

PROBABILITY AND ENTROPY: STATEMENTS AND QUESTIONS

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Abstract

I will demonstrate that probability theory and information theory are literally dual to each other via a duality relation called Birkhoff's Representation Theorem. Probability is a generalization of implication among a set of logical statements, so that probability describes the degree to which one statement implies another. Similarly, a quantity called 'relevance' generalizes the notion of answering among questions, so that relevance describes the degree to which one question answers another. The Boolean lattice of logical statements gives rise to the Free Distributive Lattice of Questions, the two lattices being related by Birkhoff's theorem. Moreover, relevances can be expressed in terms of entropies by requiring that the relevance of one question to another be expressed in terms of the probabilities of their answers. This result reveals the precise nature of the duality between probability theory and a natural generalization of information theory.