Sigpro Memo: Analysis of Stationarity

Principal Investigator and Author: Jeff Andrews
Secondary Investigator: James McNames

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1 Abstract

Torque data from flight 2 was analyzed for stationarity, by computing confidence intervals for the mean and the autocorrelation. A window size of a single rotor revolution was used for analysis, which corresponds to approximately 9300 samples at the operational sampling frequency (50 KHz). While the results presented are inconclusive, it appears that maneuvers H and M are the most stationary of the maneuvers performed in flight 2, while G, K, and N are the least stationary.

2 Introduction

AMES is in possession of a large amount of vibrational and torque data taken from eight test flights on a Cobra helicopter. For more information on this data and how it was recorded, please refer to the Healthwatch Data Acquisition System document [1] compiled by Sigpro.

In order to make meaningful statements in the analysis of statistical data such as that recorded by the Healthwatch system, it is often important to know how stationary the data is over a given window size. Ideally, all deterministic effects from helicopter flight could be accounted for and subtracted out of the recorded data. However, this requires a great deal of time and is subject to large errors due to the unpredictable nature of helicopter flight. While some of these macroscopic patterns seen in the flight data may eventually be accounted for, in the meantime it would be beneficial to be able to make statements about the stationarity of the data irrespective of such knowledge.

To this end, data from flight 2 was analyzed for stationarity. The approach used and the results follow.

3 Approach and Results

Two items were analyzed for stationarity:

1. Mean M(t)
2. Autocorrelation A(t)

In the interest of simplicity, the period used for stationarity corresponded to approximately one rotation of the helicopter rotor. This is typically in the neighborhood of 320 RPM, and since the sampling frequency is 50KHz, 9300 samples are analyzed in each batch to keep the data an even multiple of 100. For the purpose of this memo, only the torque data of flight 2 was analyzed. All maneuvers in flight 2 are considered, so it should be possible to generalize the results from this flight to that of the other seven flights.

3.1 Analysis of Mean

The mean of the torque data for flight 2 was analyzed for stationarity by using the well known method of confidence intervals [2]. For each batch of 9300 samples, the samples were placed into 93 bins of 100 samples. It is assumed that the process mean and variance does not change over 100 samples (only 2 msec), thus the law of large numbers applies [2], and these 100 samples can be assumed to follow a Gaussian distribution. Using the resulting 93 Gaussian random variables, a confidence region about the mean can be computed as in [2].

Each maneuver of the flight lasts approximately 34 seconds, and thus contains about 181 such confidence intervals about the mean. Representative plots of these confidence regions are labeled and included at the end of this memo. As can be observed, over some regions the confidence bounds don't change that quickly (thus the signal is quasistationary), while over other regions, the mean increases or decrease rapidly (nonstationary).

In order to be quantitative, let us restrict our attention to just adjacent batches of 9300 samples. We will say the mean is stationary over the adjacent batches of samples if the confidence bounds for the mean have a region of overlap. Otherwise, the mean will be said to be nonstationary for the two batches of samples. Quantitatively, the decision is made this way:

if $M_{hi}(k) > M_{lo}(k+1)$ and $M_{hi}(k+1) > M_{lo}(k)$
 overlapping
else
 non overlapping

For each maneuver and for confidence regions of 90% (tightest), 95%, and 99% (loosest), the number of overlapping confidence regions is computed as a percentage.

The results are as follows:

Maneuver letter	Number of maneuvers	Average percentage of overlaps for 90, 95, and 99% confidence intervals		
		99%	**95%**	**90%**
G	2	33.5	28.5	20.0
H	2	82.0	68.5	60.5
I	3	66.3	52.0	43.5
J	3	73.0	60.5	51.5
K	3	42.0	31.5	27.5
L	3	73.0	58.5	49.0
M	3	91.5	82.0	73.0
N	3	59.5	50.0	42.0

As can be seen, the results are not very clear cut. For simplicity, we shall restrict our discussion to the case of 99% confidence intervals, although depending on the application, other confidence intervals may be more appropriate. A 99% confidence interval is equivalent to saying: "there is a 1% chance that the mean is not in the following interval".

For the above maneuvers, three bins can be constructed.

1. Least stationary (overlap % from 0 to 50): G, K
2. Medium stationary (50 to 80%): I, J, L, N
3. Most stationary (above 80%): H, M

Thus, in analyzing the torque data from flight 2, maneuvers G and K seem ruled out as stationary over a rotor revolution, while H and M appear reasonably stationary over a revolution.

3.2 Analysis of Autocorrelation

The autocorrelation is very slow to compute, so only preliminary investigations have been made into it's stationarity. Again, only adjacent bins of 9300 samples were compared. Due to its complexity, it was only considered over 5 adjacent bins. The first 4 bins are not considered in order to make sure that transient flight behavior is not a major factor. Further simplifying the calculation, only every 50th sample was used, since the sampling frequency of 50 KHz is much greater than the window of stationarity being considered.

After the autocorrelation is estimated, it is compared to the adjacent bin's autocorrelation function. The metric used for comparison is the well known normalized mean-squared error (NMSE) criterion. The NMSE is simply the MSE normalized by the variance of the signal, or in this case the product of the standard deviations of the two signals of interest.

$$NMSE = \frac{\frac{1}{N}\sum_{n=1}^{N}(A_1[n]-A_2[n])^2}{\sigma_1\sigma_2} \quad ,$$

where σ_i is the standard deviation of the random vector A_i

The results of the autocorrelation analysis are:

Maneuver letter	Number of maneuvers	NMSE averaged over all adjacent periods for all such maneuvers
G	2	.9473
H	2	.3412
I	3	.3959
J	3	.3360
K	3	.9583
L	3	.5261
M	3	.2378
N	3	.9597

As can be seen, the autocorrelation does not appear very stationary for the bulk of the maneuvers. It is possibly stationary for maneuvers H, I, J, and M, which are Hover, Hover turn left, Hover turn right, and Forward climb, respectively. More research needs to be done to determine the stationarity of the autocorrelation.

4 Conclusions and Future Research

It is encouraging that in both analysis of the mean and autocorrelation, somewhat consistent results were seen. Maneuvers H and M showed the most stationarity, while G K and N showed the least. The other maneuvers are in grayer area.

However, the results presented still are somewhat vague. Ideally, a more illuminating approach would be to subtract out deterministic or predictable effects from the statistical data before analyzing for stationarity. As noted, this may prove difficult. Data from the other flights could be analyzed in a similar manner to that performed in this research, and the results could be compared and/or concatenated for more conclusive results.

5 References

[1] Jeff Andrews et al. Healthwatch Data Acquisition System. NASA Ames research center internal document.

[2] Alberto Leon-Garcia, Probability and Random Processes for Electrical Engineering, 2nd edition. Addison Wesley, 1994.