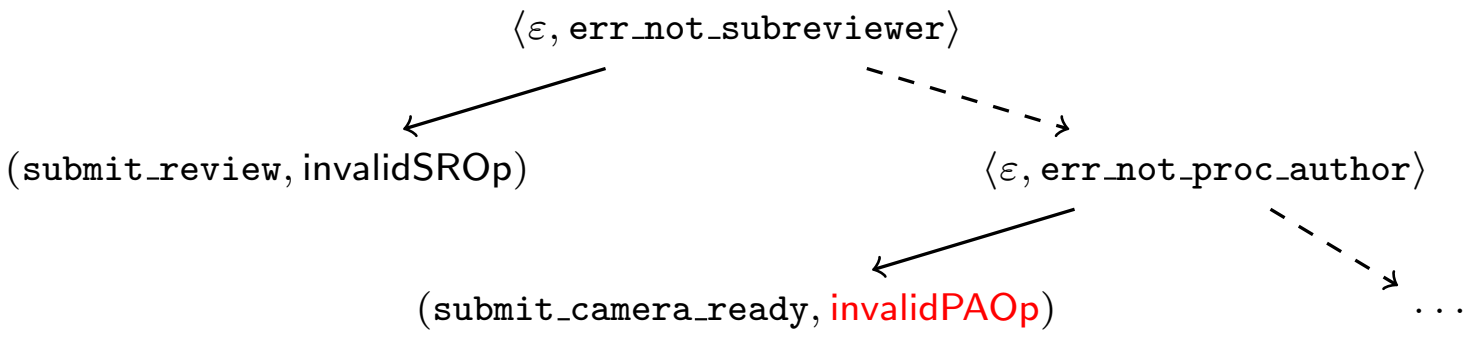

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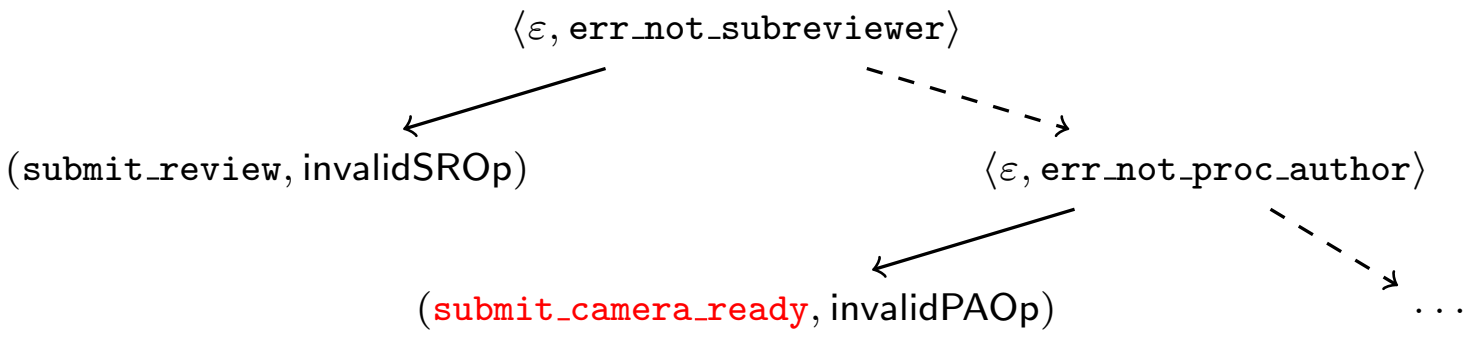

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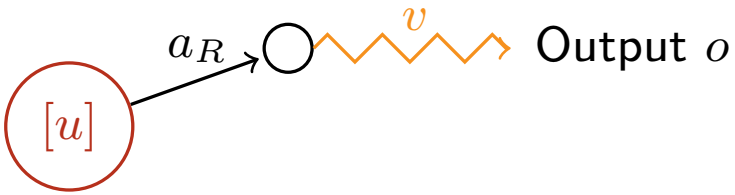

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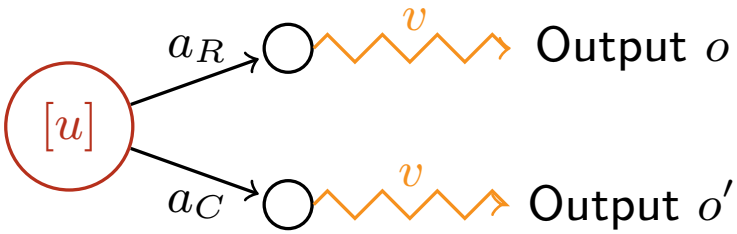

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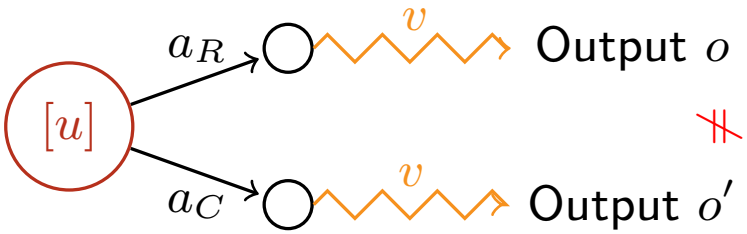

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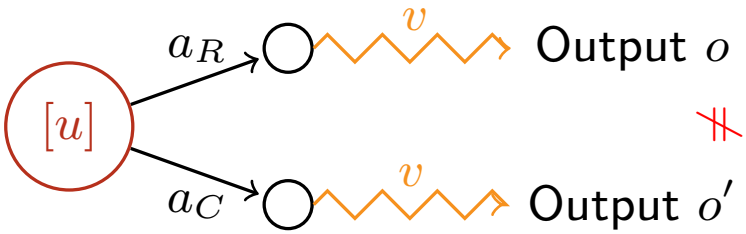

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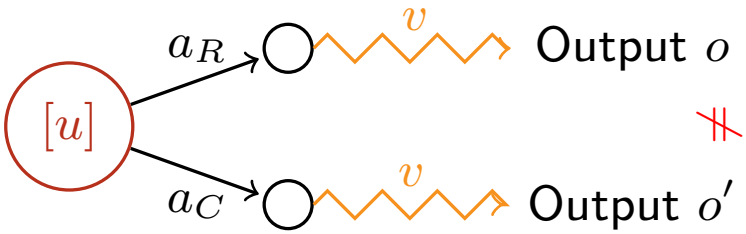

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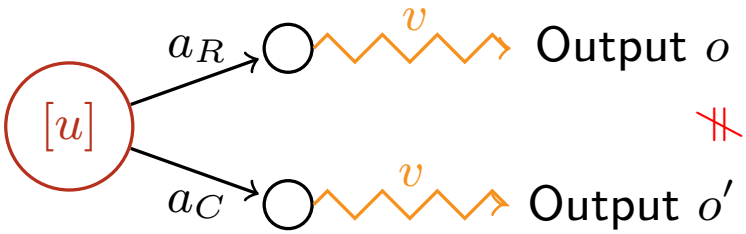

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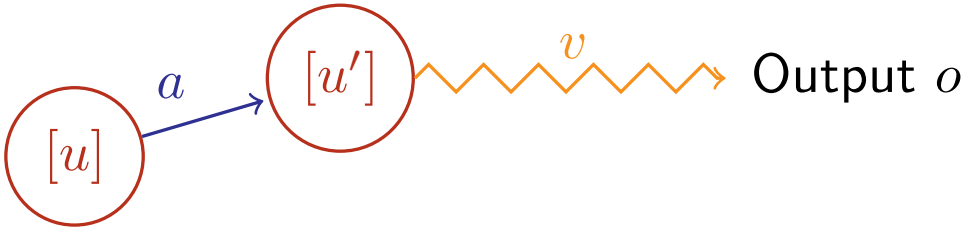

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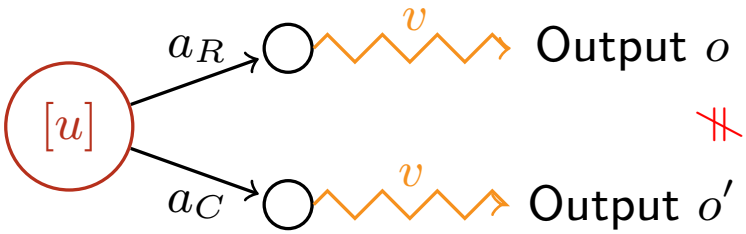

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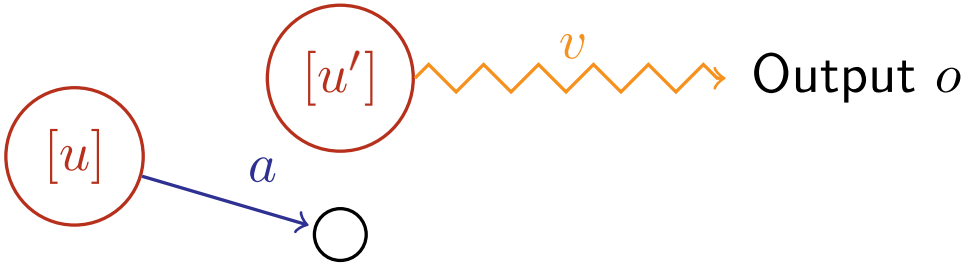

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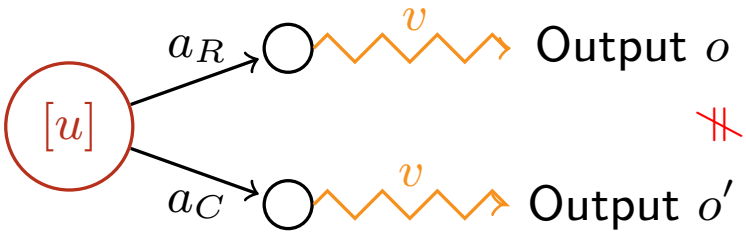

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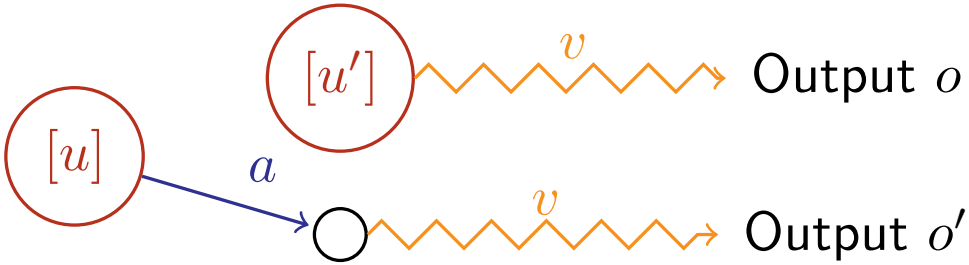

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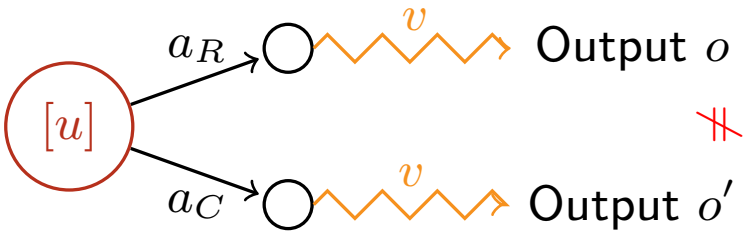

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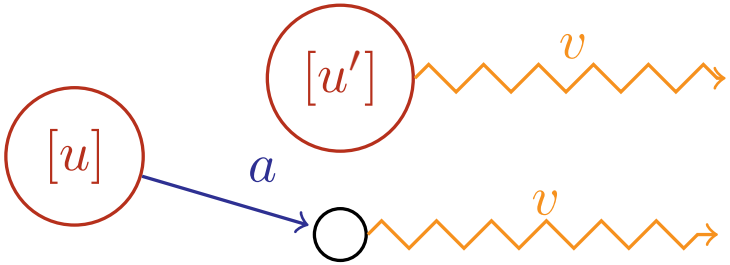

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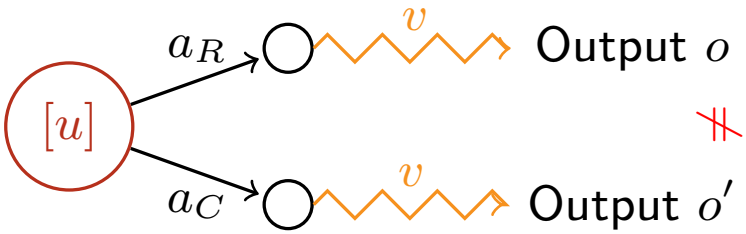

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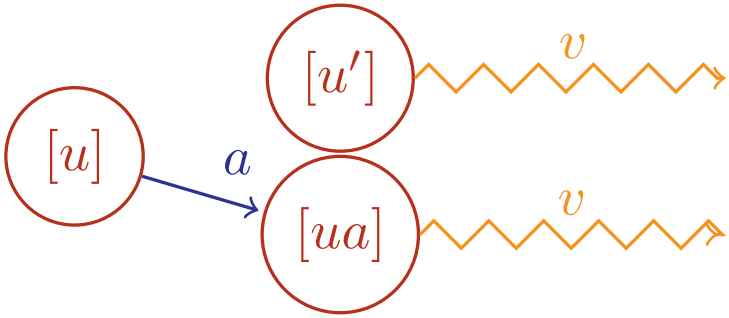

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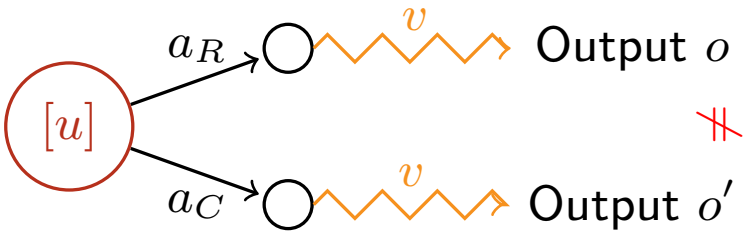

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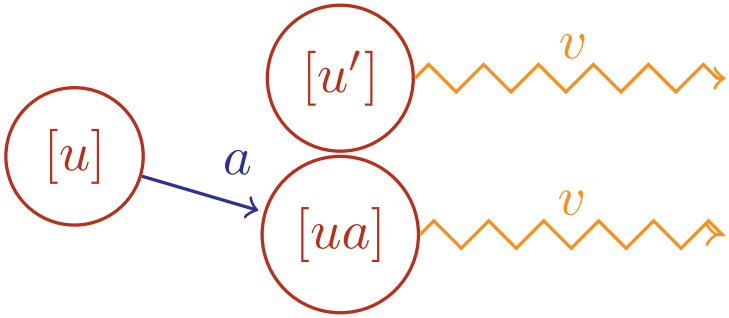

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CCC	320,200	1	51s	10,953	**201**	5m55s	2,600	204	22s
FHN1	190,122	3	28s	39,456	**93**	39s	3,798	99	4s
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BV8	5,994,280	24	25m51s	677,176	287	16m49s	25,168	116	1m43s
BV9	19,370,515	15	2h00m09s	890,475	535	53m48s	39,507	99	2m54s
BV10	80,473,959	26	13h19m02s	2,795,183	1060	8h57m32s	80,455	111	10m13s

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Example	L^*_M			Global AAR			Local AAR		
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BV7	1,269,564	14	4m27s	175,458	146	2m00s	10,926	91	26s
BV8	5,994,280	24	25m51s	677,176	287	16m49s	25,168	116	1m43s
BV9	19,370,515	15	2h00m09s	890,475	535	53m48s	39,507	99	2m54s
BV10	80,473,959	26	13h19m02s	2,795,183	1060	8h57m32s	80,455	111	10m13s

- "Model" application for local alphabet abstraction, global abstraction performs poorly

Conclusions

- Algorithm for simultaneously inferring
  - a model (Mealy machine)
  - a locally coarsest determinism-preserving alphabet abstractions
- Very natural extension of "classical" automata learning
- Pure black-box technique
  - also applicable to white-box scenarios (e.g., JPF Psyco)
- considerably reduces the number of membership queries
  - more equivalence queries, but more room for clever search heuristics

Questions?

Q & A

L^* – Correctness & Convergence

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  - Trivial correctness argument

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  - Convergence guaranteed ...
- ... for perfect equivalence oracles
  - Equivalence queries in general undecidable
  - Various heuristics, very much setting-dependent

Handling Counterexamples

- Cleverly exploiting counterexamples key to advanced learning techniques
- Counterexample triggers model refinement (new state) or alphabet refinement
- Idea: Transformation to representatives ([Rivest&Schapire, 1989] for model refinement)
  - $\lfloor \cdot \rfloor : \Sigma^* \to \Sigma^*$ maps words to short prefix of their target state (hypothesis)
  - Counterexample $w = u \cdot a \cdot v$
  - $MQ(\lfloor u \rfloor a \cdot v) \neq MQ(\lfloor ua \rfloor \cdot v)$: transition target (long prefix) $\lfloor u \rfloor a$ differs from hypothesis state (short prefix) $\lfloor ua \rfloor$ (suffix v exposes difference)
  - Provides state $\lfloor u \rfloor$ with wrong transition (not used in L^*)

Local Alphabet Refinement

- Transformation to representative symbol: $\gamma(\alpha(\cdot))$
- Perform check: $MQ(\lfloor u \rfloor \cdot a \cdot v) \neq MQ(\lfloor u \rfloor \cdot \gamma(\alpha(a)) \cdot v)$?
  - Split leaf for $\alpha(a)$ in abstraction tree for $\lfloor u \rfloor$
  - Witness: $\langle v, MQ(\lfloor u \rfloor \cdot a \cdot v) \rangle$

Alphabet vs. Model Refinement

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- Discriminate using futures v (different rows in observation table)
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Combination of both methods/analyses: interleaving (ce. $w = w_1 \dots w_n$)

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\stackrel{?}{=} \; & MQ(\lfloor \gamma(\alpha(w_1))\gamma(\alpha(w_2)) \rfloor \cdot w_3 \dots w_n)
\end{aligned}
$$

$$
\begin{aligned}
\stackrel{?}{=} \; & MQ(\gamma(\alpha(w_1)) \cdot w_2 \dots w_n) \\
\stackrel{?}{=} \; & MQ(\lfloor \gamma(\alpha(w_1)) \rfloor \gamma(\alpha(w_2)) \cdot w_3 \dots w_n) \\
\stackrel{?}{=} \; & \dots
\end{aligned}
$$