A Dynamical Physics Model of Nominal and Faulty Operational Modes of Propellant Loading (Liquid Hydrogen): From Space Shuttle to Future Missions

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Overview

In this report we develop a dynamical physics model describing the process of rocket refueling by liquid hydrogen and different possible faults in the filling system. This model describes the complex dynamics of hydrogen filling from a storage tank to an external tank and the influence of different faults on this process. The model takes into account the main physical processes such as highly non-equilibrium condensation of the hydrogen vapor, pressurization, and also the dynamics of liquid hydrogen and vapor flows inside the system in the presence of helium gas. We develop self-consistency theory of the dynamical condensation-evaporation processes and show that the effect of hydrogen vapor condensation blocking occurs during the liquid fuel filling. The model incorporates multiple faults in the system providing a suitable framework for model-based diagnostics and Heath Management algorithms. We also develop an accurate algorithm for numerical simulations of the developed model. We also provide a manual for the Matlab code simulating the refueling process.*

This report is an extension of the previous work:

Sun Hwan Lee, Vadim N. Smelyanskiy, Viatcheslav V. Osipov, Cyrill B. Muratov, Anupa Bajwa, Igor Kulikov, "A dynamical model of nominal and faulty modes of rocket refueling by liquid hydrogen: from Space Shuttle to future missions", Proceeding of 2008 Integrated Systems Health Management (ISHM) Conference, 11-15 August 2008, Northern Kentucky Convention Center, Covington, KY

V.V. Osipov and C.B. Muratov, “Dynamic Condensation Blocking in Cryogenic Refueling,” *Applied Physics Letters*, vol. 93, 2008.

* Software available by request. Contact Vadim Smelyanskiy: Vadim.N.Smelyanskiy@nasa.gov

Stages of Operation

1. **Initial purges**
   Cleaning of Transfer line, ET, Tail Service mast (TSM), Umbilical Carrier and Disconnect Cavity
2. **Chilldown**
   Allow LH2 into fueling system lines, but not into ET
3. **Slow fill** (ET is 0% to 5% fill)
4. **Fast fill and Reduced fast fill** (ET 5% to 98% fill)
5. **Topping** (ET 98%-100% fill)
6. **Replenish at 100%**
   Maintaining ET at 100% full as LH2 boils off/evaporates

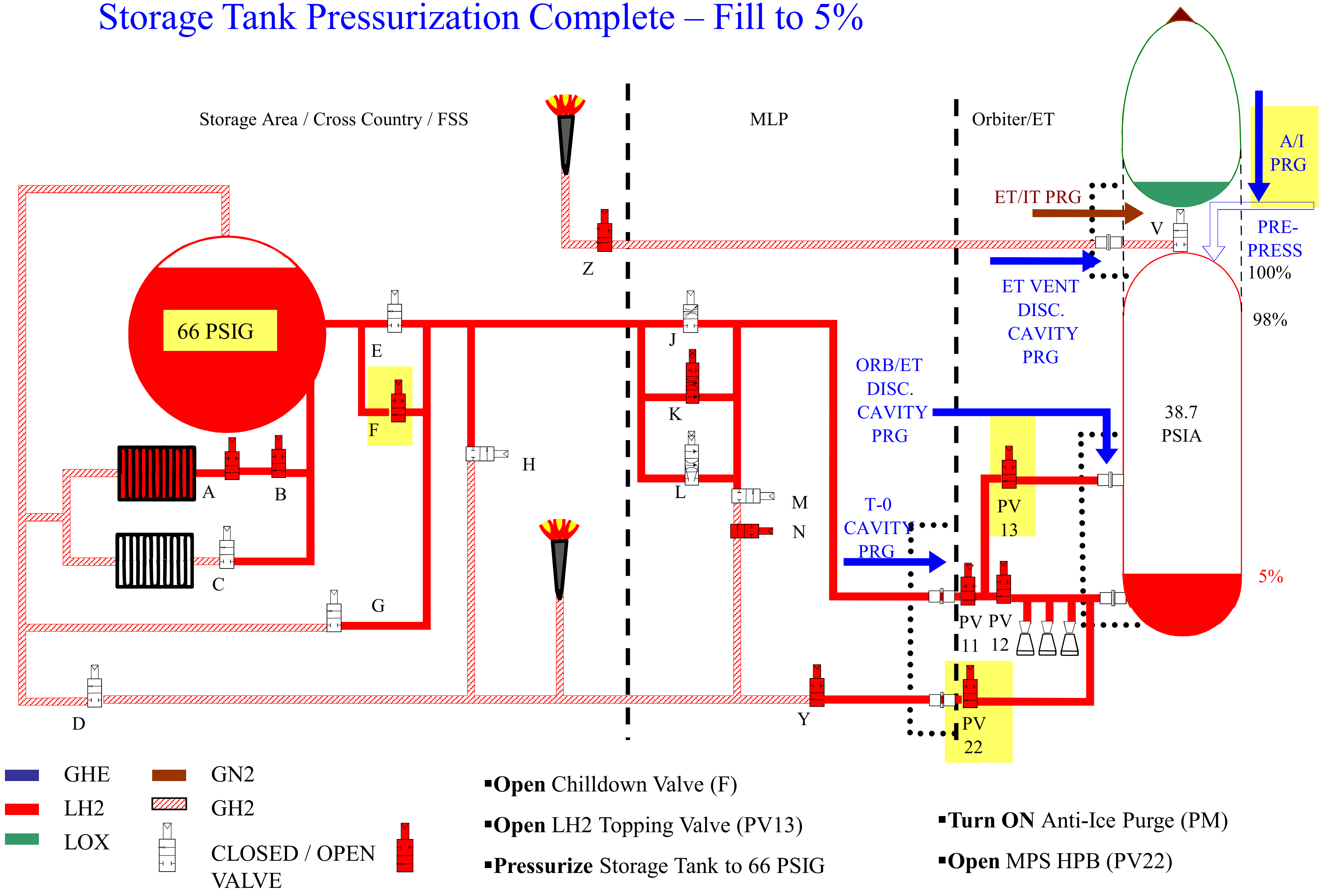
Stages described by our model

7. **Terminal Countdown Sequence**
   Timed closing of valves in order to shut down systems as they are no longer needed while launch approaches
8. **Postlaunch** A system check of "quick disconnects" integrity through the use of pressure transducers and purge lines
9. **Postlaunch Line Drain Purges**
   Cleaning of Transfer Line
10. **Postlaunch Purging**
    Cleaning of system beyond transfer line

LH2 Slow Fill

Storage Tank Pressurization Complete – Fill to 5%

Figure courtesy Shuttle Program and Marshall Space Flight Center



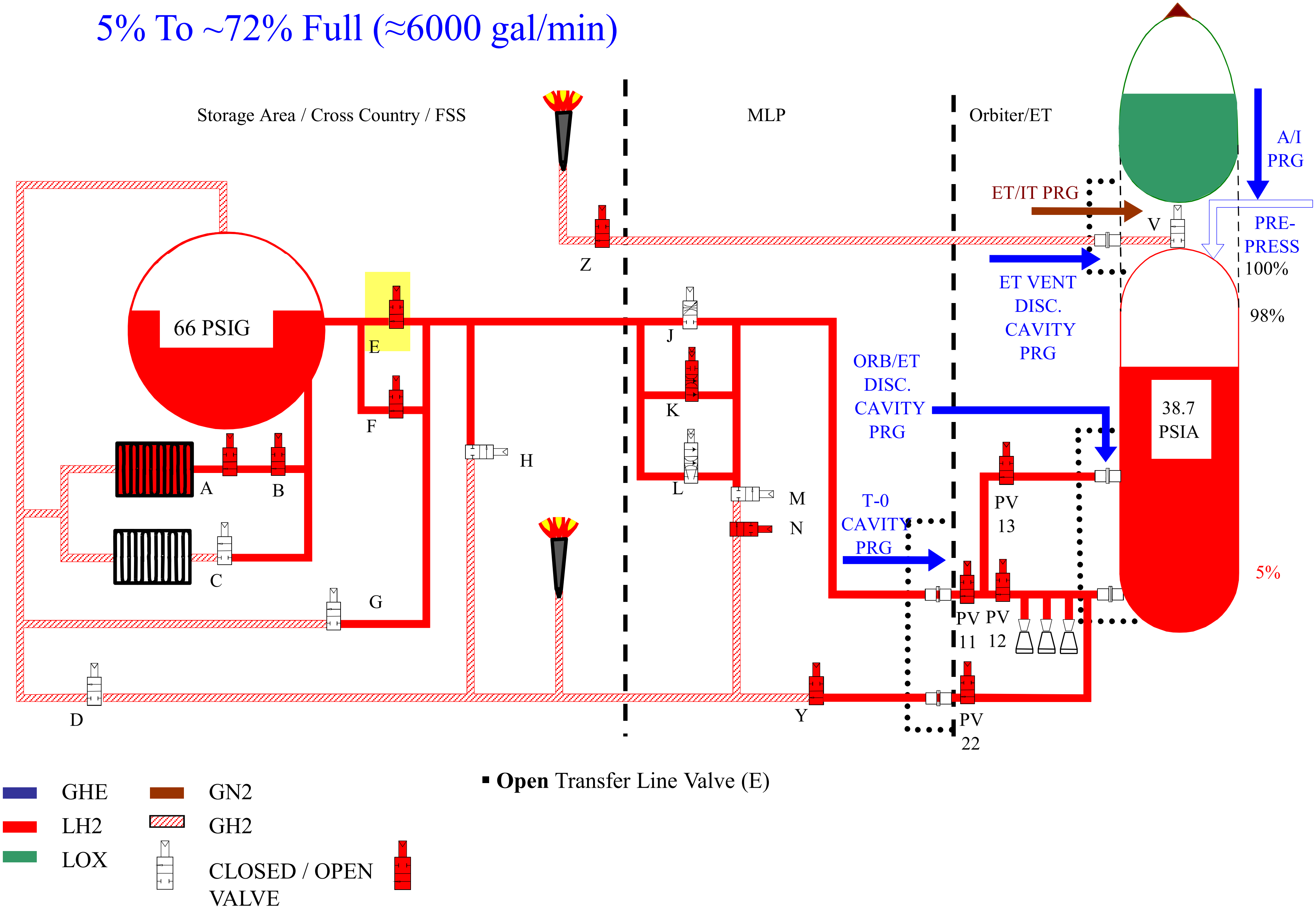
- **Open** Chilldown Valve (F)
- **Open** LH2 Topping Valve (PV13)
- **Pressurize** Storage Tank to 66 PSIG

- **Turn ON** Anti-Ice Purge (PM)
- **Open** MPS HPB (PV22)

LH2 Fast Fill

5% To ~72% Full (≈6000 gal/min)

Figure courtesy Shuttle Program and Marshall Space Flight Center

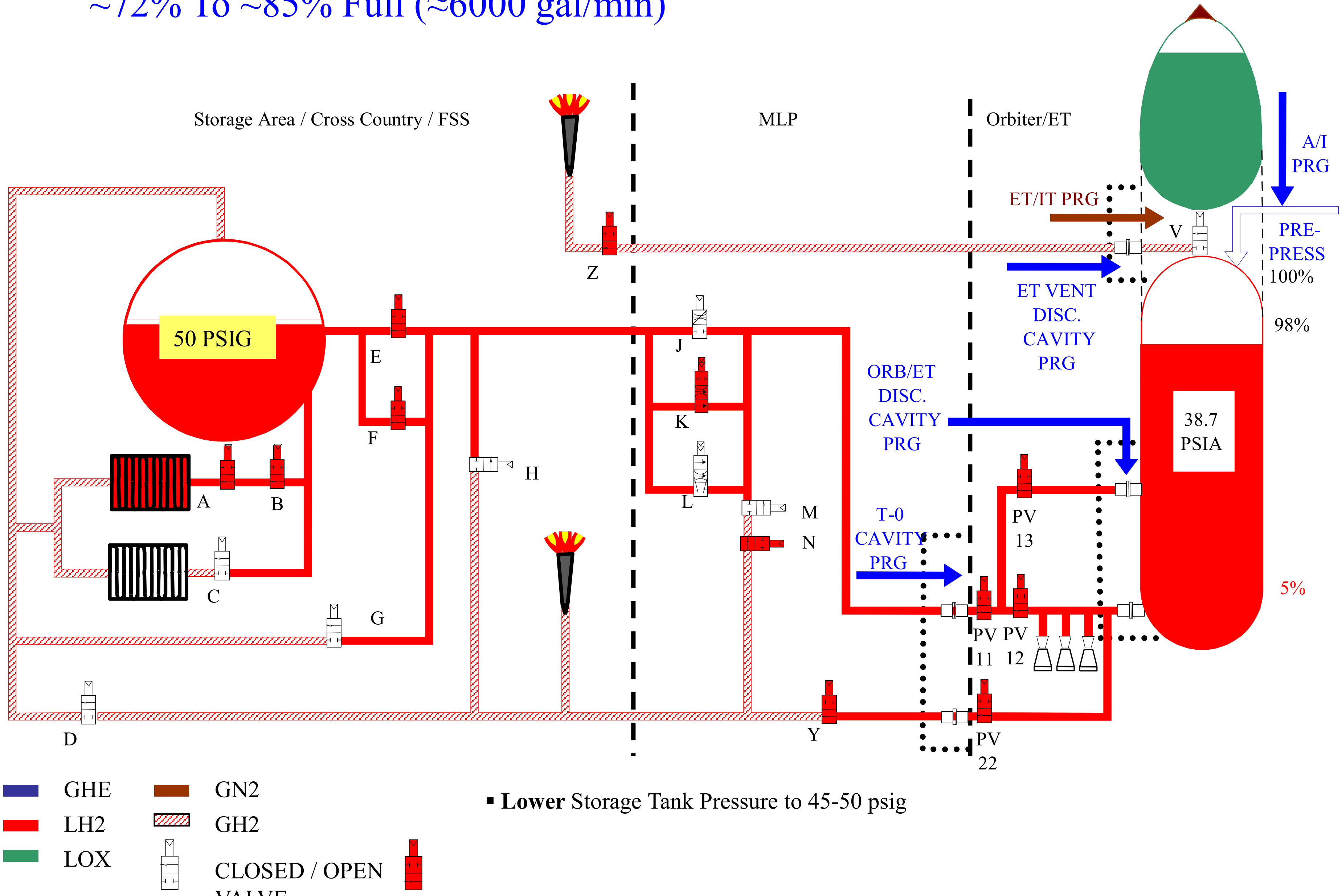


- **Open** Transfer Line Valve (E)

LH2 Fast Fill

~72% To ~85% Full (≈6000 gal/min)

Figure courtesy Shuttle Program and Marshall Space Flight Center

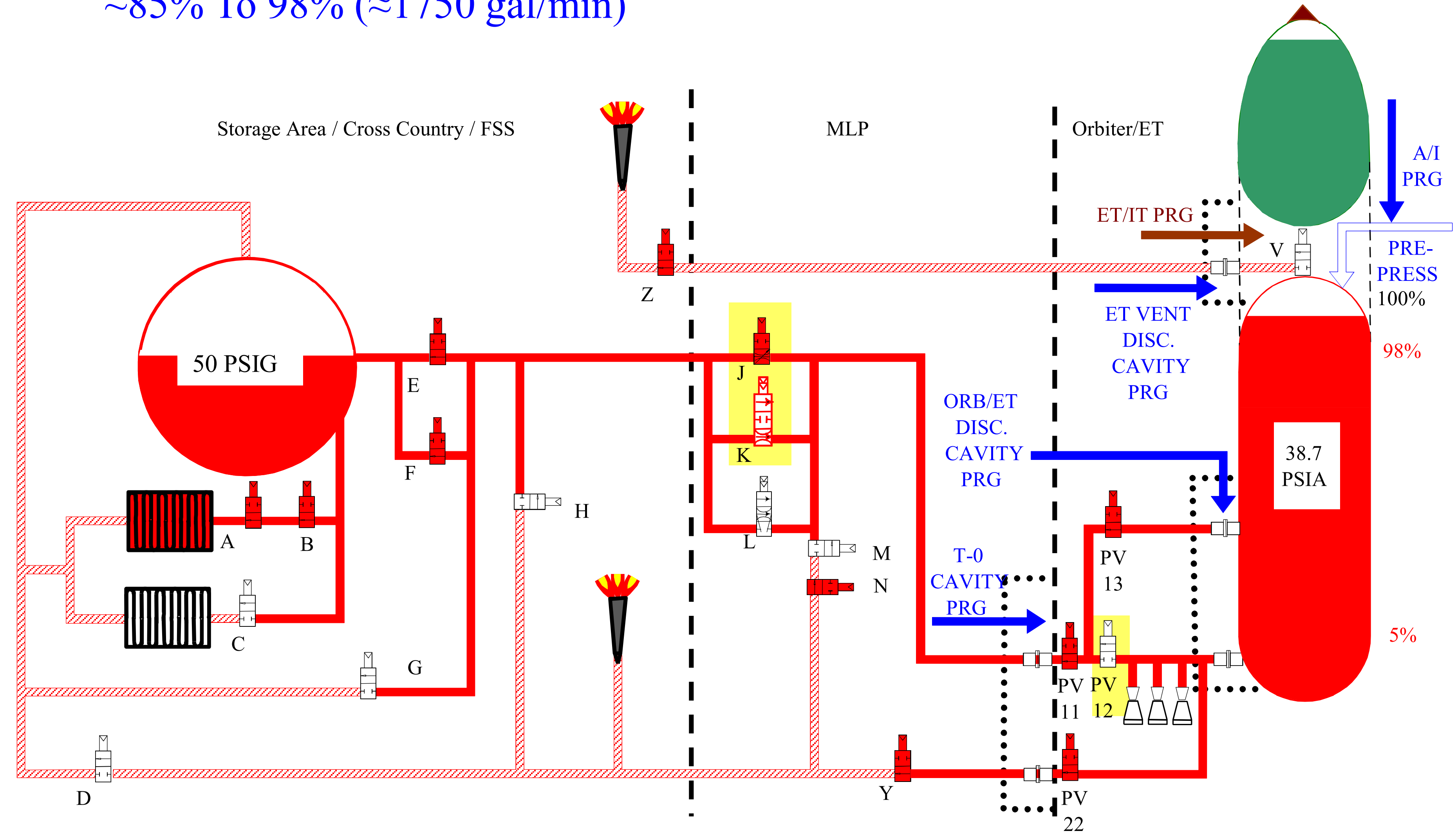


- **Lower** Storage Tank Pressure to 45-50 psig

LH2 Reduced Fast Fill

~85% To 98% (≈1750 gal/min)

Figure courtesy Shuttle Program and Marshall Space Flight Center



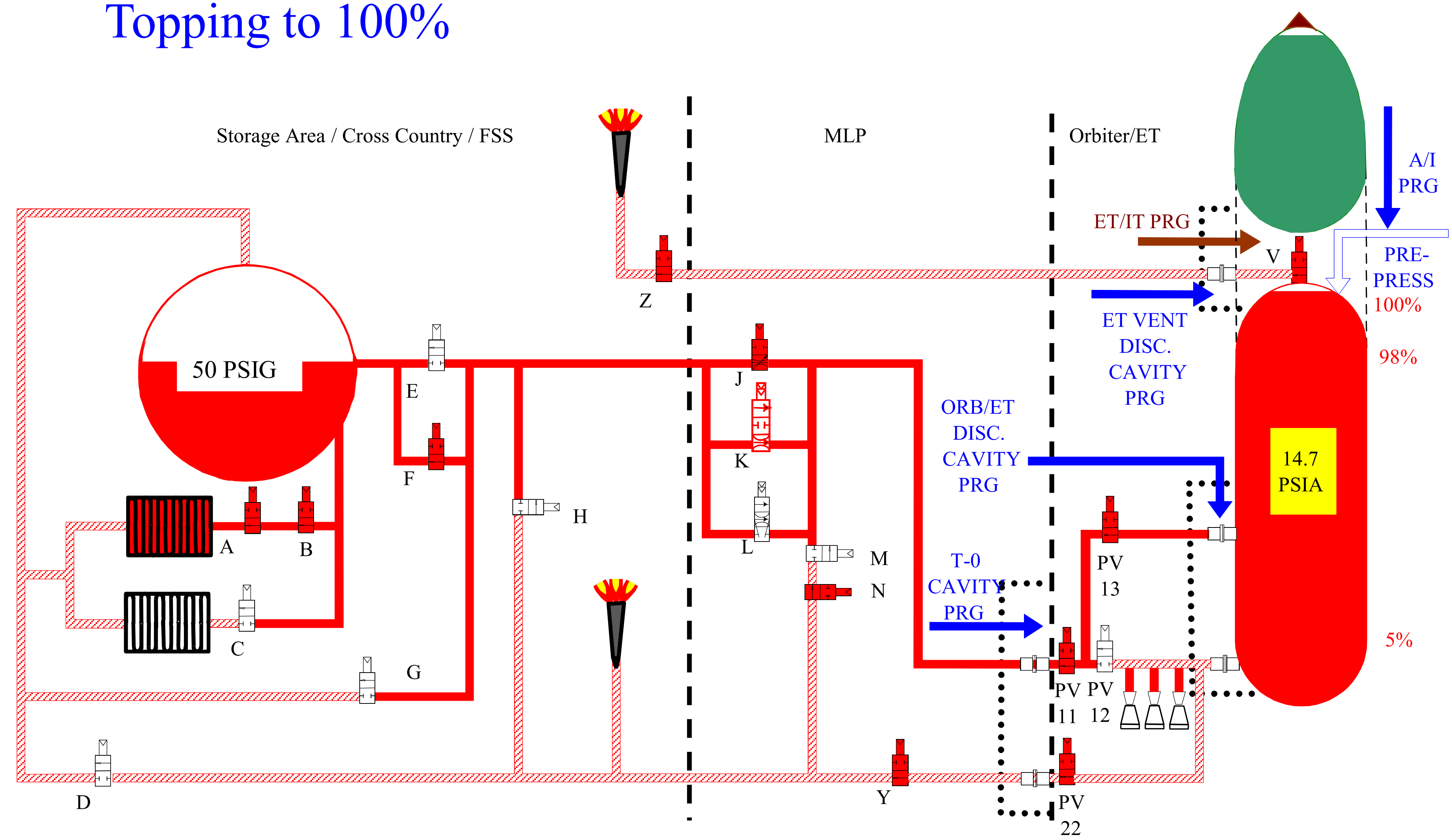
🟦 GHE
🟥 LH2
🟩 LOX
🟫 GN2
▨ GH2
CLOSED / OPEN VALVE

- **Open** Replenish Valve (J)
- **Reduce** Main Fill Valve (K)
- **Close** LH2 Inboard Fill/Drain Valve (PV12)

LH2 Topping

Topping to 100%

Figure courtesy Shuttle Program and Marshall Space Flight Center



🟦 GHE

🟥 LH2

🟩 LOX

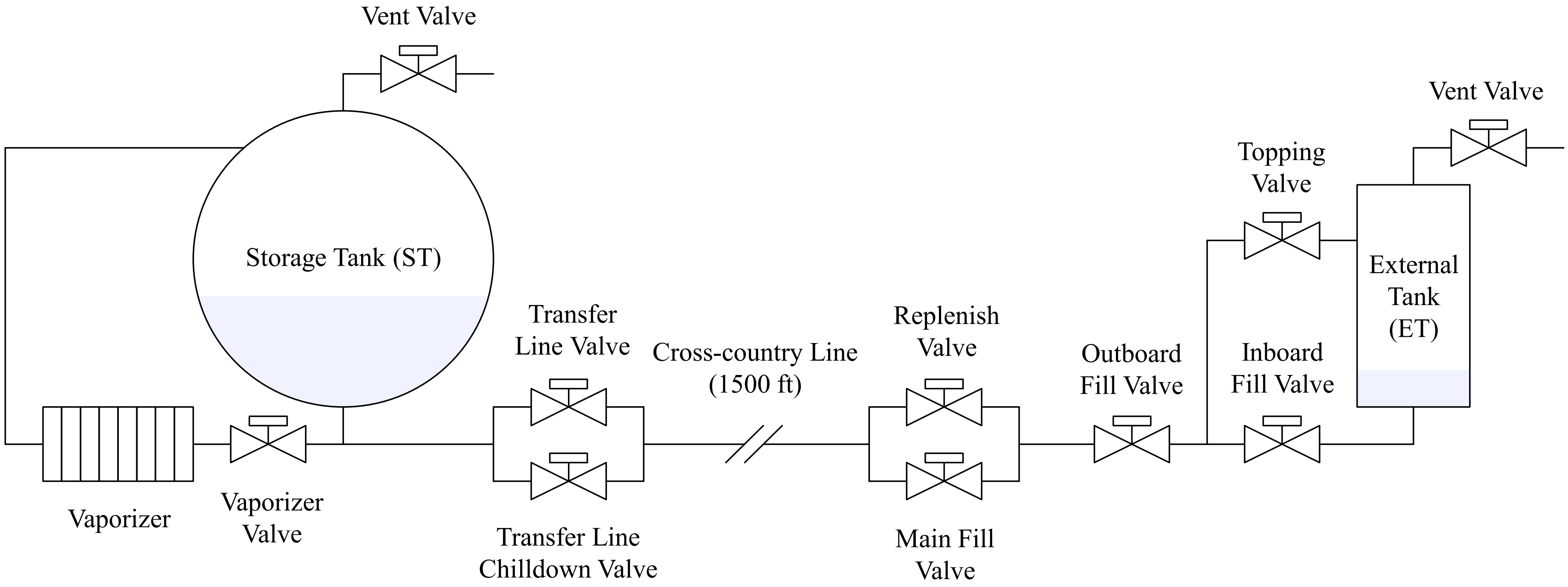
🟫 GN2

▨ GH2

CLOSED / OPEN
VALVE

- **Close** Transfer Line Valve (E)
- **Open** ET Vent Valve (V)

Simplified LH2 Propellant Loading Schematic



Summary of the Model

- Reduced dynamical model has been developed for LH2 loading system . This model consists of Storage Tank (ST) and External Tank (ET) and Transmission Line (TL) that operates in all filling regimes.

- Two vent valves were considered (in ET and ST).
  - ET vent valve operated at two threshold pressures 38.7 and 41.7 psia
  - ST vent valve operated at one threshold pressure of 64.7 psig or 50 psig (depends on filling regime)

- ET initially filled with only GH2 up 14.7 psia. Pressurization of ET performed using heated GHe.

- Heat transferred from the vaporizer to LH2 in ST goes into a latent heat of a phase transformation from LH2 to GH2 thus supporting the flows of vapor mass and enthalpy from the LH2 control volume into the control volume of GH2 in ST.

- Transmission line is modeled by a cross-country transfer pipe and 7 valves to control mass flow rate

Summary of the Model (cont.)

- Major and minor head (pressure) losses due to friction were considered in laminar and turbulent flow regimes with linear interpolation between them. Our loss model can be easily extended to consider a realistic pipe network given the specifications for all system elements such as, pipes, tees, elbows, expansions/contractions, vent valves, etc.
- The model accounts for radiation, conduction and convection heat exchange with the environment for both tanks and the transmission line.
- The model describes nominal regime and various faults, including gas, liquid and heat leaks, vent valve clogging/leakage and others

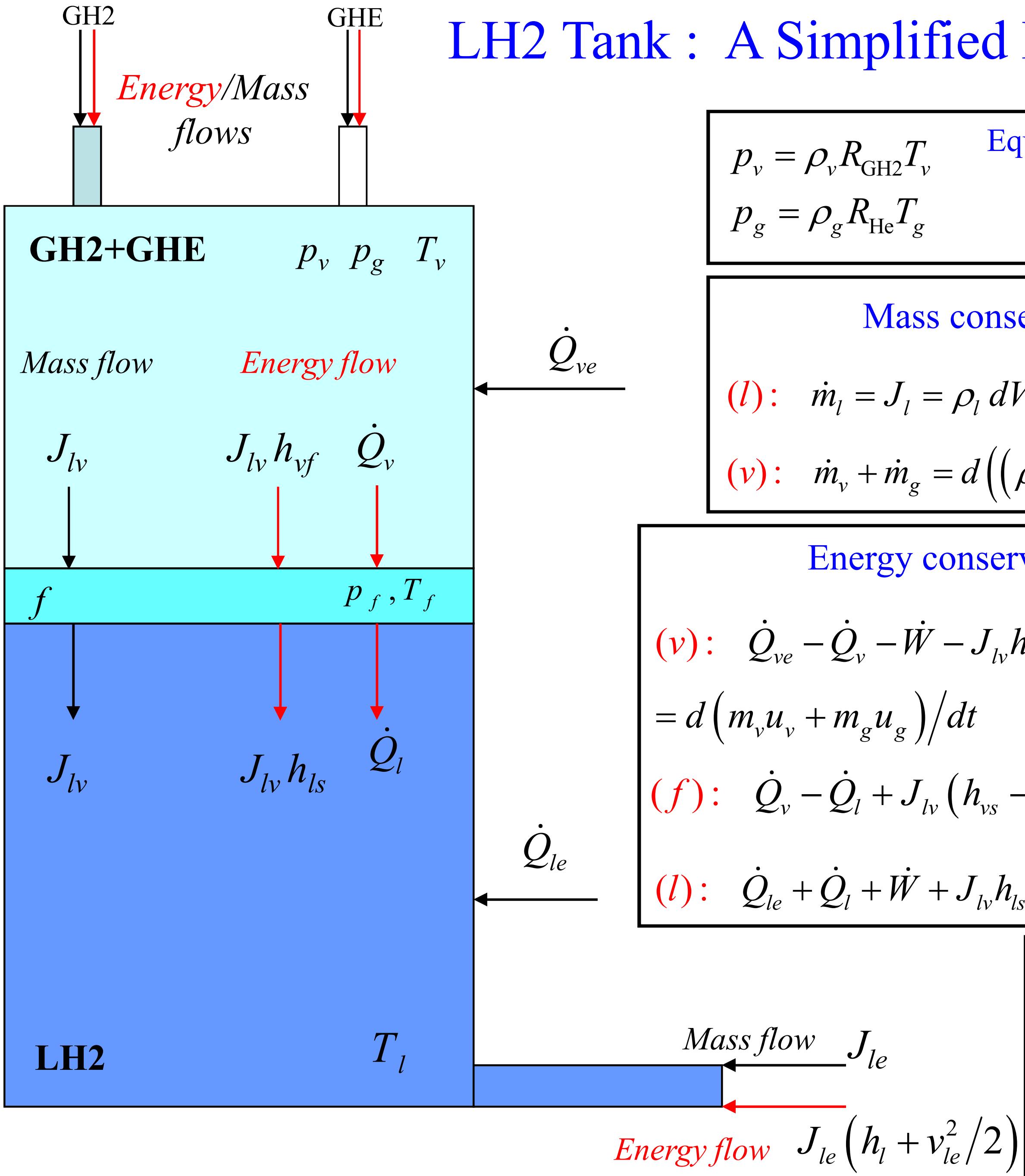
Summary of the Model (cont.)

- The reduced model is described by 18 state variables
  - LH2 , GH2 and GHE masses in Storage Tank (ST)
  - LH2 , GH2 and GHE masses in External Tank (ET)
  - Partial pressures of GH2 and GHE in ET
  - Partial pressures of GH2 and GHE in ET
  - Gas volumes in ST and ET
  - Gas and liquid temperatures in ST and ET (4 variables)
  - Film temperatures in ST and ET

- There are total 6 constraints on state variables
  - Equations of state for LH2, GHE in ST
  - Equations of state for LH2, GHE in ST
  - Gas volume in ST is expressed via LH2 mass and ST volume
  - Gas volume in ET is expressed via LH2 mass and ST volume

- There are total 12 ordinary differential and integral equations for 12 state variables

LH2 Tank : A Simplified Model



Equations of state:

$$p_v = \rho_v R_{\mathrm{GH2}} T_v$$

$$p_g = \rho_g R_{\mathrm{He}} T_g$$

$$p_f\left(T_f\right) = p_C\left(T_f / T_C\right)^{\lambda}$$

Mass conservation:

(l): $\dot{m}_l = J_l = \rho_l\, dV_l / dt = J_{le} + J_{lv}$

(v): $\dot{m}_v + \dot{m}_g = d\left(\left(\rho_v + \rho_g\right)V_v\right) / dt = J_{ve} + J_{ge} - J_{lv}$

Energy conservation:

(v): $\dot{Q}_{ve} - \dot{Q}_v - \dot{W} - J_{lv} h_{vs} + J_{ve}\left(h_{ve} + v_{ve}^2/2\right) + J_{ge}\left(h_{ge} + v_{ge}^2/2\right)$
$= d\left(m_v u_v + m_g u_g\right)/dt$

(f): $\dot{Q}_v - \dot{Q}_l + J_{lv}\left(h_{vs} - h_{ls}\right) = d\left(m_f u_f\right)/dt = 0$

(l): $\dot{Q}_{le} + \dot{Q}_l + \dot{W} + J_{lv} h_{ls} + J_{le}\left(h_l + v_{le}^2/2\right) = d\left(m_l u_l\right)/dt$

a) Conduction:

$$\dot{Q}(t) = A\left(\frac{\kappa c \rho}{\pi}\right)^{1/2} \int_0^t \frac{d\tau}{\left(t-\tau\right)^{1/2}} \frac{\partial T_f(\tau)}{\partial \tau}$$

b) Convection: $\dot{Q} = A\alpha_f\left(T_f - T\right)$

Vaporizer

- When vaporizer valve open, LH2 flows into vaporizer
- LH2 accumulates in vaporizer and vaporizes into GH2
- Vaporizer valve position depends on signal pressure and flow regime pressure set point

Flow into vaporizer depends on tank pressure and valve position

$$J_{vap} = c_{vap} \lambda_{vap} \sqrt{p_1 - p_{vap}}$$

$$\dot{J}_{boil} = \frac{1}{\tau_{vap}} \left(J_{vap} - J_{boil} \right)$$

Vaporizer has a lag defined by τ_{vap}

Conservation of mass in the vaporizer

$$\dot{m}_{vap} = J_{vap} - J_{boil}$$

External Tank Chilling

- When slow fill begins, ET is not chilled
- As ET is filled, tank chills and loses heat to LH2

Tank wall heat balance

$$\dot{T}_w = \frac{1}{m_w c_w}\left(\dot{Q}_{ext} - \dot{Q}_{wl} - \dot{Q}_{wv}\right)$$

$$\dot{Q}_{ext} = \alpha_{ext} A\left(T_{amb} - T_w\right)$$

$$\dot{Q}_{wl} = \alpha_{wl}(A + Rh_l)(T_w - T_L)$$

Heat transfer through natural convection

$$\dot{Q}_{wv} = \frac{Nu\kappa_v}{L_v}\left(A + R\left(H - h_l\right)\right)\left(T_w - T_v\right)$$

$$J_{boil} = \frac{\dot{Q}_{wl}}{h_v}$$

All heat from wall to LH2 goes into boiling

Liquid and Gas Flow in Transmission Line Pipes

a) Viscous Effects: Friction Head Losses in Fluids

Steady flow pressure difference: $\Delta p = \rho g\left(h_L + \Delta z\right)$

$$h_L = h_f + \sum h_m = f\,\frac{l}{2R}\,\frac{U^2}{2g} + \sum K_m \frac{U_m^2}{2g}$$

Minor head losses

Major head loss

Mass flow rate:

$$\dot{m} = J = \pi R^2 \rho U$$

Resistance coefficient:

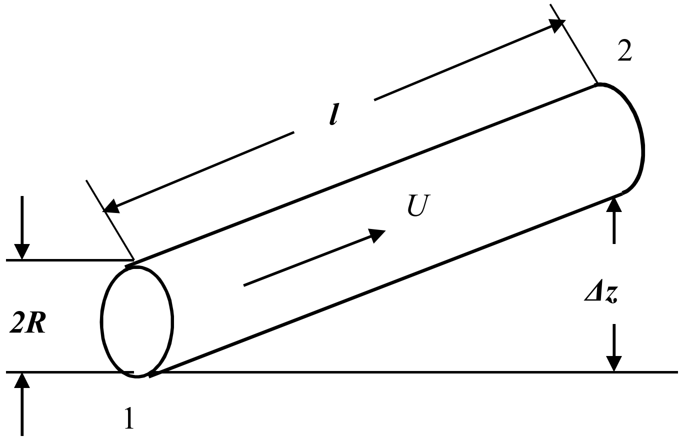
Laminar regime: $$f\left(\mathrm{Re}\right) = \frac{32\nu}{UR} = \frac{64}{\mathrm{Re}}, \quad \left(\mathrm{Re} \le 3\cdot 10^3\right)$$

Turbulent regime: $$\frac{1}{\sqrt{f}} = -0.87\log\left(\frac{d_r}{7.4R} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right), \quad \left(\mathrm{Re} > 3\cdot 10^3\right)$$

b) Viscous Laminar Flow of Compressible Gases

$$\dot{m} = J = \frac{\pi R^4}{16\mu l R_g T}\left(p_1^2 - p_2^2\right)$$ *Isothermal*

$$\dot{m} = J = \frac{\pi\gamma}{\gamma+1}\frac{R^4\rho_1 p_1}{\mu}\left(1-\left(\frac{p_2}{p_1}\right)^{1+1/\gamma}\right)$$ *Adiabatic*



Liquid and Gas Flow in Transmission Line Pipes

c) Thermal Effects in the Absence of Phase Transformations

i) Variations of Bulk Temperature T_b along the Pipe Axis x : $$Jc_P \frac{dT_b}{dx} = 2\pi R q_w$$

Uniform Heat Flux through the Wall : $$T_b\left(x\right) = T_{bi} + \frac{2\pi R q_w}{Jc_P} x$$

Temperature Drop due to Thermal Conduction: $$\Delta T_b = T_{bd} - T_{bi} = \frac{2\pi R \kappa_{ins} \Delta T_{ins} l}{Jc_P d_{ins}}$$

ii) Variations of the Wall Temperature T_w : $$\Delta T_w = T_w - T_b \cong \frac{2R\kappa_{ins}}{Nu_{av} \cdot d_{ins} \kappa_l} \Delta T_{ins}$$

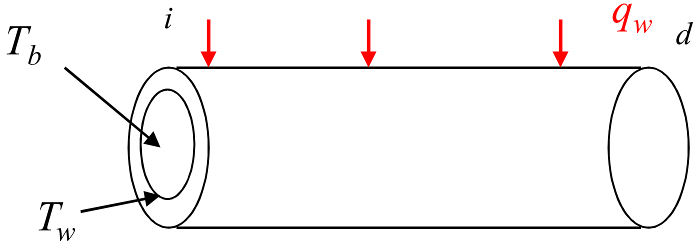
- *Laminar Flow* $(\mathrm{Re} \leq 3 \cdot 10^3)$ *:*

$$Nu_{av} = 1.95\left(Pe \cdot d / l\right)^{1/3} \qquad Pe \cdot d / l > 10^2$$

$$Nu_{av} = 4.36 \qquad Pe \cdot d / l < 10$$

$$Pe = \mathrm{Re} \cdot \mathrm{Pr}$$

- *Turbulent Flow* $(\mathrm{Re} > 3 \cdot 10^3)$ *:* $$Nu_{av} = \frac{f}{8} \mathrm{Re} \cdot \mathrm{Pr}^{1/3}$$



For LH2 $\Delta T_b \cong \left(10^{-2} - 10^{-1}\right) K$ if $l \cong (10 - 100) m$

Transmission Line Model for Different Filling Modes

Turbulent regime: $\mathrm{Re}_{pipe} > \mathrm{Re}_{cr} \approx 10^3$

$$\boxed{J_{tr} = \alpha_{eff} \Delta p_{tot}^{1/2}}$$

Laminar regime: $\mathrm{Re}_{pipe} < \mathrm{Re}_{cr} \approx 10^3$

$$\boxed{J_{tr} = \frac{\alpha_V^2}{2k_{pipe}} \left[\sqrt{1 + \frac{4k_{pipe}^2 \Delta p_{tot}}{\alpha_V^2}} - 1 \right]}$$

$$\alpha_{eff} = \left(\alpha_V^{-2} + \alpha_{pipe}^{-2} \right)^{-1/2}$$

$$\alpha_m = A_{Vm} \frac{x}{l_m} \left(\frac{2\rho_l}{K_m} \right)^{1/2} = \lambda_m(x) \alpha_m^0$$

$$\alpha_V = \left[\left(\alpha_E + \alpha_F \right)^{-2} + \left(\alpha_J + \alpha_K \right)^{-2} \right]^{-1/2}$$

$$k_{pipe} = \frac{\pi R^4}{8 \nu l}$$

$$\alpha_{pipe} = 2\pi R^2 \left(\frac{\rho_l R}{fl} \right)^{1/2}$$

$\{\lambda_m = x/l_m\}$ set defines a filling mode:

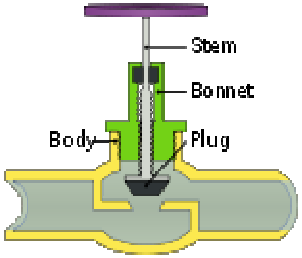
a) slow fill,

b) fast fill,

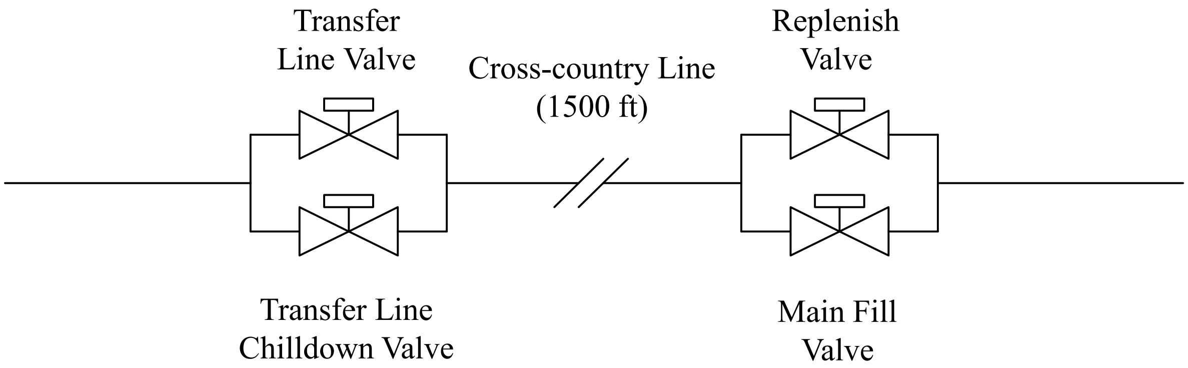
c) reduced fast fill,

d) topping,

e) replenish



LH2 Flow Network



Assume turbulent flow through valves. For valve *i*: $J_i = \alpha_i \lambda_i \sqrt{p_{i1} - p_{i2}}$

Allow transfer line to be in laminar or turbulent regime:

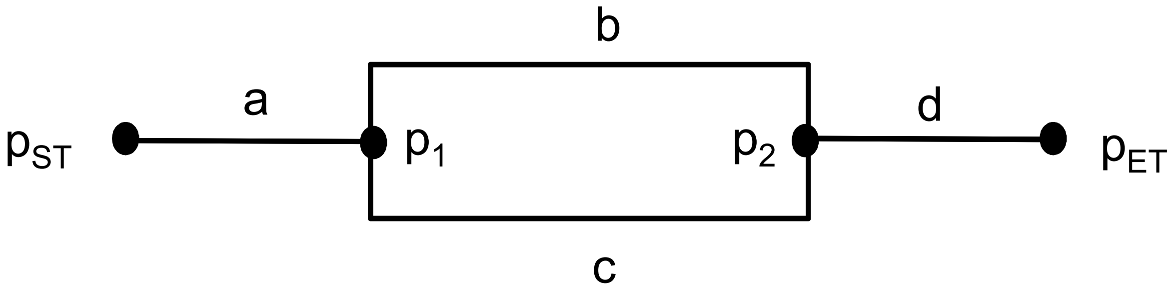
Turbulent regime: $\mathrm{Re}_{pipe} > \mathrm{Re}_{cr} \approx 10^3$ $\quad J_{tr} = \alpha_{eff} \Delta p_{tot}^{1/2}$

Laminar regime: $\mathrm{Re}_{pipe} < \mathrm{Re}_{cr} \approx 10^3$ $\quad J_{tr} = \dfrac{\alpha_V^2}{2k_{pipe}}\left[\sqrt{1 + \dfrac{4k_{pipe}^2 \Delta p_{tot}}{\alpha_V^2}} - 1\right]$

Flow network was solved for each filling regime based on identified valve and pipe parameters. In each case, we find the pipe is always in the turbulent flow regime - Reynolds number calculated to be 2-4 orders of magnitude larger than critical value.

Flow Network Solution Procedure

Inputs: pipe and valve configuration and parameters, and pressures on the ends of the flow network. Configuration defines set of constraints for conservation of mass.



$$J_a = J_b + J_c$$

$$J_d = J_b + J_c$$

Outputs: mass flow through each flow segment, and internal pressures

Each pipe/valve segment may individually be in either the laminar or turbulent regime. Therefore each segment is described by the following general expression:

$$J_i = \begin{cases} k_L(p_{in} - p_{out}) & \text{if } k_l(p_{in} - p_{out})\dfrac{D}{\rho A k_v} < \mathrm{Re}_{critical} \\ k_T\sqrt{p_{in} - p_{out}} & \text{if } k_T\sqrt{p_{in} - p_{out}}\dfrac{D}{\rho A k_v} > \mathrm{Re}_{critical} \end{cases}$$

Solution approach: Must solve simultaneous set of nonlinear algebraic equations. Mass flows and internal pressures are unknown. Use optimization solver to compute these variables.

Reduced Dynamical Model for LH2 Loading System

Equations of state for Tank 1 (ST)

$p_{v1} = \rho_{v1} \; R_{\mathrm{GH2}} \; T_{v1}$ GH2

$p_{g1} = \rho_{g1} \; R_{\mathrm{He}} \; T_{v1}$ GHE

$p_{\mathrm{f1}}\left(T_{\mathrm{f1}}\right) = p_C \; \left(T_{\mathrm{f1}}/T_C\right)^{\lambda}$ Saturated GH2 film

Mass conservation for Tank 1 (ST)

$$\dot{m}_{l1} = J_{le1} + J_{lv1}$$

$$\dot{m}_{v1} + \dot{m}_{g1} = J_{ve1} + J_{ge1} - J_{lv1}$$

$$J_{le1} = J_{boil} - J_{tr}$$

Mass flow rates into Tank 1 (ST)

$$J_{ve1} = J_{valve}^{v,1} + J_{boil} \qquad J_{ge1} = J_{valve}^{g,1}$$

Energy Conservation for Tank 1 (ST)

LH2

$$m_{l1} c_l \dot{T}_{l1} = \dot{Q}_{le1} + \dot{Q}_{l1} + \dot{W}_1 + J_{lv1} c_l \left(T_{f1} - T_{l1}\right) - \frac{1}{2} \sum J_{le1} v_{le1}^2$$

TL, Boilup
Liquid leak

GH2+GHE

$$(m_{v1} c_V^{\mathrm{GH2}} + m_{g1} c_V^{\mathrm{He}}) \dot{T}_{v1} = \dot{Q}_{ve1} - \dot{Q}_{v1} - \dot{W} +$$

$$\left(\sum J_{ve1} - J_{lv1}\right) R_{\mathrm{GH2}} T_{v1} + J_{ge1} R_{\mathrm{He}} T_{g1} + J_{lv1} c_P^{\mathrm{GH2}} \left(T_{v1} - T_{f1}\right) + \frac{1}{2} \sum J_{ve1} v_{ve1}^2$$

Saturated GH2 film

$$\dot{Q}_{v1} - \dot{Q}_{l1} + J_{lv1} \left(h_{vs} - h_{ls}\right) = 0$$

Vent valve,
Boilup, Gas Leak

ST body

$$m_{w1} c_{w1} \dot{T}_{w1} = \dot{Q}_{w1} - \dot{Q}_{le1} - \dot{Q}_{ve1}$$

$\dot{W} = -(p_{v1} + p_{g1}) \dot{V}_{l1}$ -- Work performed by gas (LH2+GHE)

$v_{ge1} = J_{ge1} / \rho_{g1} A_{ge1}$ -- GHE mass flow velocity for Vent Valve (VV)

$v_{ve1} = J_{ve1} / \rho_{v1} A_{ve1}$ -- GH2 mass flow velocity for each source (VV, BOILUP, Gas leak)

Heat Exchange Modes

conduction --- conduction

$T_{v1} > T_{f1} > T_{l1}$

$$\dot{Q}_{l1}(t) = A\left(\frac{\kappa_l c_l \rho_l}{\pi}\right)^{1/2} \int_0^t \frac{d\tau}{(t-\tau)^{1/2}} \frac{\partial T_{f1}(\tau)}{\partial \tau} \qquad \dot{Q}_{v1}(t) = A\left(\frac{\kappa_v c_P \rho_v}{\pi}\right)^{1/2} \int_0^t \frac{d\tau}{(t-\tau)^{1/2}} \frac{\partial T_{f1}(\tau)}{\partial \tau}$$

$T_{v1}, T_{l1} > T_{f1}$

$$\dot{Q}_{l1}(t) = A_1\, \alpha_{fl1}\left(T_{f1} - T_{l1}\right) \qquad \dot{Q}_{v1}(t) = A\left(\frac{\kappa_v c_P \rho_v}{\pi}\right)^{1/2} \int_0^t \frac{d\tau}{(t-\tau)^{1/2}} \frac{\partial T_{f1}(\tau)}{\partial \tau}$$

convection --- conduction

$T_{f1} > T_{l1}, T_{v1}$

$$\dot{Q}_{l1}(t) = A\left(\frac{\kappa_l c_l \rho_l}{\pi}\right)^{1/2} \int_0^t \frac{d\tau}{(t-\tau)^{1/2}} \frac{\partial T_{f1}(\tau)}{\partial \tau} \qquad \dot{Q}_{v1} = A_1\, \alpha_{fv1}\left(T_{f1} - T_{v1}\right)$$

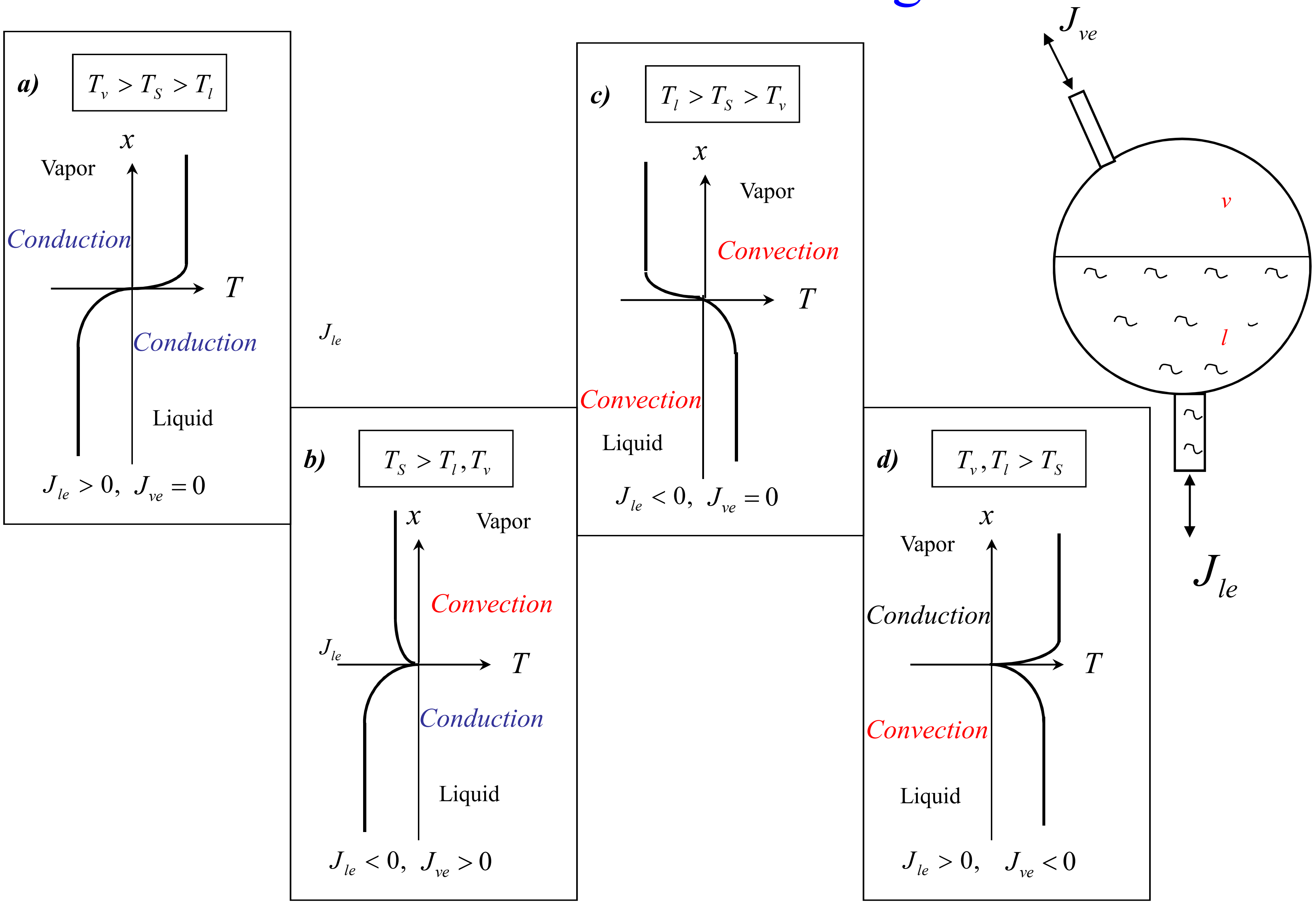
conduction --- convection

$T_{f1} > T_{l1}, T_{v1}$

$$\dot{Q}_{l1}(t) = A_1\, \alpha_{fl1}\left(T_{f1} - T_{l1}\right) \qquad \dot{Q}_{v1} = A_1\, \alpha_{fv1}\left(T_{f1} - T_{v1}\right)$$

convection --- convection

Classification of Heat Exchange Modes



Model for Vent Valve – Choked Flow

Vent valve parameters are thresholds pressure p_{th1} and p_{th2} , valve cross-section S_{valve}

$$p_{th1} < p_{th2}$$

$$J_{valve}^{v,g} = \begin{cases} \dfrac{\rho_{v,g}\sqrt{\gamma(p_v + p_g)}}{\Gamma\sqrt{\rho_v + \rho_g}} S_{valve} & \text{if} \quad (p_v + p_g > p_{th2}) \,\|\, ((p_{th1} < p_v + p_g < p_{th2})\,\&\,(\text{last crossing } p_{th2} \text{ down})) \\ 0 & \text{if} \quad (p_v + p_g < p_{th1}) \,\|\, ((p_{th1} < p_v + p_g < p_{th2})\,\&\,(\text{last crossing } p_{th1} \text{ up})\,) \end{cases}$$

$$\Gamma = \left(\frac{\gamma+1}{2}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \quad , \quad \gamma = \frac{c_P}{c_V} = 1.4$$

Choked Flow Condition

$$\frac{p_v + p_g}{p_{\text{ambient}}} > \left(\frac{\gamma+1}{2}\right)^{\frac{\gamma}{\gamma-1}} = 1.9, \quad \gamma = \frac{c_P}{c_V} = 1.4$$

If we assume that $p_{ambient} = p_{atm} = 14.7$ psia

then the choked flow condition will be satisfied

since $p_g + p_v > 39$ psia

Faults in Reduced Dynamical Model for LH2 Loading System

Vent valve fault

$$J_{valve} = \frac{\sqrt{\gamma(p_v + p_g)(\rho_v + \rho_g)}}{\Gamma} S_{valve}$$

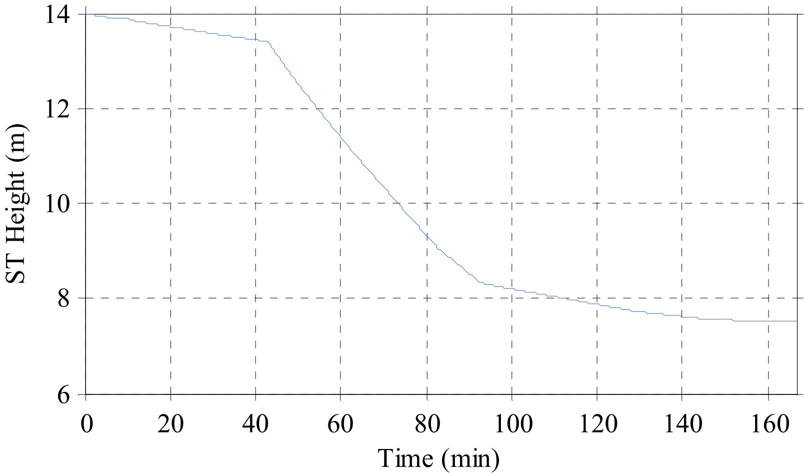
Model for the liquid leak:
Bernoulli's law

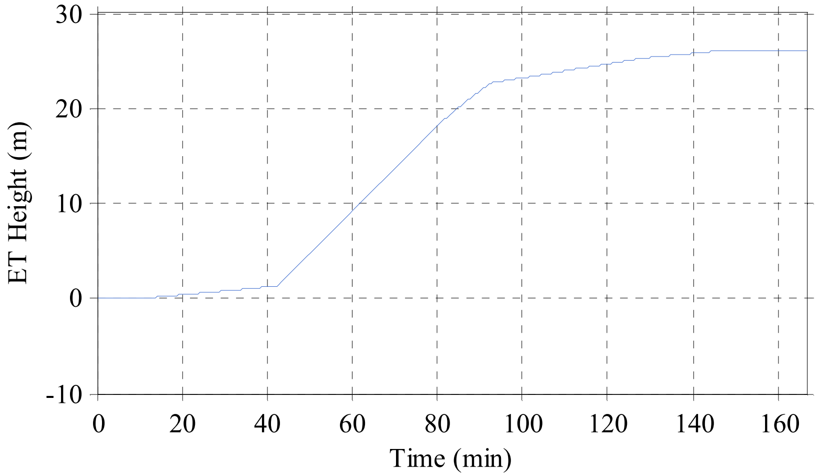
$$J_{Ll} = \sqrt{2\rho_L\left(p_v + p_g - p_{atm}\right)} S_{Ll}(t)$$

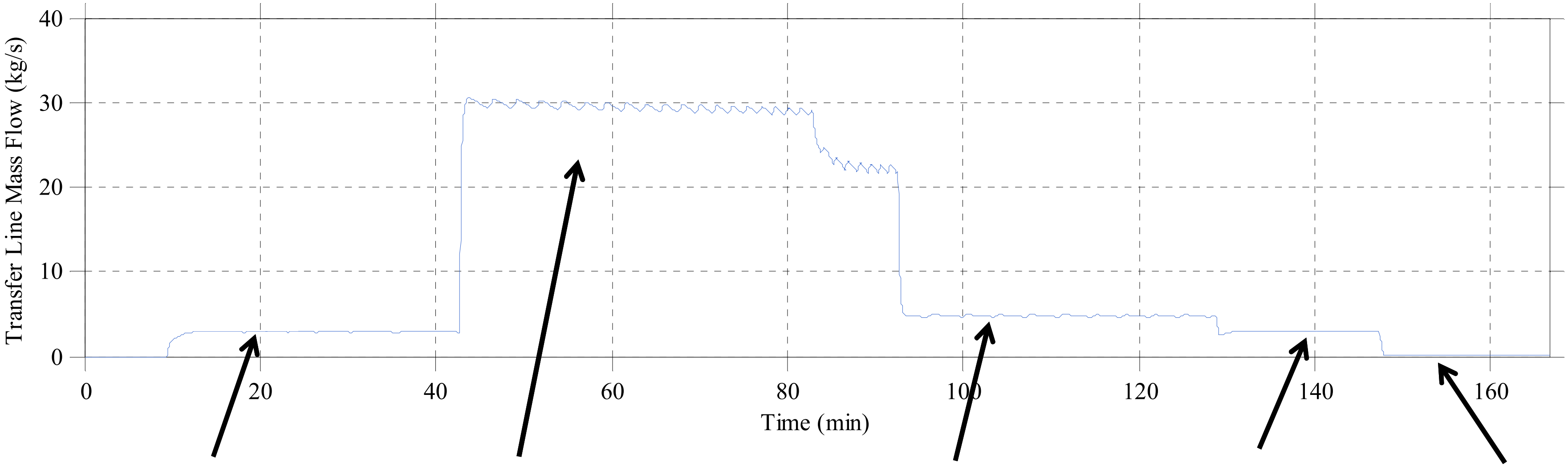
Model for vapor and gas leaks
assuming chocking flow

$$J_{vl} = \frac{\rho_v \sqrt{\gamma(p_v + p_g)}}{\Gamma\sqrt{\rho_v + \rho_g}} S_{gl}(t), \quad J_{gl} = \frac{\rho_g \sqrt{\gamma(p_v + p_g)}}{\Gamma\sqrt{\rho_v + \rho_g}} S_{gl}(t).$$

Nominal Regime of the LH2 Loading System







Slow Fill to 5% ET Height

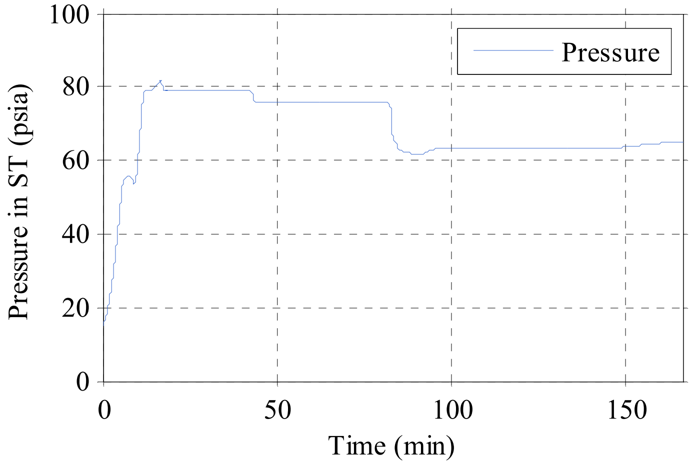
Fast Fill to 85% ET Height (≈6000 gal/min)

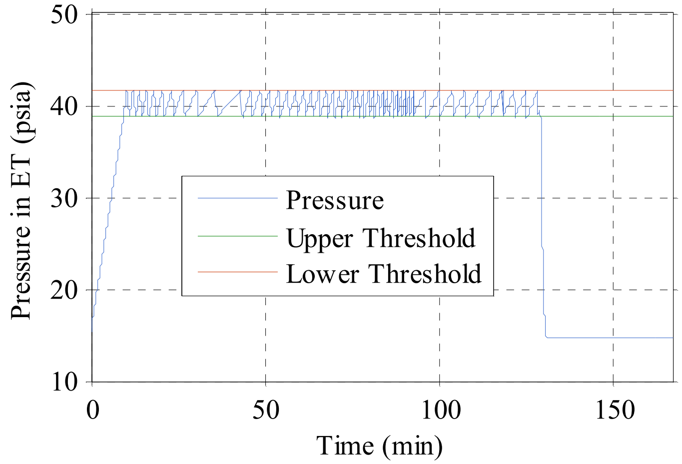
Fast Fill to 98% ET Height (≈1750 gal/min)

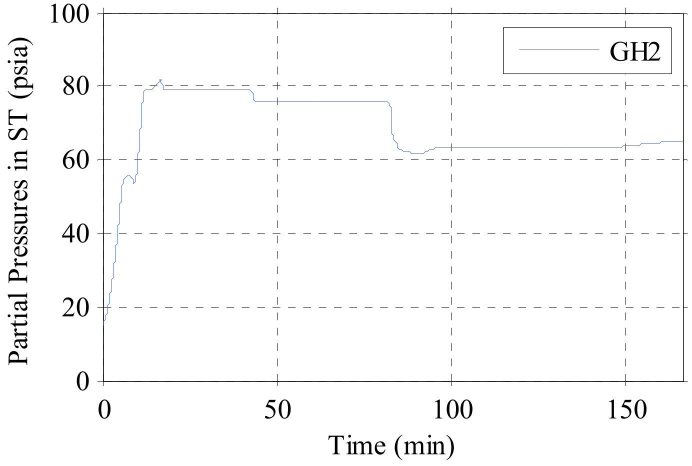
Topping to 100% ET Height (slow fill rate)

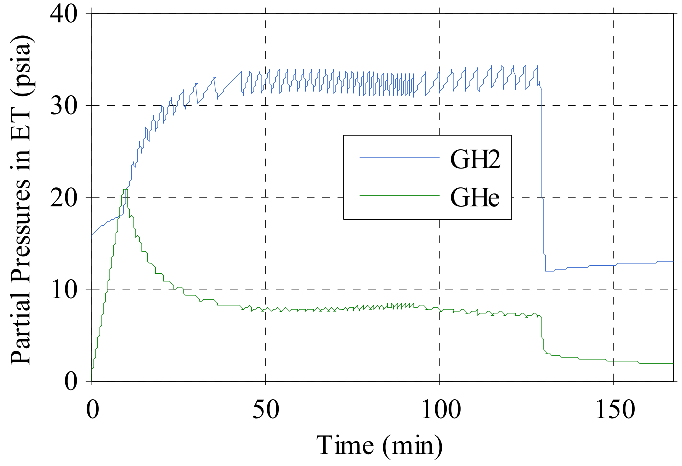
Continuous Replenish

Nominal Regime of the LH2 Loading System







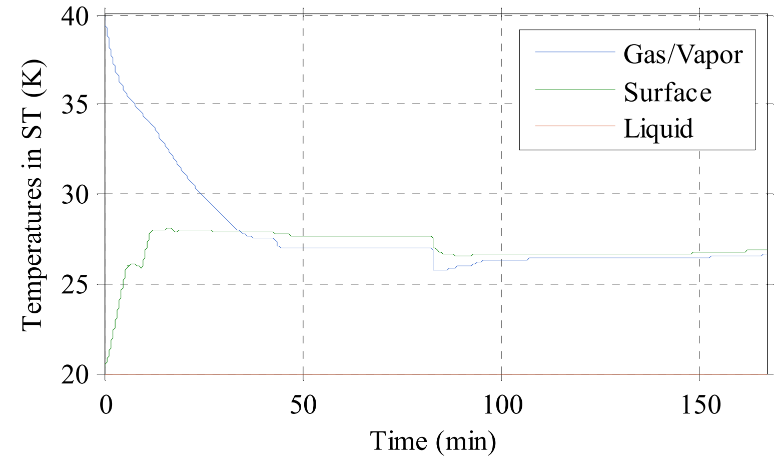


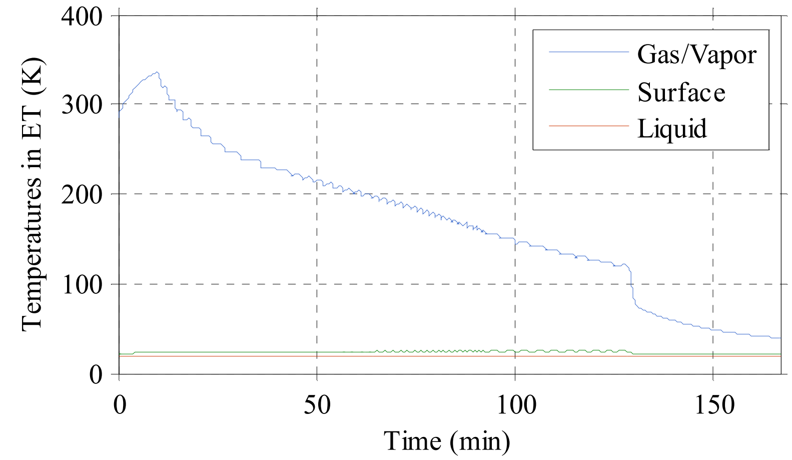
ST pressure maintained at 80.7 psia up to reduced fast fill, then 64.7 psia

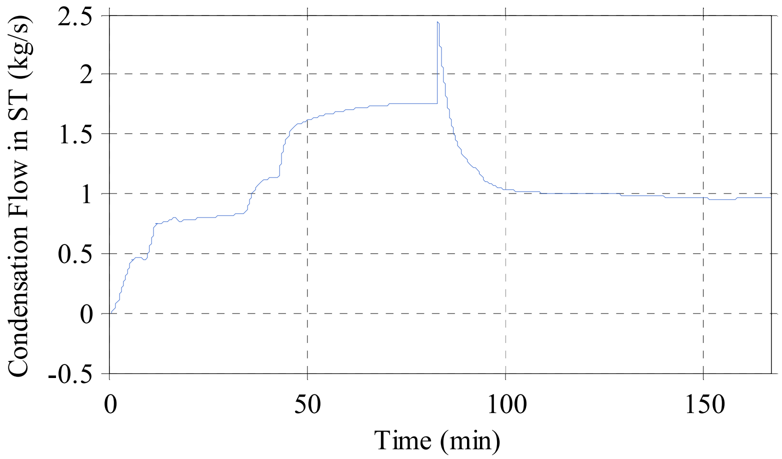
ET pressure maintained between 38.7 and 41.7 psia

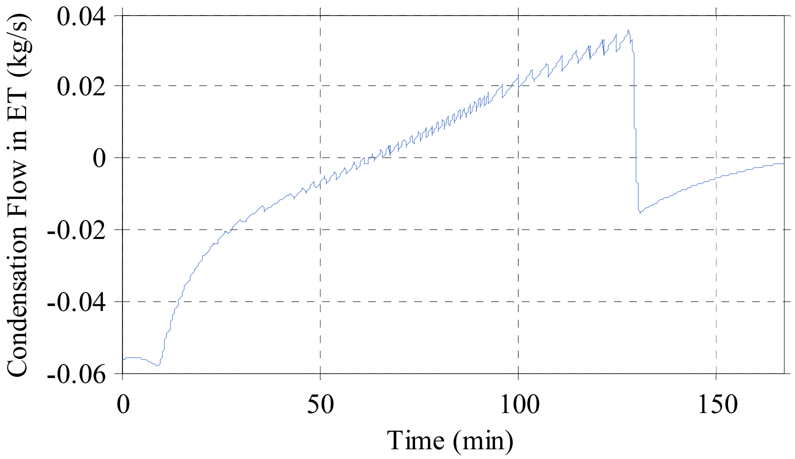
The pressure p_1 in ST is determined by loading dynamics and vaporizer

Nominal Regime of the LH2 Loading System









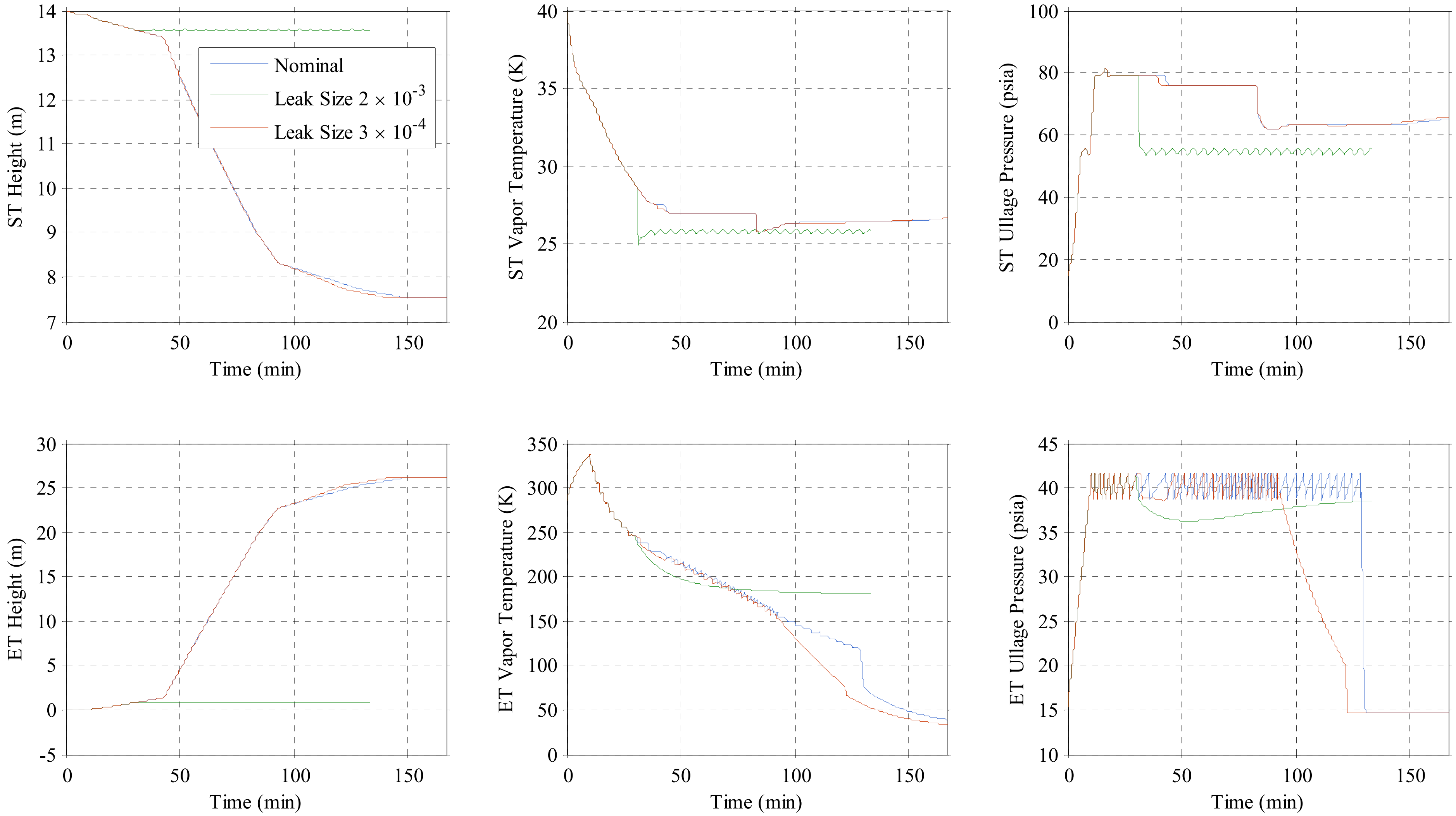
Liquid surface temperature T_{s1} in ST increases due to vapor condensation and gas temperature T_1 decreases because of increase of the gas volume during loading.

The condensation mass flow in ET is small, temperature of gas T_2 drops to due venting and boiling creating cold gas.

The condensation mass flow in ST sharply grows due to increase of the vapor pressure.

Gas Leak Fault in ET

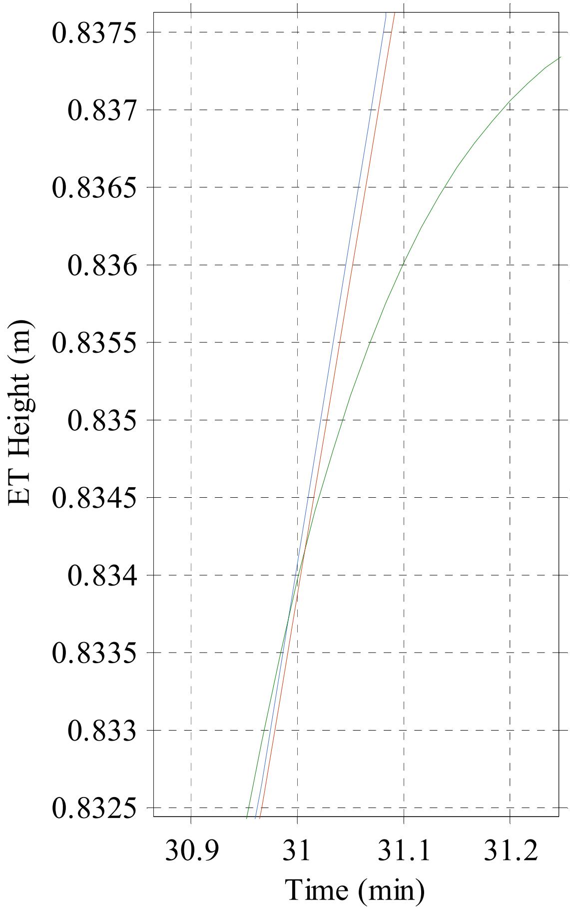
Initiation of a leak hole in the upper part of ET

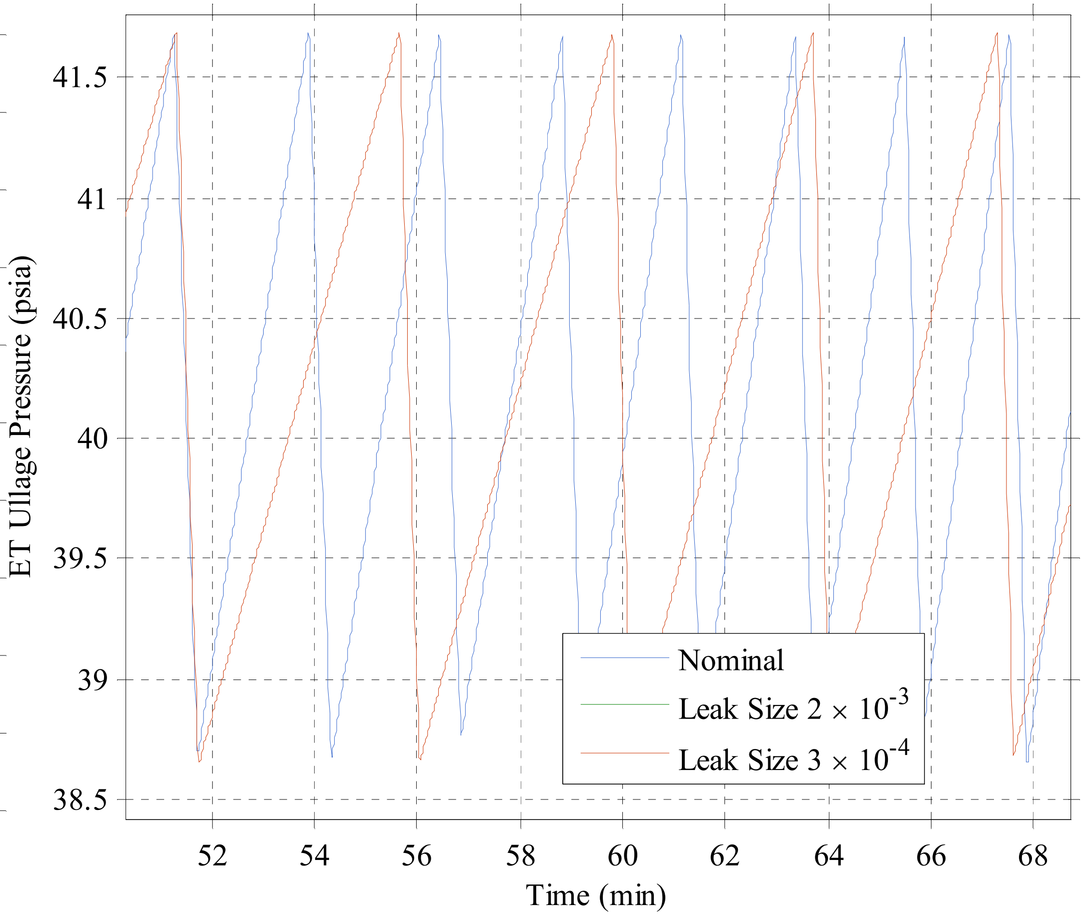


Faults injected at t=30 min, change becomes most significant during fast fill

Gas Leak Fault in ET

Initiation of a leak hole in the upper part of ET

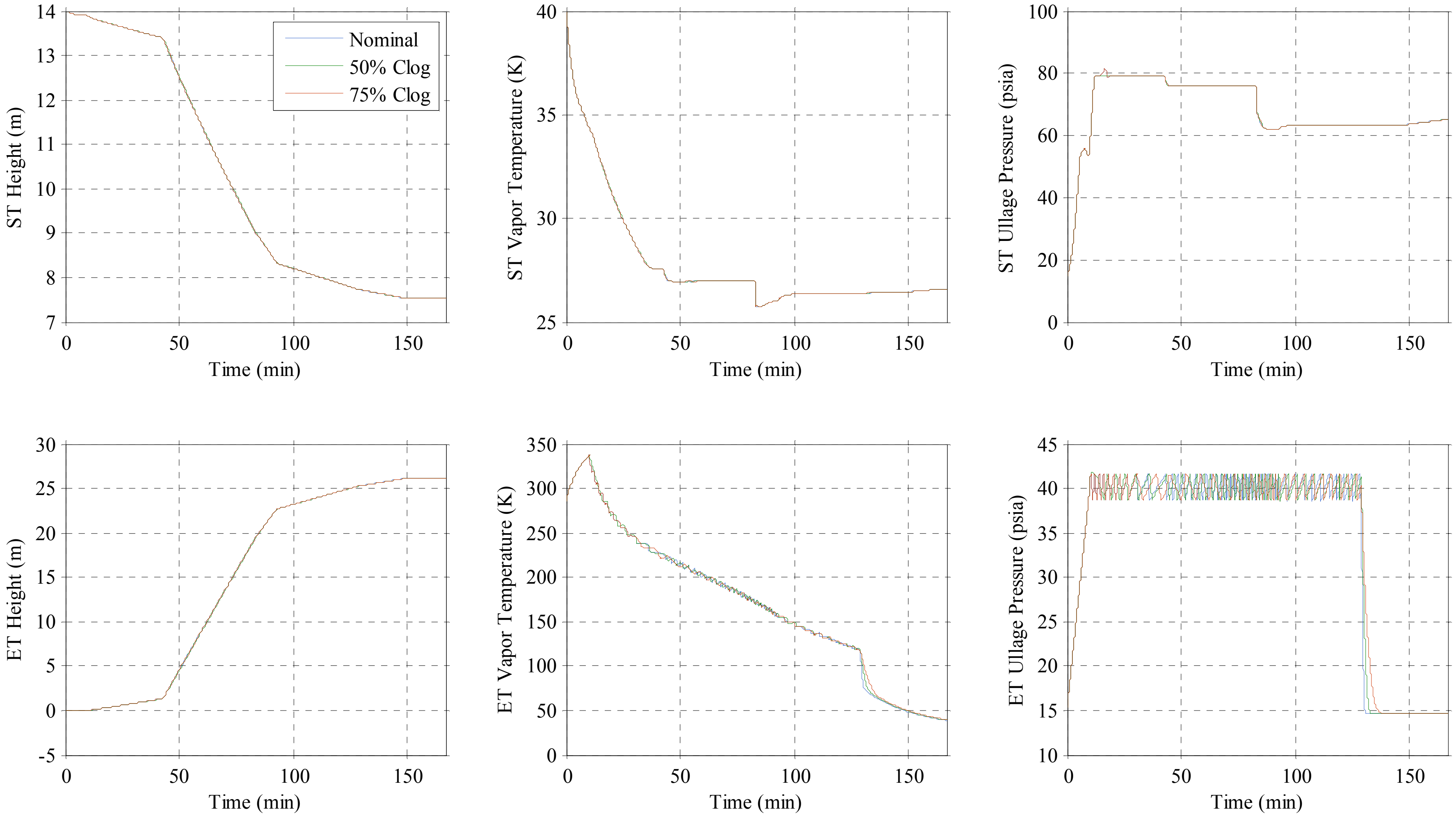




Presence of leak can be detected by observing rates of ullage pressure increase and decrease.

Vent Valve Clogging Fault in ET

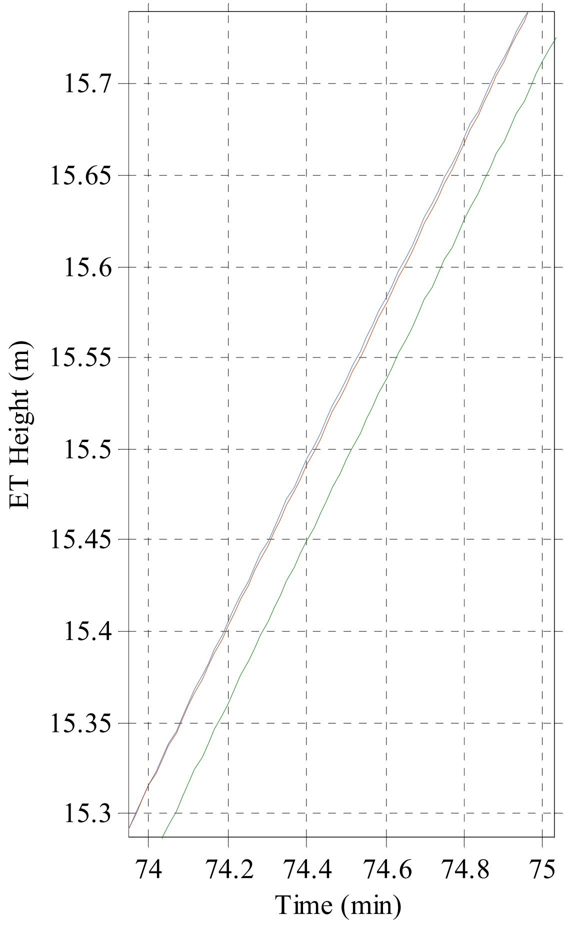
Decreases of the cross section of the vent valve: S_{valve}

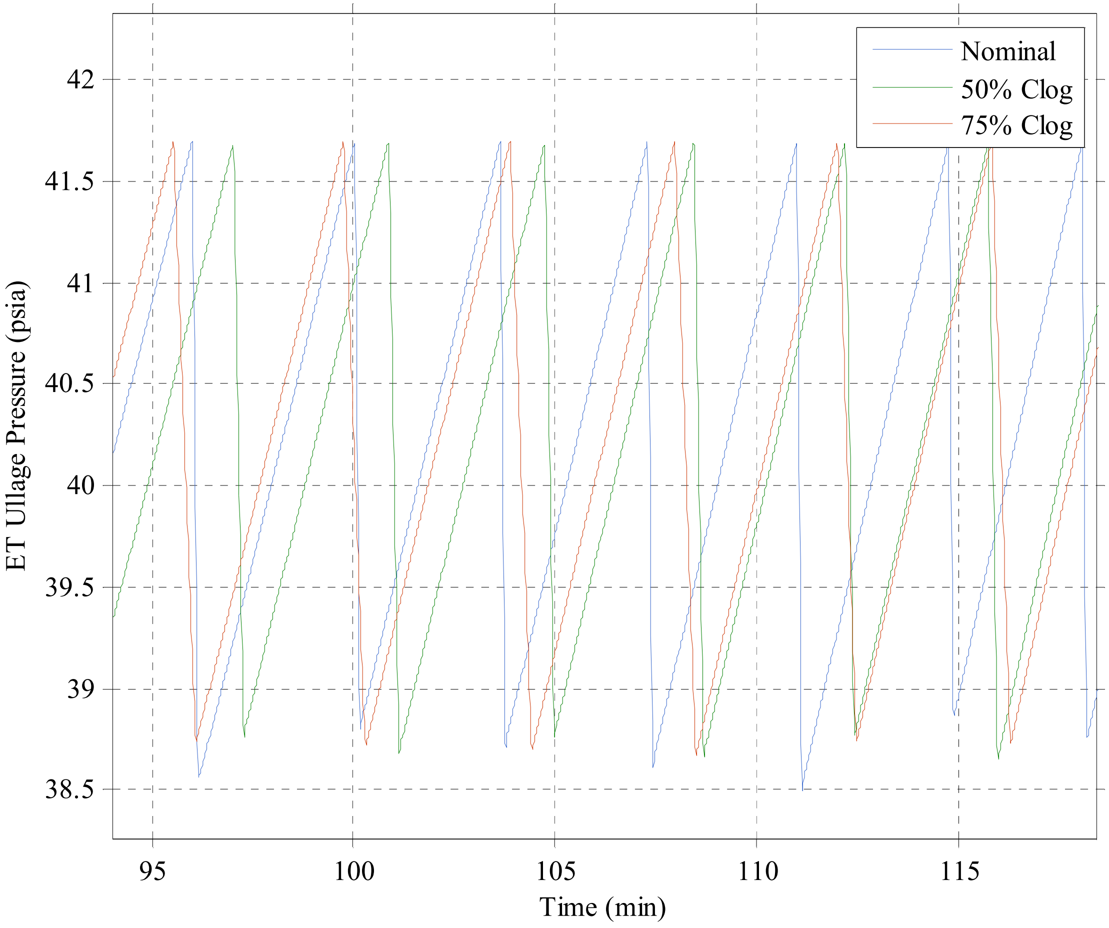


Mass flow through vent valve is proportional to $(p-p_{valve})S_{valve}$

- Loading can still be accomplished with 50%, 75% reduction in S occurring at t=0 ($S_{nominal} = 0.025\ m^2$)
- Significant increase in pressure obtained with full clog occurring at t=30 min (abort required).

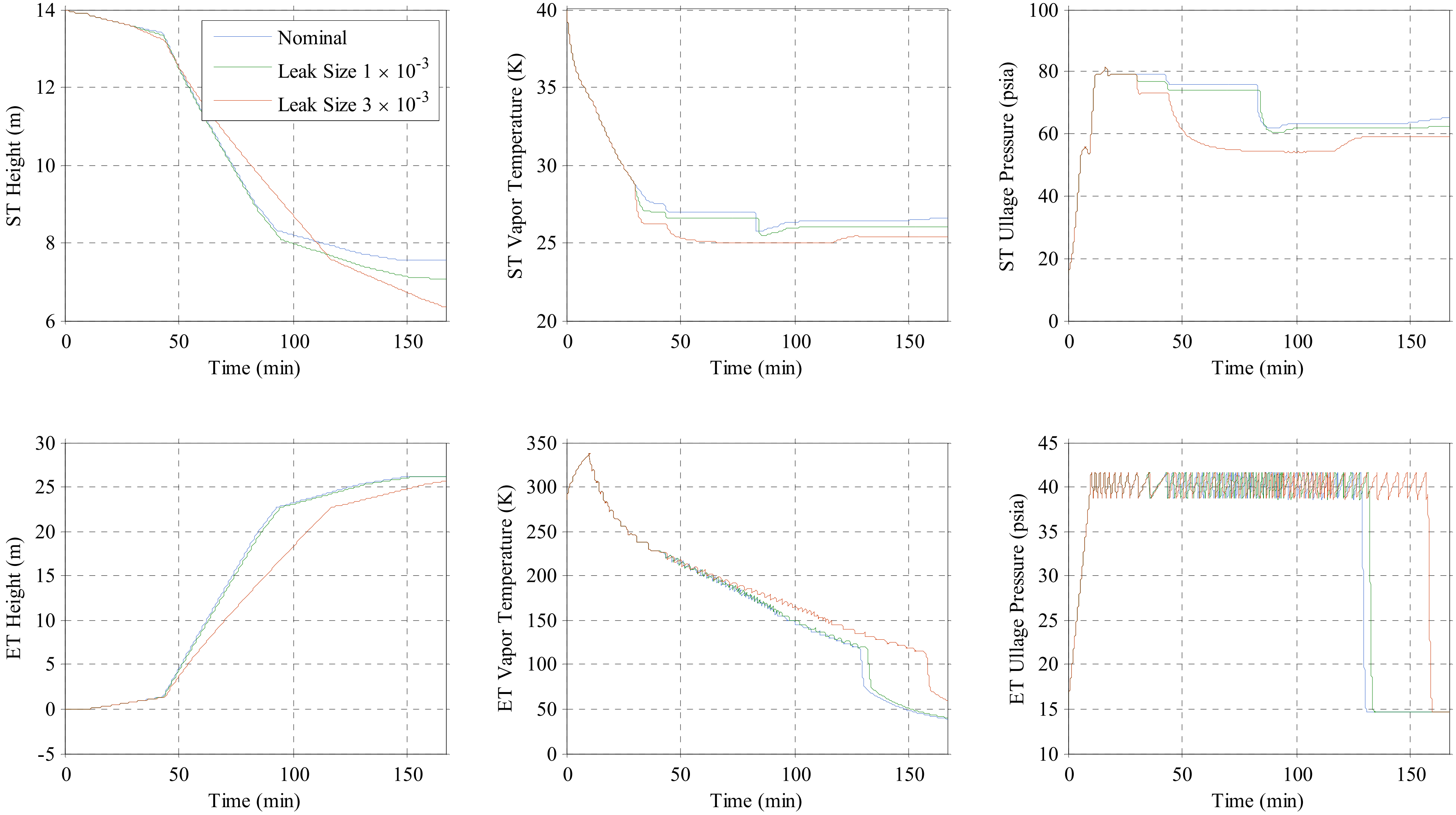
Vent Valve Clogging Fault in ET





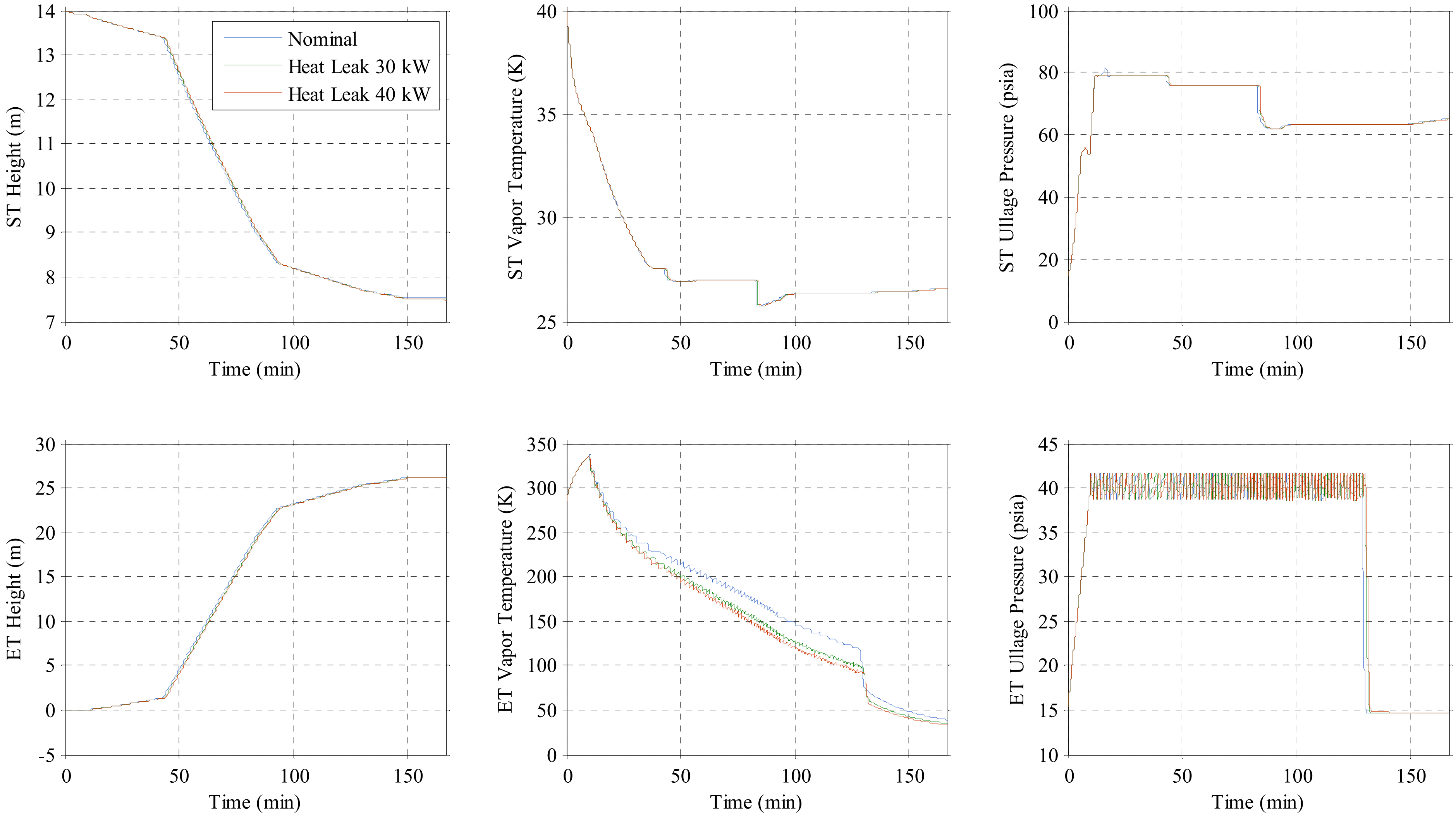
- Loading is slightly slower with clog.
- Clog can be detected by observing difference in pressure relief rate.

Gas Leak Fault in ST



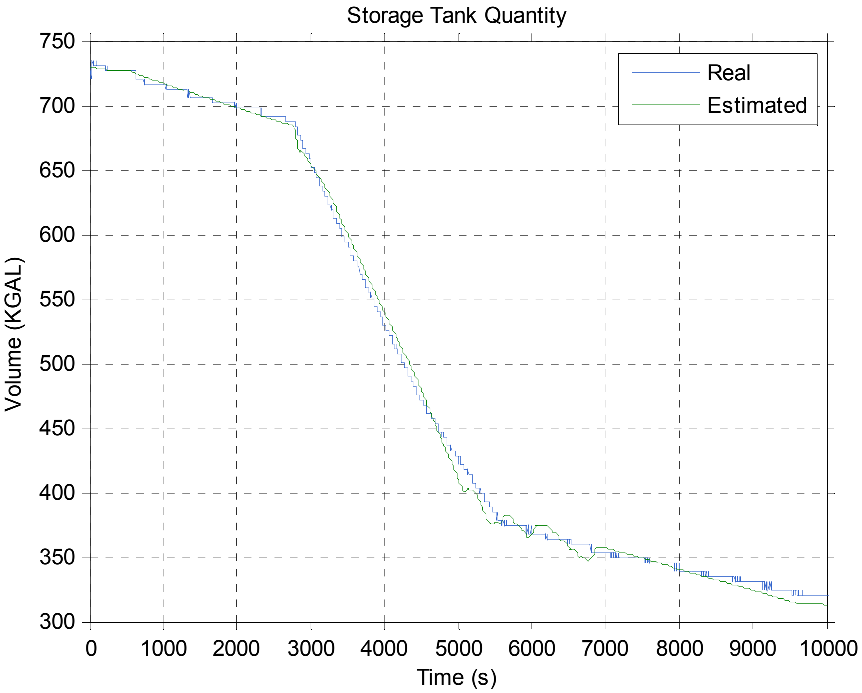
- Vaporizer cannot maintain pressure at high flow rates in presence of leak
- Reduced pressure results in slower loading

Heat Leak in ET



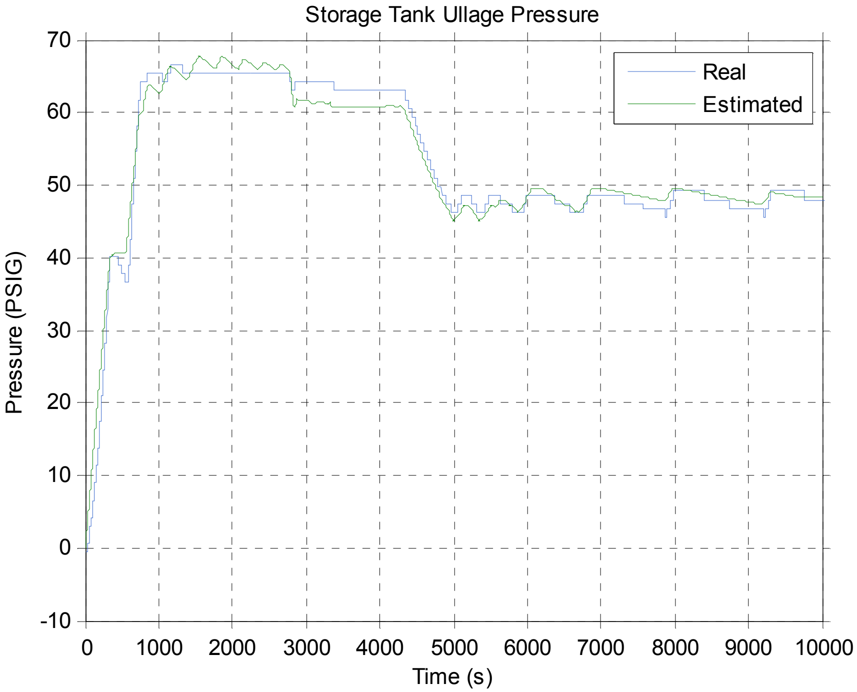
- Heat leak causes more boiling from the tank walls, introduction of cold vapor produced from this reduces ullage temperature

Comparison with Real Data: ST

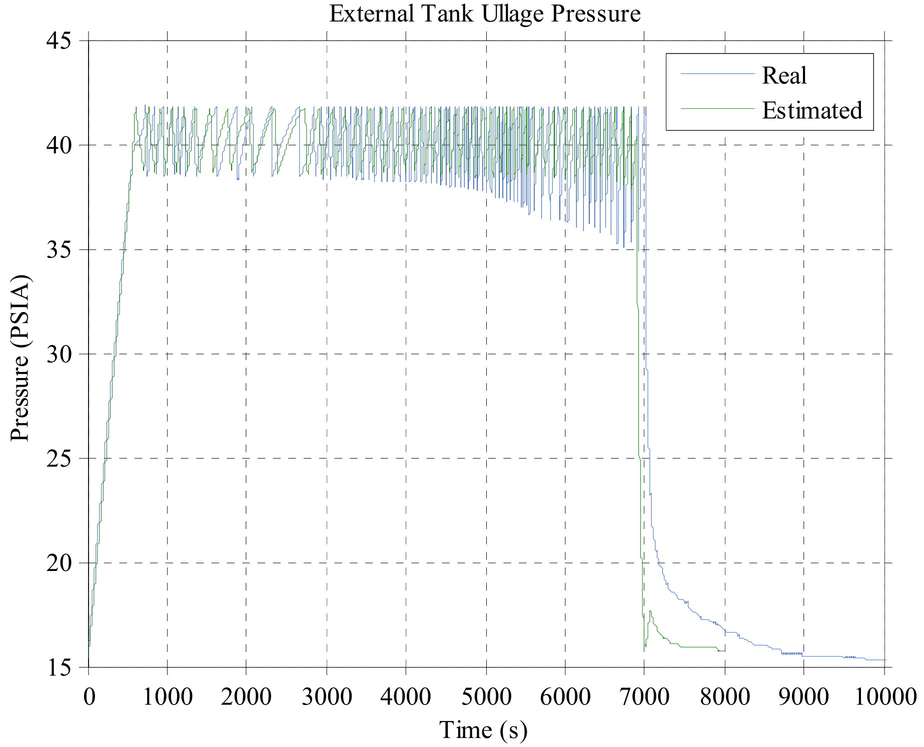


Storage tank quantity depends largely on two factors: (1) transmission line flow, (2) vaporizer rate

Storage tank pressure depends most strongly on vaporizer rate

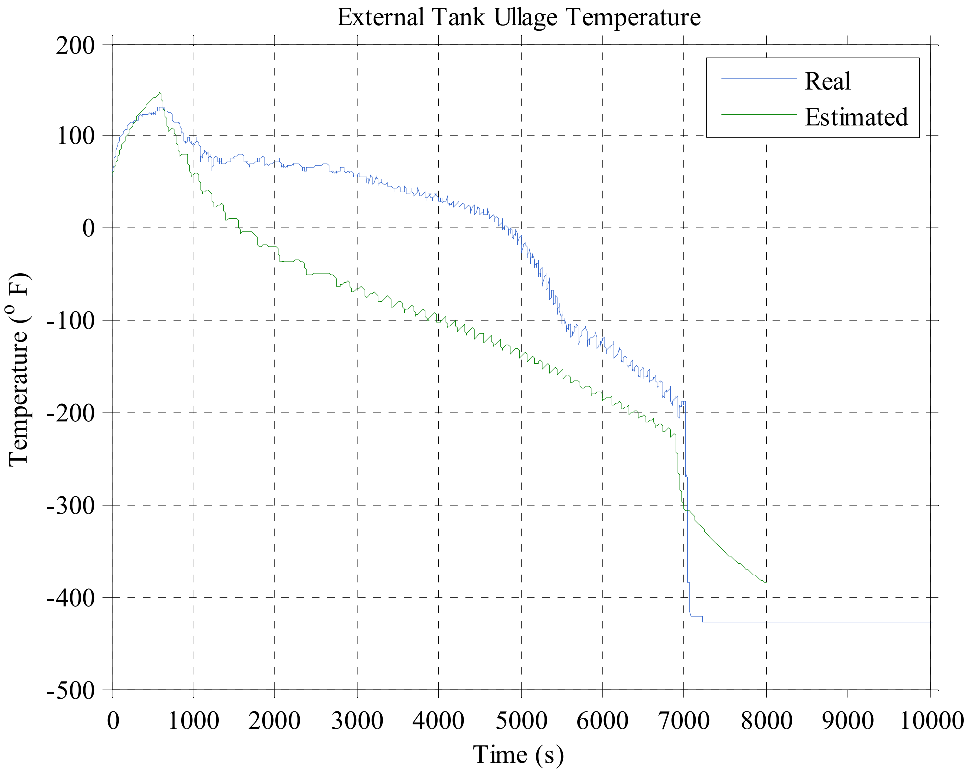


Comparison with Real Data: ET

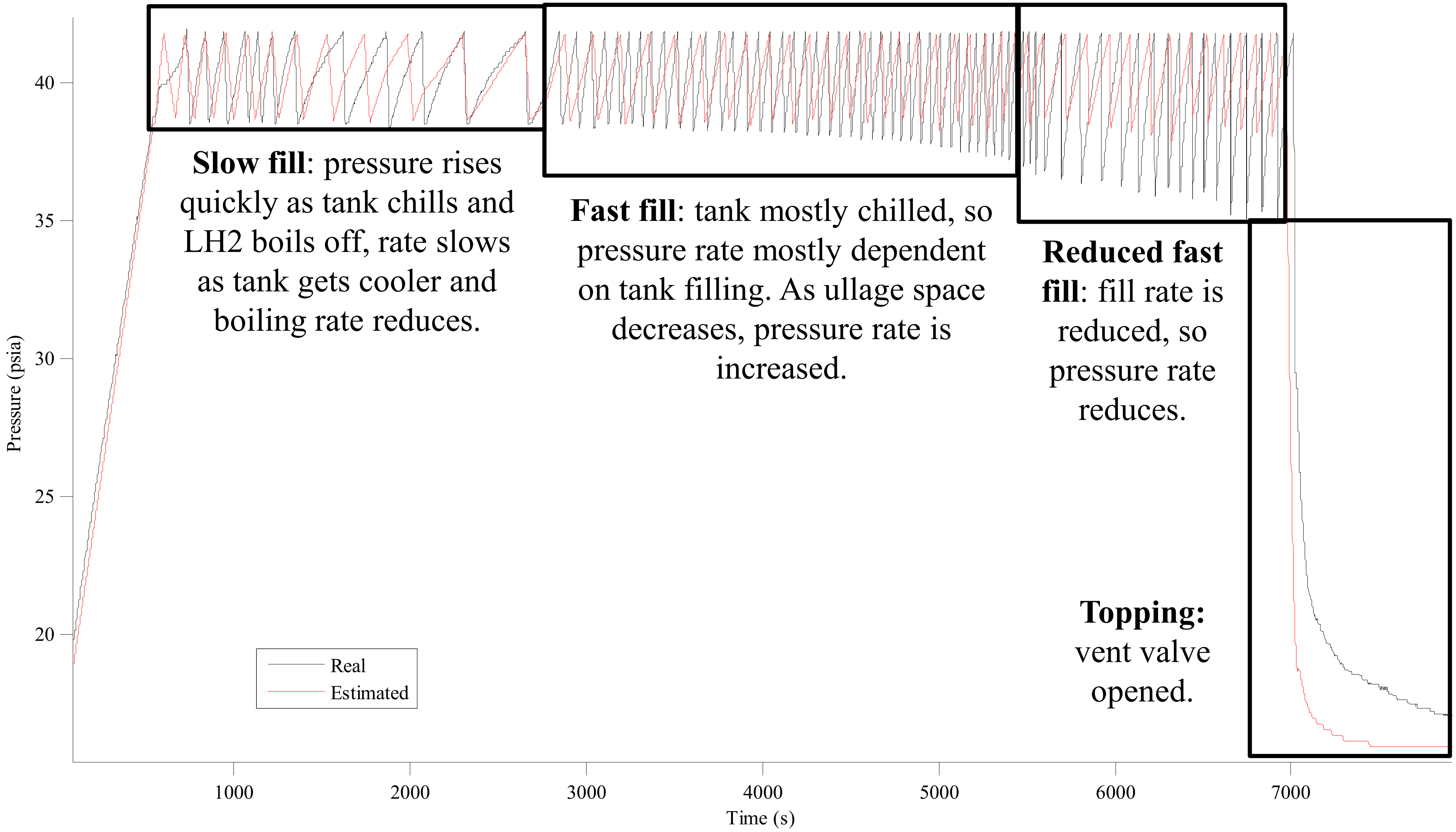


Vent valve regulates ET pressure. Pressure rate changes due to (1) fill rate, (2) boil rate, (3) evaporation/condensation rate, (4) vent valve behavior.

Temperature rises initially due to pressurization GHe, cools due to (1) tank chilling, (2) cold vapor from boiling, (3) release of vapor through vent



Comparison with Real Data: ET



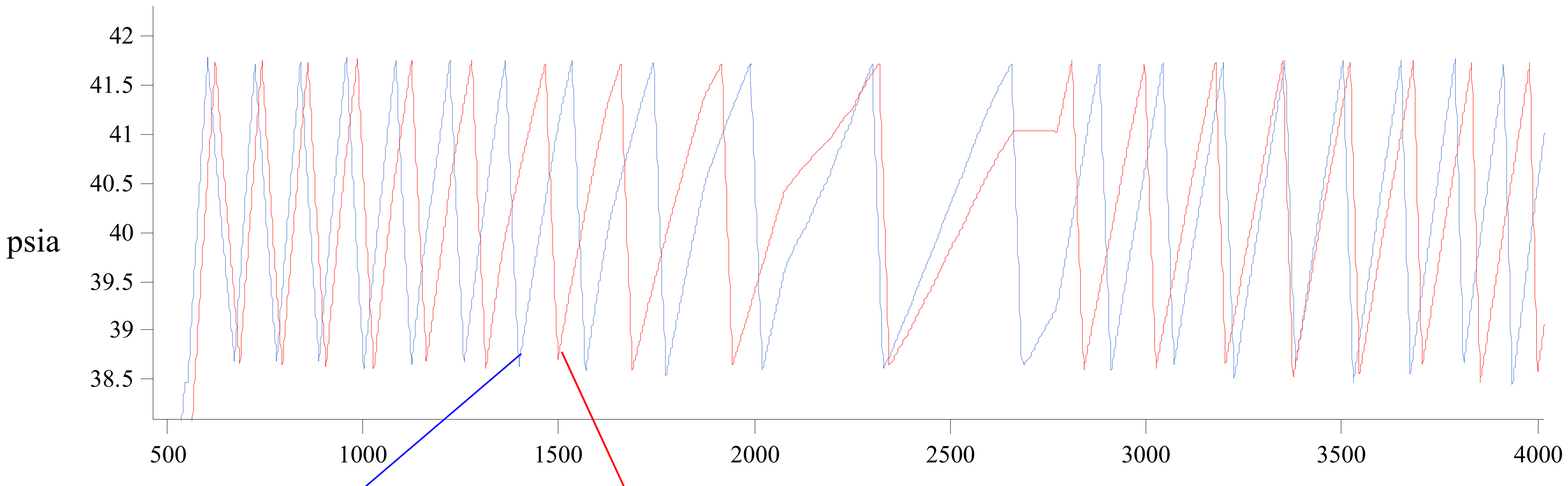
Slow fill: pressure rises quickly as tank chills and LH2 boils off, rate slows as tank gets cooler and boiling rate reduces.

Fast fill: tank mostly chilled, so pressure rate mostly dependent on tank filling. As ullage space decreases, pressure rate is increased.

Reduced fast fill: fill rate is reduced, so pressure rate reduces.

Topping: vent valve opened.

Analysis of ET Leak on Vent Valve Frequency



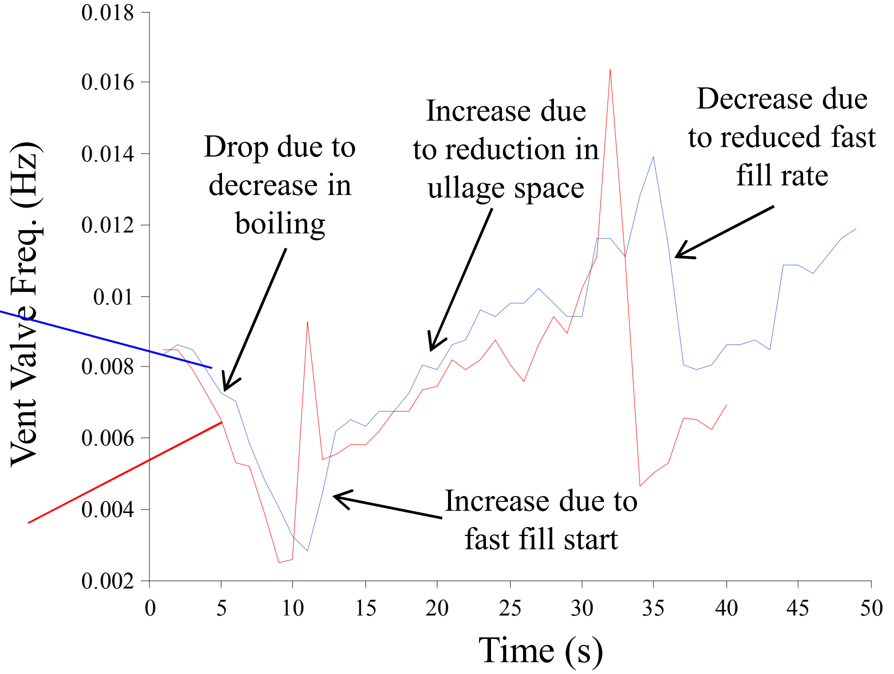
ET Pressure without Leak

ET Pressure with Leak

ET Pressure increases more slowly with a leak, thus vent valve frequency is reduced.

Peak to Peak Frequency without Leak

Peak to Peak Frequency with Leak



Drop due to decrease in boiling

Increase due to reduction in ullage space

Decrease due to reduced fast fill rate

Increase due to fast fill start

Possible Faults in LH2 Loading System

Fault	Effects caused by fault	Reconstructed parameters
1. Vent valve fault: decrease (increases) of threshold pressure p_{valve}	Decreases (increase) of pressures, gas temperatures in both Tanks, and increase filling rate and vaporizer rate	The value of p_{valve}
2. Vent valve choking fault: decreases of the cross section area S_{valve}	Decrease of filling rate and vaporizer rate, and increase of pressure p_2 and T_2 in Tank 2 (more than p_1 and T_1 in Tank 1)	The value of S_{valve}
3. Valve choking fault; valve stuck fault	Change of filling dynamics	Valve area; valve position
4.Gas leak fault in external tank (2)	Decrease of pressure and gas temperature in both tanks and increases of filling rate and vaporizer rate	Cross-section of the gas leak hole area in the external tank
5. Liquid leak fault in tank 2 or transmission line after the valves	Decrease of filling rate, pressure and gas temperature in external tank	Cross-section of the liquid leak hole area
6. Gas leak fault in the storage tank (1)	Increase of heater current and decreases of all other parameters during of fast filling	Cross-section of the gas leak hole area
7. Liquid leak fault in tank 1 or transmission line before the valves	Increase of heater current and decrease the rates of slow and fast fillings	Cross-section of the liquid leak hole area
8. Heater feedback fault	Sharply change of filling dynamics	Failure of heater feedback circut
9. Heat insulation fault of the external tank	Increase of liquid and gas temperature and pressure in the external tank	The power of heating
10. Heat insulation fault of the storage tank	Increase of liquid and gas temperature and pressure in the storage tank	The power of heating

Fault detection is based on sensor data for gas pressure, temperature, and liquid height in Storage and External Tanks (p_1, p_2, T_1, T_2, h_{L1}, h_{L2})

LH2 Matlab Simulation User Manual

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May 24, 2010

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Contents

1 Introduction

This document describes the liquid hydrogen (LH2) model and MATLAB simulation, including both nominal and faulty regimes. A brief description of the system is provided, followed by the model equations, and instructions for using the MATLAB code.

2 System Description

The goal of the LH2 propellant loading system is to move LH2 from the storage tank (ST) to the external tank (ET). A schematic outlining the scope of the current modeling efforts is shown below as Fig. 1.

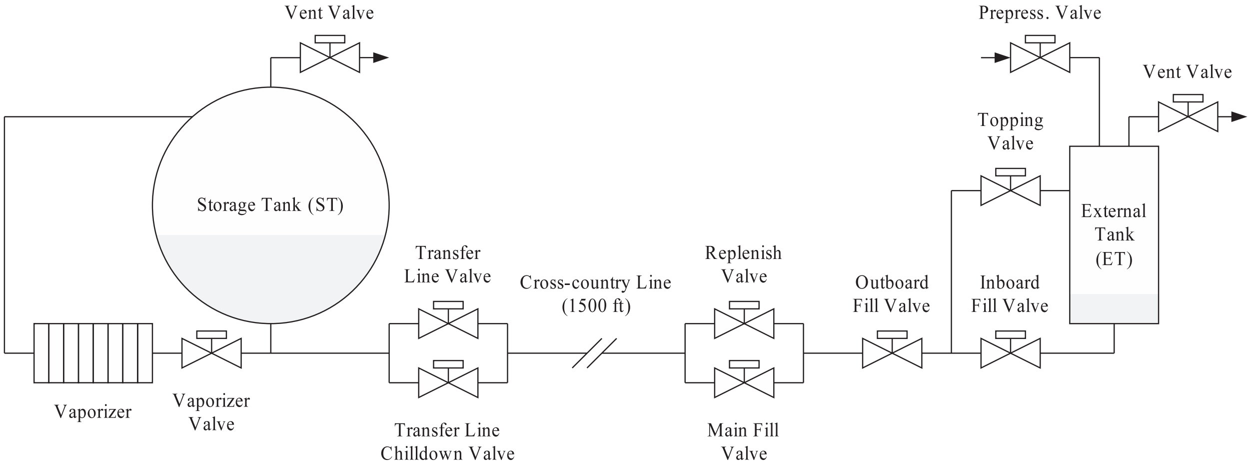

Figure 1: LH2 propellant loading schematic.

Filling progresses in seven stages: (1) Pressurization, (2) slow fill, (3) fast fill, (4) fast fill at reduced pressure , (5) reduced flow fast fill, (6) topping, and (7) replenish.

1. **Pressurization:** The ST begins with a large amount of LH2 and enough gaseous hydrogen (GH2) for an ullage pressure equal to atmospheric pressure. The ET begins with no liquid and enough GH2 for an ullage pressure equal to atmospheric pressure. Initially, there is no flow path between the tanks. The tanks are individually pressurized before any transfer of LH2 begins. The ST is pressurized first to about 40 psig, and then to 66 psig solely through the use of the vaporizer. The vaporizer valve opens, allowing LH2 to flow through the vaporizer, which boils off LH2, and the GH2 created feeds back into the ST. Concurrently, the ET is pressurized to 38.7 psia using gaseous helium (GHe), fed in through the prepress. valve.

2. **Slow Fill:** After pressurization is complete, slow fill begins. The transfer line chilldown valve, main fill valve, outboard fill valve, inboard fill valve, and topping vales are all opened. The ullage pressure in the ST drives fluid to the ET. The ullage pressure in the ST is constantly maintained by the vaporizer. The flow through the vaporizer valve is modulated based on the ST pressurization set point. The ullage pressure in the ET is maintained using its vent valve, which opens and closes to maintain the pressure between 38.7 and 41.7 psia.

3. **Fast Fill:** Fast fill begins when the ET is 5% full. The transfer line valve opens to increase flow from about 1500 gallons per minute (GPM) to around 7500 GPM.

4. **Fast Fill (Reduced Pressure):** At 72% full, the ullage pressure of the ST is reduced to 50 psig.

5. **Fast Fill (Reduced Flow):** At 85% full, reduced flow fast fill begins. The main fill valve is set at a reduced flow state.
6. **Topping:** At 98% full, topping begins. The transfer line valve is closed and the replenish valve fully opens. The ET vent valve is also opened, reducing the ET ullage pressure to 14.7 psia. The inboard fill valve closes, forcing the remaining liquid to pass into the ET through the topping valve.
7. **Replenish:** At around 100% full, topping ends and the tank is continuously replenished to replace boil off before launch. During replenish, the transfer line chilldown valve remains open, the main fill valve is closed, and the replenish valve is modulated to maintain the ET level at 100%.

3 Mathematical Model Description

Nomenclature

Subscripts

1	storage tank
2	external tank

Dynamical Variables

m_L	Mass of LH2; kg
m_v	Mass of H2 vapor; kg
m_g	Mass of GH2; kg
h_L	Height of liquid; m
ρ_v, p_v	Density and partial pressure of vapor; kg/m^3, Pa
ρ_g, p_g	Density and partial pressure of pressurizing gas; kg/m^3, Pa
T_v	Temperature of the vapor-gas mixture; K
T_s	Temperature of the liquid surface; K
T_w	Temperature of tank wall; K
J_{boil}	Vapor flow generated by vaporizer; kg/s
J_{tr}	Flow through transmission line; kg/s
J_{vvalve}, J_{gvalve}	Flow of vapor and gas through vent valve; kg/s
J_{vl}, J_{gl}	Vapor and gas leak flows; kg/s
J_{Ll}	Liquid leak flow; kg/s
J_{cd}	Vapor condensation flow; kg/s
$\dot{Q}_{Ls}, \dot{Q}_{vs}$	heat transfer from liquid to surface, vapor to surface; W
$\dot{Q}_{wL}, \dot{Q}_{wv}$	heat transfer from wall to liquid, wall to vapor; W
$\dot{Q}_{ev}, \dot{Q}_{ew}$	heat transfer from environment to vapor, environment to wall; W
$\lambda_{vent,ST}$	Discrete state of storage tank vent valve
$\lambda_{vent,ET}$	Discrete state of external tank vent valve

Liquid Parameters

ρ_L	Density of liquid; kg/m^3
c_L	Specific heat of liquid; J/kg/K
T_L	Temperature of the bulk liquid; K
κ_L	Thermal conductivity of liquid; W/m/K
h_{fg}^0	Specific heat of evaporation; J/kg
T_c, p_c	Critical temperature and pressure of the liquid; K, Pa
λ	Dimensionless saturated vapor pressure exponent
μ	Dynamic viscosity; Pa s

Vapor and Gas Parameters

R_v, R_g	Vapor and pressurizing agent gas constants; J/kg/K
c_v, c_g	Specific heat of vapor and pressurizing gas at constant volume; J/kg/K
κ_v, κ_g	Thermal conductivities of vapor and gas at $T = T_L$; W/m/K
γ	Ratio of specific heats
Γ	Parameter characterizing the flow through a nozzle

Tank Parameters

S_1, V_1, R_1	Cross section area, volume and radius of storage tank; m^2, m^3, m
R_2, H	Radius and height of external tank; m^2, m
H_{max}	Maximum filling height of external tank; m
p_{01}, p_{02}	Initial pressures in storage and external tanks; Pa
p_{atm}	Atmospheric pressure; Pa

Transfer Valve Parameters

d_E	Transfer line valve diameter; m
d_F	Transfer line chilldown valve diameter; m
d_J	Replenish valve diameter; m
d_K	Main fill valve diameter; m
d_{PV11}	Outboard fill valve diameter; m
d_{PV12}	Inboard fill valve diameter; m
d_{PV13}	Topping valve diameter; m
k_E	Transfer line valve coefficient
k_F	Transfer line chilldown valve coefficient
k_J	Replenish valve coefficient
k_K	Main fill valve coefficient
k_{PV11}	Outboard fill valve coefficient
k_{PV12}	Inboard fill valve coefficient
k_{PV13}	Topping valve coefficient
λ_i	Position input of valve i

Vent Valve Parameters

S_{valve}	Valve cross-section; m^2
$p_{slow}, p_{fast}, p_{reduced}, p_{topping}, p_{replenish}$	ST pressure thresholds for different loading regimes
$p_{ET_{low}}, p_{ET_{high}}$	Thresholds for ET ullage pressure

Cross-country Line Parameters

τ_{tr}	Transmission line time constant;
D_{pipe}	Pipe diameter; m
L_{pipe}	Pipe length; m
$d_{r,pipe}$	Pipe roughness; m
Re^*	Critical Reynolds number for the pipe

Vaporizer Parameters

T_{boil}	Temperature of vapor inside bubbles; K
τ_{vap}	Vaporizer time constant
c_{vap}	Vaporizer valve flow coefficient
λ_{vap}	Vaporizer valve position

Fault Parameters

$f_{S,valve1}$	Multiplication factor of ST vent valve orifice area (valve choking)
$t_{S,valve1}$	Time of ST vent valve choking fault; s

$f_{S,valve2}$	Multiplication factor of ET vent valve orifice area (valve choking)
$t_{S,valve2}$	Time of ET vent valve choking fault; s
S_{gl1}	Hole cross-section of gas leak fault in storage tank; m^2
t_{gl1}	Time of gas leak fault in storage tank; s
S_{gl2}	Hole cross-section of gas leak fault in external tank; m^2
t_{gl2}	Time of gas leak fault in external tank; s
S_{Ll1}	Hole cross-section of liquid leak fault in storage tank; m^2
t_{Ll1}	Time of liquid leak fault in storage tank; s
S_{Ll2}	Hole cross-section of liquid leak fault in external tank; m^2
t_{Ll2}	Time of liquid leak fault in external tank; s
Q_{l2}	Heat leak to ET ullage area
t_{AE}	Time of transfer line valve blockage; s
f_{AE}	Multiplication factor for valve area
t_{AF}	Time of transfer line chilldown valve blockage; s
f_{AF}	Multiplication factor for valve area
t_{AJ}	Time of replenish valve blockage; s
f_{AJ}	Multiplication factor for valve area
t_{AK}	Time of main fill valve blockage; s
f_{AK}	Multiplication factor for valve area
f_{APV11}	Multiplication factor for valve area
f_{APV12}	Multiplication factor for valve area
f_{APV13}	Multiplication factor for valve area
t_{StuckE}	Time of transfer line valve stuck fault; s
$stuckE$	Valve position for stuck fault (in [0,1])
t_{StuckF}	Time of transfer line chilldown valve stuck fault; s
$stuckF$	Vvalve position for stuck fault (in [0,1])
t_{StuckJ}	Time of replenish valve stuck fault; s
$stuckJ$	Valve position for stuck fault (in [0,1])
t_{StuckK}	Time of main fill valve stuck fault; s
$stuckK$	Valve position for stuck fault (in [0,1])
$t_{StuckPV11}$	Time of main fill valve stuck fault; s
$stuckPV11$	Valve position for stuck fault (in [0,1])
$t_{StuckPV12}$	Time of main fill valve stuck fault; s
$stuckPV12$	Valve position for stuck fault (in [0,1])
$t_{StuckPV13}$	Time of main fill valve stuck fault; s
$stuckPV13$	Valve position for stuck fault (in [0,1])
$t_{StuckST}$	Time of ST vent valve stuck fault; s
$stuckST$	Valve position for stuck fault (in [0,1])
$t_{StuckET}$	Time of ET vent valve stuck fault; s
$stuckET$	Valve position for stuck fault (in [0,1])

The mathematical model is described by the following set of equations. Here, we assume that the temperature of the LH2 is constant at 20 K.

3.1 Storage Tank

$$
\begin{align}
h_{L1} &= f(m_{L1}/\rho_L) \tag{3.1.1}\\
\rho_{v1} &= m_{v1}/(V_1 - m_{L1}/\rho_L) \tag{3.1.2}\\
p_{v1} &= m_{v1}R_vT_{v1}/(V_1 - m_{L1}/\rho_L) \tag{3.1.3}\\
\nu &= \mu/\rho_{v1} \tag{3.1.4}\\
S_1 &= \pi(R_1^2 - (R_1 - h_{L1})^2) \tag{3.1.5}\\
T_{s1} &= T_c\left(\frac{p_{v1}}{p_c}\right)^{\dfrac{1}{\lambda}} \tag{3.1.6}\\
h_{fg1} &= h_{fg}^0\sqrt{\frac{T_c - T_{s1}}{T_c - T_{L1}}} \tag{3.1.7}\\
h_{Ls1} &= \sqrt{\kappa_L c_L \rho_L/\pi} \tag{3.1.8}\\
\dot{Q}_{Ls1} &= h_{Ls1}S_1(T_{L1} - T_{s1}) \tag{3.1.9}\\
h_{vs1} &= \begin{cases} \sqrt{\kappa_v c_v \rho_{v1}/\pi}, & T_{v1} \geq T_{s1} \\ 0.156\sqrt[3]{\dfrac{g\beta c_p \rho_{v1}^2(T_{s1} - T_{v1})}{\kappa_v \nu}}, & T_{v1} < T_{s1} \end{cases} \tag{3.1.10}\\
\dot{Q}_{vs1} &= h_{vs1}S_1(T_{v1} - T_{s1}) \tag{3.1.11}\\
J_{cd1} &= \frac{-(\dot{Q}_{Ls1} + \dot{Q}_{vs1}}{h_{fg1}} \tag{3.1.12}\\
\dot{m}_{L1} &= -J_{tr} - J_{vap} - J_{L,Leak1} + J_{cd1} \tag{3.1.13}\\
\dot{m}_{v1} &= J_{boil} - J_{v,valve1} - J_{v,Leak1} - J_{cd1} \tag{3.1.14}\\
\dot{Q}_{v1} &= \dot{Q}_{ev1} - \dot{Q}_{vs1} + p_1\dot{m}_{L1}/\rho_L - c_pT_{v1}(J_{v,Leak1} + J_{v,valve1}) \tag{3.1.15}\\
&\quad - c_pT_{s1}J_{cd1} + c_pT_{boil}J_{boil} \tag{3.1.16}\\
&\quad - J_{v,valve}v_{v1}^2/2 - J_{v,Leak1}v_{v,Leak1}^2/2 \tag{3.1.17}\\
\dot{T}_{v1} &= \frac{1}{m_{v1}c_v}(\dot{Q}_{v1} - \dot{m}_{v1}c_vT_{v1}) \tag{3.1.18}\\
& \tag{3.1.19}
\end{align}
$$

Here, $f(m_{L1})$ is the algebraic function which computes liquid height for a sphere for a given amount of volume.

3.2 Vaporizer

$$
\begin{align}
J_{vap} &= c_{vap}\lambda_{vap}\sqrt{2\rho_L(p_1 - p_{atm})} \tag{3.2.1}\\
\dot{J}_{boil} &= \begin{cases} \frac{1}{\tau_{vap}}(J_{vap} - J_{boil}), & m_{vap} > 0 \\ 0, & \text{otherwise} \end{cases} \tag{3.2.2}\\
\dot{m}_{vap} &= J_{vap} - J_{boil} \tag{3.2.3}\\
& \tag{3.2.4}
\end{align}
$$

3.3 Transmission Line

$$f = \frac{1.3}{\log\left(\frac{D_{pipe}}{2d_{r,pipe}}\right)^2} \tag{3.3.1}$$

$$\alpha_{pipe} = 2\pi\left(\frac{D_{pipe}}{2}\right)^2\sqrt{\frac{\rho_L D_{pipe}}{2L_{pipe}f}} \tag{3.3.2}$$

$$\alpha_i = 2\pi\left(\frac{d_i}{2}\right)^2\sqrt{\frac{2\rho_L}{k_i}}, \quad i = \{E, F, J, K, PV11, PV12, PV13\} \tag{3.3.3}$$

$$\alpha_{eff} = ((\lambda_E\alpha_E + \lambda_F\alpha_F)^{-2} + \alpha_{pipe}^{-2} + \tag{3.3.4}$$

$$(\lambda_J\alpha_J + \lambda_K\alpha_K)^{-2} + \lambda_{PV11}\alpha_{PV11}^{-2} + \tag{3.3.5}$$

$$(\lambda_{PV12}\alpha_{PV12} + \lambda_{PV13}\alpha_{PV13})^{-2})^{-1/2} \tag{3.3.6}$$

$$\dot{J}_{tr} = \frac{1}{\tau_{tr}}(\alpha_{eff}\sqrt{|p_1 - p_2|}\text{sign}(p_1 - p_2) - J_{tr}) \tag{3.3.7}$$

$$\tag{3.3.8}$$

3.4 External Tank

$$h_{L2} = f(m_{L2}/\rho_L) \tag{3.4.1}$$

$$\rho_{v2} = m_{v2}/(V_2 - m_{L2}/\rho_L) \tag{3.4.2}$$

$$\rho_{g2} = m_{g2}/(V_2 - m_{L2}/\rho_L) \tag{3.4.3}$$

$$\rho_2 = (m_{v2} + m_{g2})/(V_2 - m_{L2}/\rho_L) \tag{3.4.4}$$

$$p_{v2} = m_{v2}R_vT_{v2}/(V_2 - m_{L2}/\rho_L) \tag{3.4.5}$$

$$p_{g2} = m_{g2}R_vT_{v2}/(V_2 - m_{L2}/\rho_L) \tag{3.4.6}$$

$$p_2 = p_{v2} + p_{g2} \tag{3.4.7}$$

$$\nu_2 = \mu/\rho_2 \tag{3.4.8}$$

$$A_2 = \pi R_2^2 \tag{3.4.9}$$

$$T_{s2} = T_c \left(\frac{p_{v2}}{p_c}\right)^{\frac{1}{\lambda}} \tag{3.4.10}$$

$$h_{fg2} = h_{fg}^0 \sqrt{\frac{T_c - T_{s2}}{T_c - T_{L1}}} \tag{3.4.11}$$

$$h_{Ls2} = \sqrt{\kappa_L c_L \rho_L/\pi} \tag{3.4.12}$$

$$\dot{Q}_{Ls2} = h_{Ls2}A_2(T_{L2} - T_{s2}) \tag{3.4.13}$$

$$h_{vs2} = \sqrt{\kappa_v c_v \rho_2/\pi} \tag{3.4.14}$$

$$\dot{Q}_{vs2} = h_{vs2}A_2(T_{v2} - T_{s2}) \tag{3.4.15}$$

$$J_{cd2} = \frac{-(\dot{Q}_{Ls2} + \dot{Q}_{vs2}}{h_{fg2}} \tag{3.4.16}$$

$$\dot{Q}_{wL2} = h_{wL2}(T_{w2} - T_{L2})(A_2 + R_2h_{L2}) \tag{3.4.17}$$

$$Pr = c_p\mu/\kappa_v \tag{3.4.18}$$

$$Ra = |g\beta(T_{w2} - T_{v2})(A_2 + R_2h_{L2})^3 Pr/\nu_2| \tag{3.4.19}$$

$$\Psi = (1 + \left(\frac{0.492}{Pr}\right)^{9/16})^{-16/9} \tag{3.4.20}$$

$$Nu = 0.68 + 0.503\sqrt[4]{Ra * \Psi} \tag{3.4.21}$$

$$h_{wv2} = \frac{Nu\kappa_v}{H - h_{L2}} \tag{3.4.22}$$

$$\dot{Q}_{wv2} = h_{wv2}(T_{w2} - T_{v2})(A_2 + (H_2 - R_2h_{L2})) \tag{3.4.23}$$

$$J_{boil2} = \dot{Q}_{wL2}/h_{fg2} \tag{3.4.24}$$

$$\dot{m}_{L2} = J_{tr} - J_{L,Leak2} + J_{cd2} - J_{boil2} \tag{3.4.25}$$

$$\dot{m}_{v2} = J_{boil2} - J_{v,valve2} - J_{v,Leak2} - J_{cd2} \tag{3.4.26}$$

$$J_{g,in} = \lambda_{prepress}c_{prepress}\sqrt{p_{g,in} - p_2} \tag{3.4.27}$$

$$\dot{Q}_{v2} = \dot{Q}_{wv2} - \dot{Q}_{vs2} + p_2\dot{m}_{L2}/\rho_L + c_{pg}T_{g,in}J_{g,in} \tag{3.4.28}$$

$$- c_pT_{v2}(J_{v,Leak2} + J_{v,valve2}) - c_{pg}T_{g2}(J_{g,Leak2} + J_{g,valve2}) \tag{3.4.29}$$

$$- c_pT_{s2}J_{cd2} + c_pT_{boil}J_{boil2} \tag{3.4.30}$$

$$- J_{v,valve2}v_{v2}^2/2 - J_{v,Leak2}v_{v,Leak2}^2/2 \tag{3.4.31}$$

$$- J_{g,valve2}v_{g2}^2/2 - J_{g,Leak2}v_{g,Leak2}^2/2 \tag{3.4.32}$$

$$\dot{T}_{v2} = \frac{1}{m_{v2}c_v + m_{g2}c_{vg}}(\dot{Q}_{v2} - \dot{m}_{v2}c_vT_{v2}) \tag{3.4.33}$$

$$\dot{T}_{w2} = \frac{1}{m_{w2}c_{w2}}(\dot{Q}_{ew2} - \dot{Q}_{wL2} - \dot{Q}_{wv2} \tag{3.4.34}$$

$$\tag{3.4.35}$$

3.5 Vent Valves

$$J_{vvalve1} = \lambda_{vent,ST} \frac{\rho_{v1}\rho_1 \sqrt{\gamma\left(p_1 - p_{atm}\right)}}{\Gamma\sqrt{\rho_{v1}}} S_{valve1} \tag{3.5.1}$$

$$J_{vvalve2} = \lambda_{vent,ET} \frac{\frac{\rho_{v2}}{\rho_2} \sqrt{\gamma\left(p_2 - p_{atm}\right)}}{\Gamma\sqrt{\rho_{v2} + \rho_{g2}}} S_{valve2} \tag{3.5.2}$$

$$J_{gvalve2} = \lambda_{vent,ET} \frac{\frac{\rho_{g2}}{\rho_2} \sqrt{\gamma\left(p_2 - p_{atm}\right)}}{\Gamma\sqrt{\rho_{v2} + \rho_{g2}}} S_{valve2} \tag{3.5.3}$$

3.6 Leaks

$$J_{L,Leak1,2} = \sqrt{2\rho_L\left(p_{1,2} - p_{atm}\right)} S_{Ll1,2} \tag{3.6.1}$$

$$J_{v,Leak1,2} = \frac{\rho_{v1,2}\sqrt{\gamma\left(p_{1,2}\right)}}{\Gamma\sqrt{\rho_{1,2}}} S_{gl1,2} \tag{3.6.2}$$

$$J_{g,Leak2} = \frac{\rho_{g2}\sqrt{\gamma\left(p_2\right)}}{\Gamma\sqrt{\rho_2}} S_{gl2} \tag{3.6.3}$$

3.7 Filling Protocol

Pressurization:

$$\begin{aligned}
\lambda'_E &= 0 \\
\lambda'_F &= 0 \\
\lambda'_J &= 0 \\
\lambda'_K &= 1 \\
\lambda'_{PV11} &= 1 \\
\lambda'_{PV12} &= 1 \\
\lambda'_{PV13} &= 0 \\
\lambda'_{vent,ST} &= f_{vent,ST}(p_1, p_{press}) \\
\lambda'_{vent,ET} &= f_{vent,ET}(p_2) \\
\lambda'_{vap} &= f_{vap}(p_1, p_{press}) \\
\lambda'_{prepress} &= 1
\end{aligned}$$

Slow Fill:

$$\begin{aligned}
\lambda'_E &= 0\\
\lambda'_F &= 1\\
\lambda'_J &= 0\\
\lambda'_K &= 1\\
\lambda'_{PV11} &= 1\\
\lambda'_{PV12} &= 1\\
\lambda'_{PV13} &= 1\\
\lambda'_{vent,ST} &= f_{vent,ST}(p_1, p_{slow})\\
\lambda'_{vent,ET} &= f_{vent,ET}(p_2)\\
\lambda'_{vap} &= f_{vap}(p_1, p_{slow})\\
\lambda'_{prepress} &= 0
\end{aligned}$$

Fast Fill:

$$\begin{aligned}
\lambda'_E &= 1\\
\lambda'_F &= 1\\
\lambda'_J &= 0\\
\lambda'_K &= 1\\
\lambda'_{PV11} &= 1\\
\lambda'_{PV12} &= 1\\
\lambda'_{PV13} &= 1\\
\lambda'_{vent,ST} &= f_{vent,ST}(p_1, p_{fast})\\
\lambda'_{vent,ET} &= f_{vent,ET}(p_2)\\
\lambda'_{vap} &= f_{vap}(p_{v1}, p_{fast})\\
\lambda'_{prepress} &= 0
\end{aligned}$$

Fast Fill (Reduced Pressure):

$$\begin{aligned}
\lambda'_E &= 1\\
\lambda'_F &= 1\\
\lambda'_J &= 0\\
\lambda'_K &= 1\\
\lambda'_{PV11} &= 1\\
\lambda'_{PV12} &= 1\\
\lambda'_{PV13} &= 1\\
\lambda'_{vent,ST} &= f_{vent,ST}(p_1, p_{reduced})\\
\lambda'_{vent,ET} &= f_{vent,ET}(p_2)\\
\lambda'_{vap} &= f_{vap}(p_{v1}, p_{reduced})\\
\lambda'_{prepress} &= 0
\end{aligned}$$

Fast Fill (Reduced Flow):

$$\begin{aligned}
\lambda'_E &= 1\\
\lambda'_F &= 1\\
\lambda'_J &= 1\\
\lambda'_K &= 0.1\\
\lambda'_{PV11} &= 1\\
\lambda'_{PV12} &= 1\\
\lambda'_{PV13} &= 1\\
\lambda'_{vent,ST} &= f_{vent,ST}(p_1, p_{reduced})\\
\lambda'_{vent,ET} &= f_{vent,ET}(p_2)\\
\lambda'_{vap} &= f_{vap}(p_1, p_{reduced})\\
\lambda'_{prepress} &= 0
\end{aligned}$$

Topping:

$$\begin{aligned}
\lambda'_E &= 0\\
\lambda'_F &= 1\\
\lambda'_J &= 1\\
\lambda'_K &= 0.1\\
\lambda'_{PV11} &= 1\\
\lambda'_{PV12} &= 0\\
\lambda'_{PV13} &= 1\\
\lambda'_{vent,ST} &= f_{vent,ST}(p_1, p_{topping})\\
\lambda'_{vent,ET} &= 1\\
\lambda'_{vap} &= f_{vap}(p_1, p_{topping})\\
\lambda'_{prepress} &= 0
\end{aligned}$$

Replenish:

$$\begin{aligned}
\lambda'_E &= 0 \\
\lambda'_F &= 1 \\
\lambda'_J &= \begin{cases} 0, & h_{L2} > H \\ 1, & \min\left(1, 0.1\dfrac{0.999H - h_{L2}}{0.999H}\right) \end{cases} \\
\lambda'_K &= 0 \\
\lambda'_{PV11} &= 1 \\
\lambda'_{PV12} &= 0 \\
\lambda'_{PV13} &= 1 \\
\lambda'_{vent,ST} &= f_{vent,ST}(p_1, p_{replenish}) \\
\lambda'_{vent,ET} &= 1 \\
\lambda'_{vap} &= f_{vap}(p_1, p_{replenish}) \\
\lambda'_{prepress} &= 0
\end{aligned}$$

where,

$$f_{vent,ST}(p_1, p_{set}) = \begin{cases} 0, & p_1 < 1.05p_{set} \\ 1, & p_1 > 0.95p_{set} \\ \lambda^-_{vent,ST}, & \text{otherwise} \end{cases} \tag{3.7.1}$$

$$f_{vent,ST}(p_2) = \begin{cases} 0, & p_1 < p_{ET,low} \\ 1, & p_1 > p_{ET,high} \\ \lambda^-_{vent,ST}, & \text{otherwise} \end{cases} \tag{3.7.2}$$

$$f_{vap}(p_1, p_{set}) = \begin{cases} min(1, max(0, 10(p_{set} - p_1)/p_{set})), & p_1 < 0.98p_{set} \\ 0, & p_1 > 1.02p_{set} \\ \lambda^-_{vap}, & \text{otherwise} \end{cases} \tag{3.7.3}$$

Here, λ^- refers to the previous value of λ. The actual valve inputs λ_i are functions of commanded position λ'_i as follows.

$$\lambda_i = \begin{cases} stuck_i, & t \geq t_{Stuck,i} \\ \lambda'_i, & \text{otherwise} \end{cases} \tag{3.7.4}$$

4 MATLAB Functions

The set of available MATLAB functions and scripts are described below. For additional information, type `help x` in the MATLAB prompt, where `x` is the script or function name (e.g. `help LH2Simulate`).

- `LH2ModelParams`: This is a script that defines all the nominal parameters of the LH2 model. It defines a variable in the MATLAB workspace called `'LH2Model'` which stores these parameters and is used by the simulation and control functions. Fault parameters may then be set after this script is called. See `runHeatLeakET` for an example of its usage.
- `LH2Simulate`: This is a function that simulates the LH2 model given the parameters defined in the workspace variable `'LH2Model'`. If given no inputs, it runs to the default time specified in

`'LH2Model.tFinal'`. Optionally, a time vector can be provided as the first argument (e.g., `0:10000` or `0:1:10000`). An optional second argument specifies the name of the scenario (e.g. `'Nominal'`). It returns a structure which contains time and relevant dynamical variables and outputs as fields. See `plotLH2Data` for an example of how to use the data structure.

- `LH2Control`: This is an internal function which encodes the filling protocol.
- `plotLH2`: This is a function which takes two arguments and a third optional argument. The first argument is the name of the data field to plot (which must be a valid data field in the data structure returned by `LH2Simulate`). The second argument is a single data structure or a cell array of data structures (returned by calls to `LH2Simulate`). The same variable from each data structure are plotted on the same figure. The third argument is a gain term (used for unit conversions). See `runHeatLeakET` for an example of how to use this function.
- `plotLH2Data`: This is a utility function that plots data returned by `LH2Simulate` in a particular format, showing the most useful dynamical variables as subplots on 3 separate figures.
- `runNominal`: This is an example script which runs the nominal scenario. It demonstrates the use of `LH2ModelParams`, `LH2Simulate`, and `plotLH2Data`.
- `runHeatLeakET`: This is an example script which runs a nominal scenario and two different heat leak scenarios. Key variables are plotted together for comparison. It demonstrates the use of `LH2ModelParams`, `LH2Simulate`, and `plotLH2`.
- `SampleSimulations`: This is an example script which contains simulations for various fault scenarios and the parameter settings used to accomplish them. The different data sets are saved to workspace variables.

The general procedure for running a nominal or faulty scenario is the following (see also `runNominal`, `runHeatLeakET`, and `SampleSimulations`). First, run `LH2ModelParams`. This loads the model and initializes all parameters to nominal. Next, set the specific fault parameter values (e.g., `LH2Model.t_StuckE = 100; LH2Model.stuckE = 0.5;`). Next, run `LH2Simulate` to obtain the data from the simulation of this scenario. Before running a new scenario, run `LH2ModelParams` first to reinitialize parameters to nominal values. New fault parameters can then be set and the simulation can be run again for the new parameters. Data returned by a scenario can be plotted using `plotLH2Data(data)`, where `data` is the variable that contains the data returned by `LH2Simulate`.