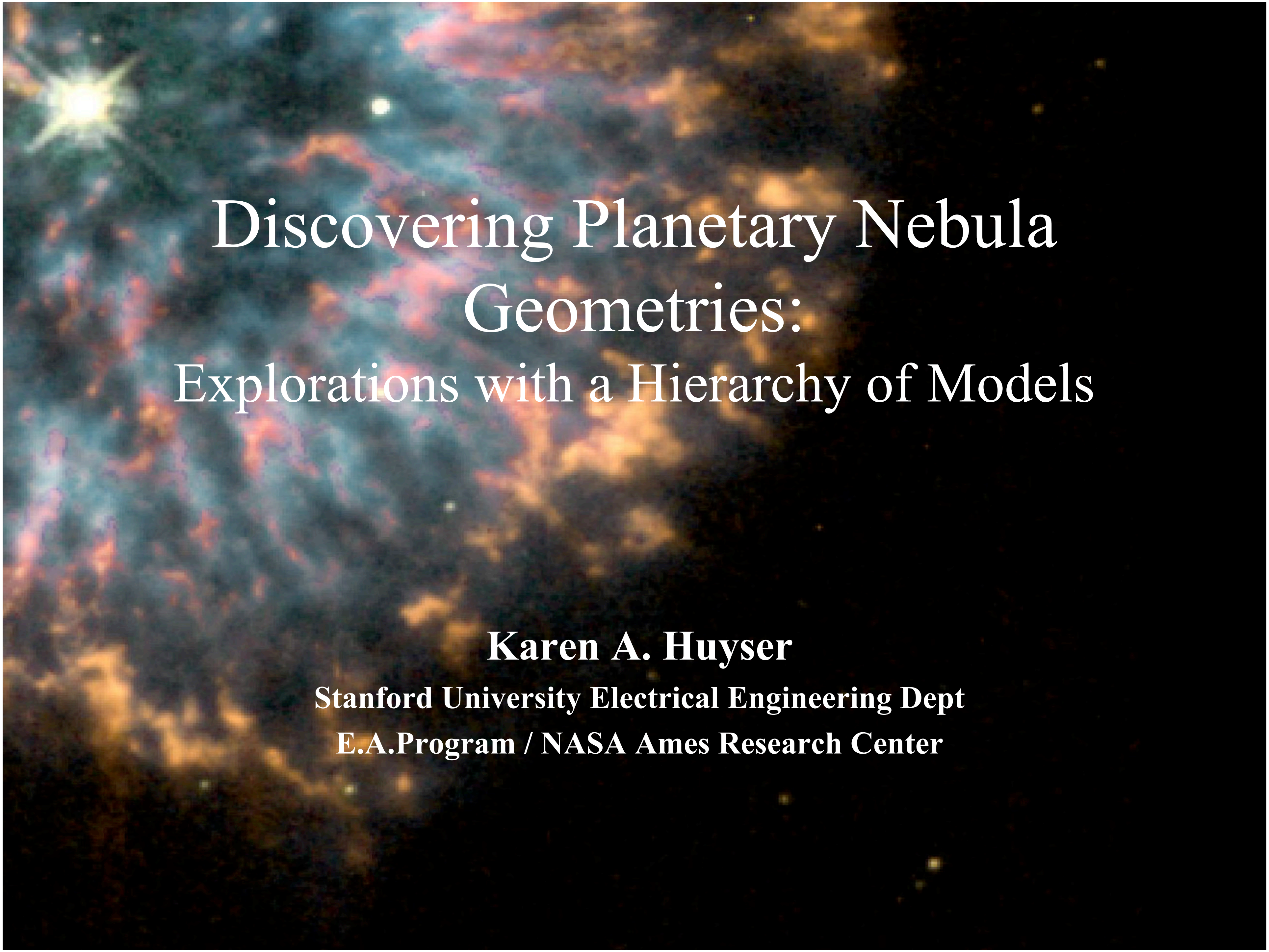

Discovering Planetary Nebula Geometries:

Explorations with a Hierarchy of Models

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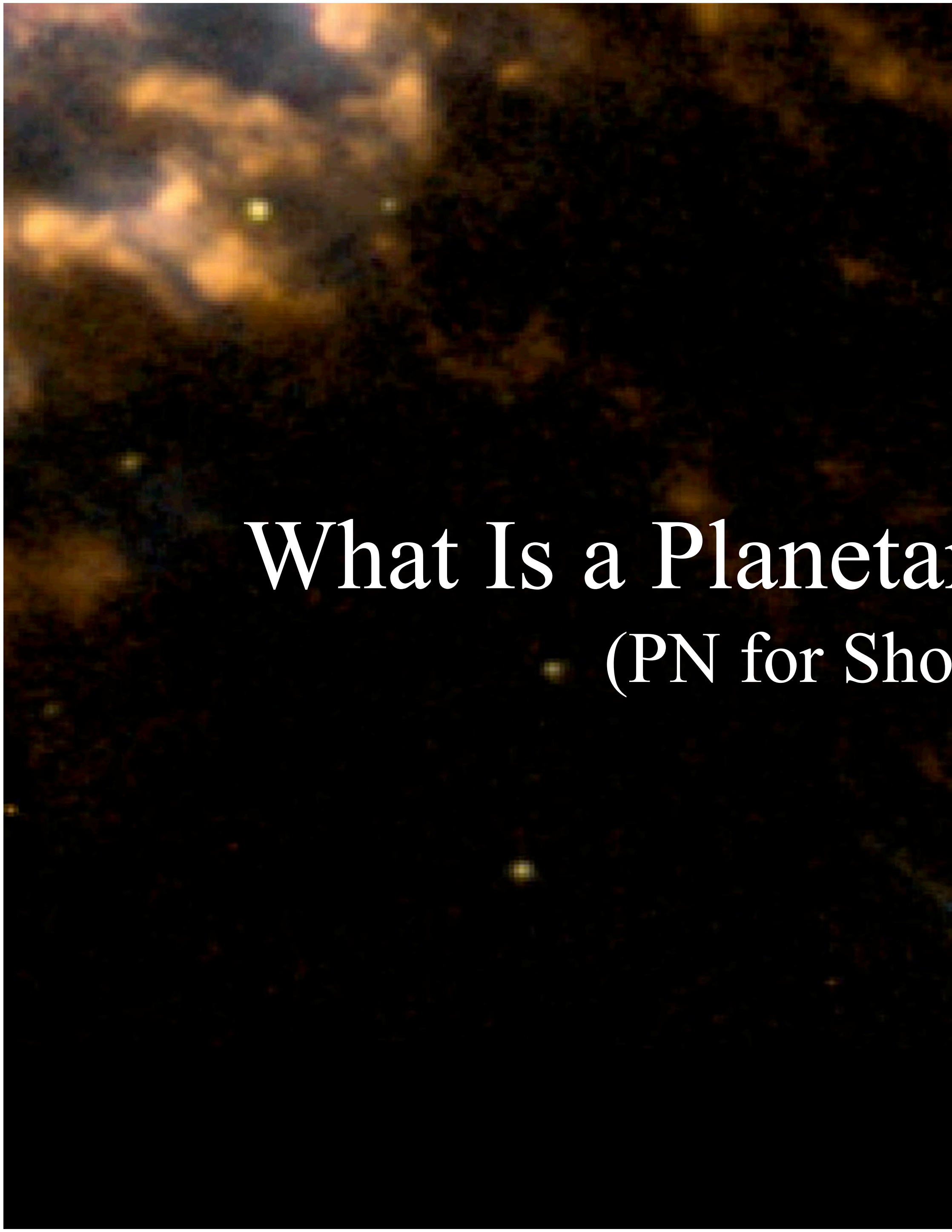
Outline

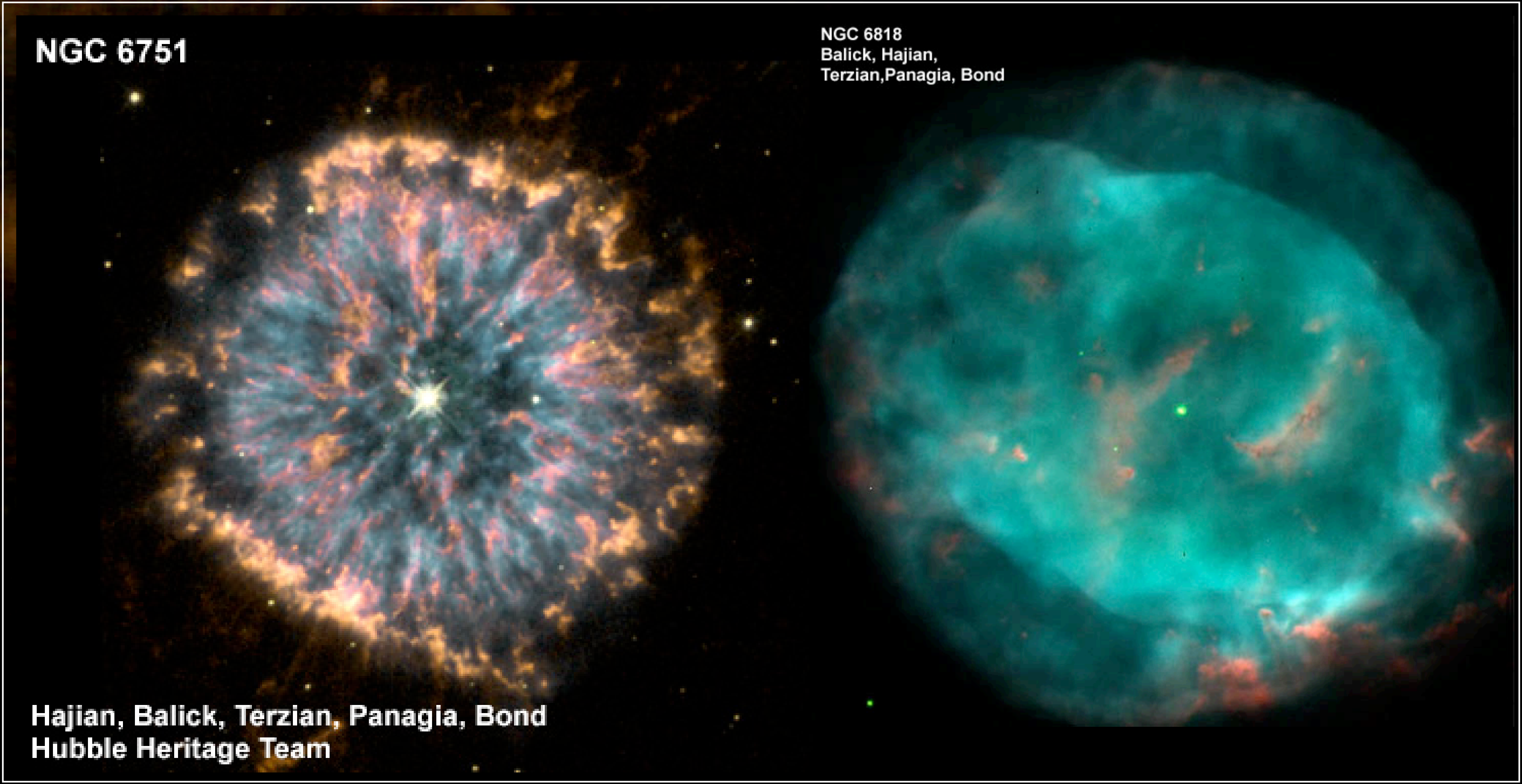
- What is a Planetary Nebula?
- Constructing a Quantitative Model
- Estimating Model Parameters from Data
- Exploring the Parameter Space
- Application to IC418
- Conclusions



What Is a Planetary Nebula?

(PN for Short)

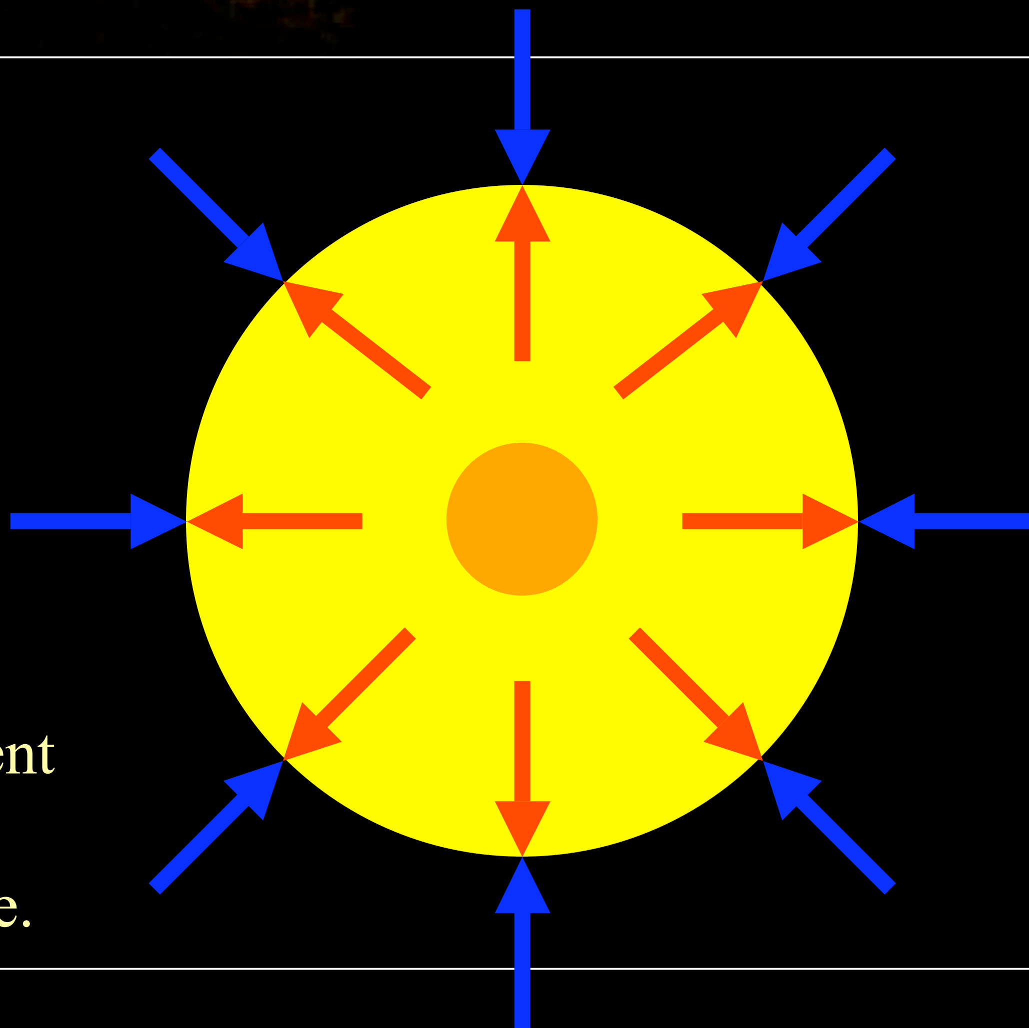
"Planetary" = Round (usually)
"Nebula" = Glowing Gases (around a star)



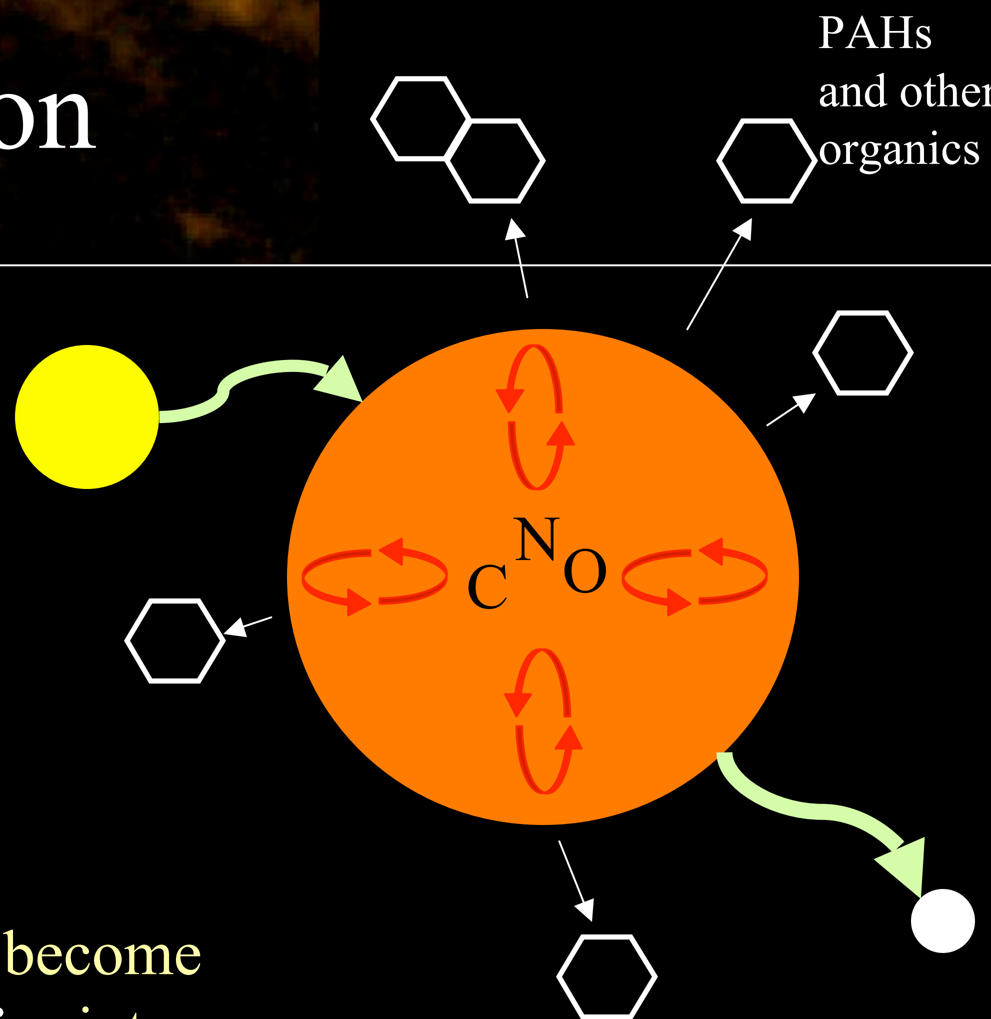
Forces Governing Stellar Structure

Nuclear fusion occurs
in the high temperature
and density present
in the **core** and generates
thermal pressure, which
acts against the attractive
gravitational forces.

As long as there is sufficient
fuel in the core, the star
remains in this steady-state.



Stellar Evolution



Stars eventually run out of Hydrogen to fuse, and begin to collapse cramming more matter into the core.

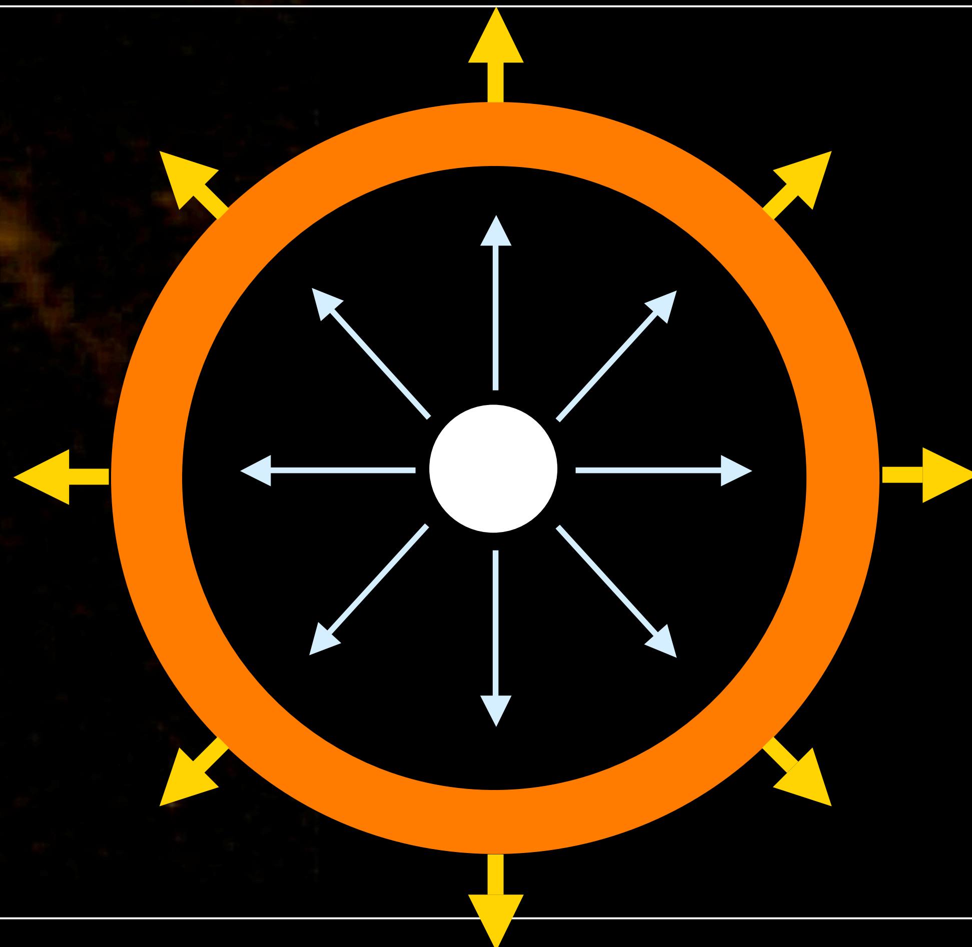
The greater densities and pressures allow creation of C N and O.
The star swells and cools to become a Red Giant spewing **organics** into space.

When fuel finally runs out, the star collapses into a **hot** White Dwarf

Planetary Nebula

During the collapse of the Red Giant into a White Dwarf much of the tenuous outer atmosphere of the star is thrown off.

A fast stellar wind from the now **hot star** forms a bubble inside the shell of gas.



Planetary Nebula

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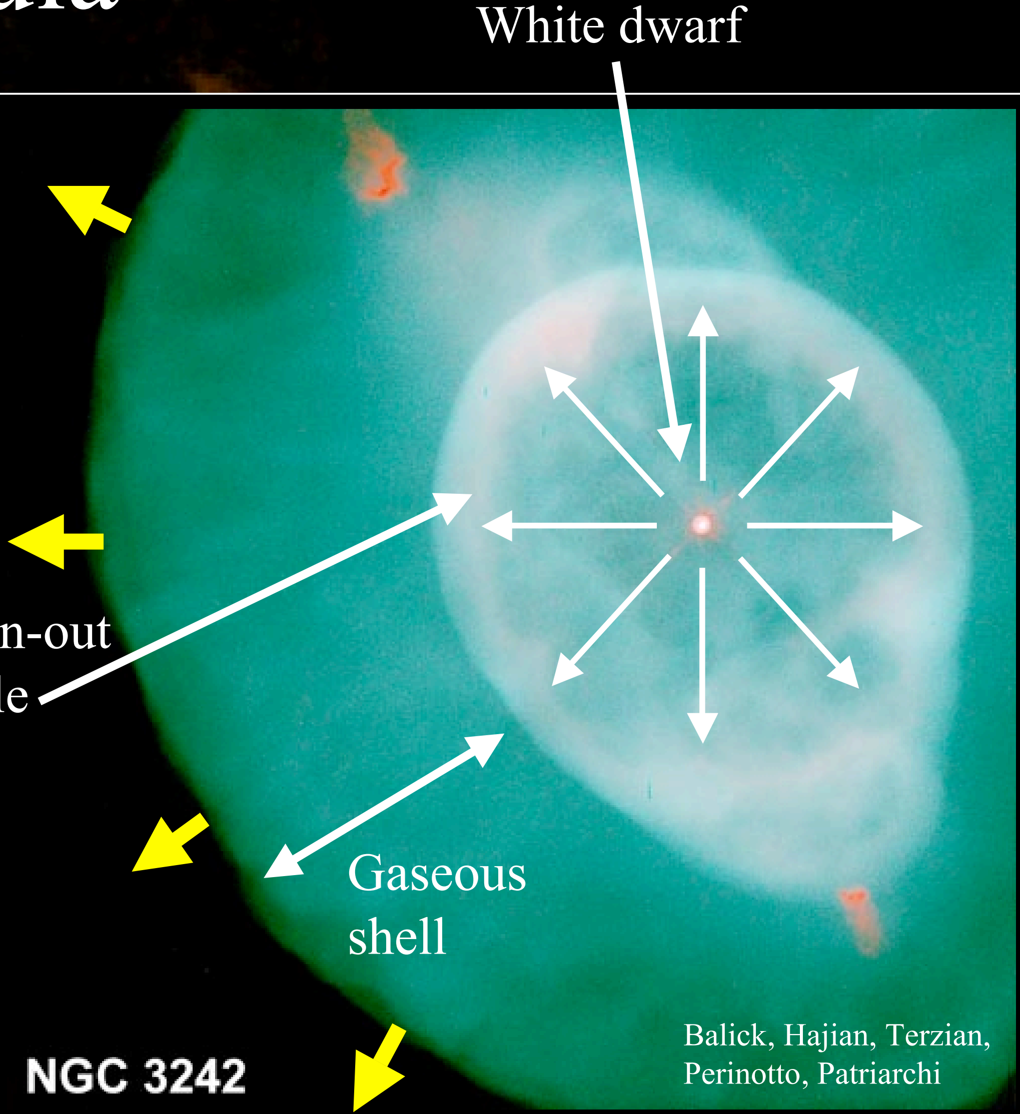
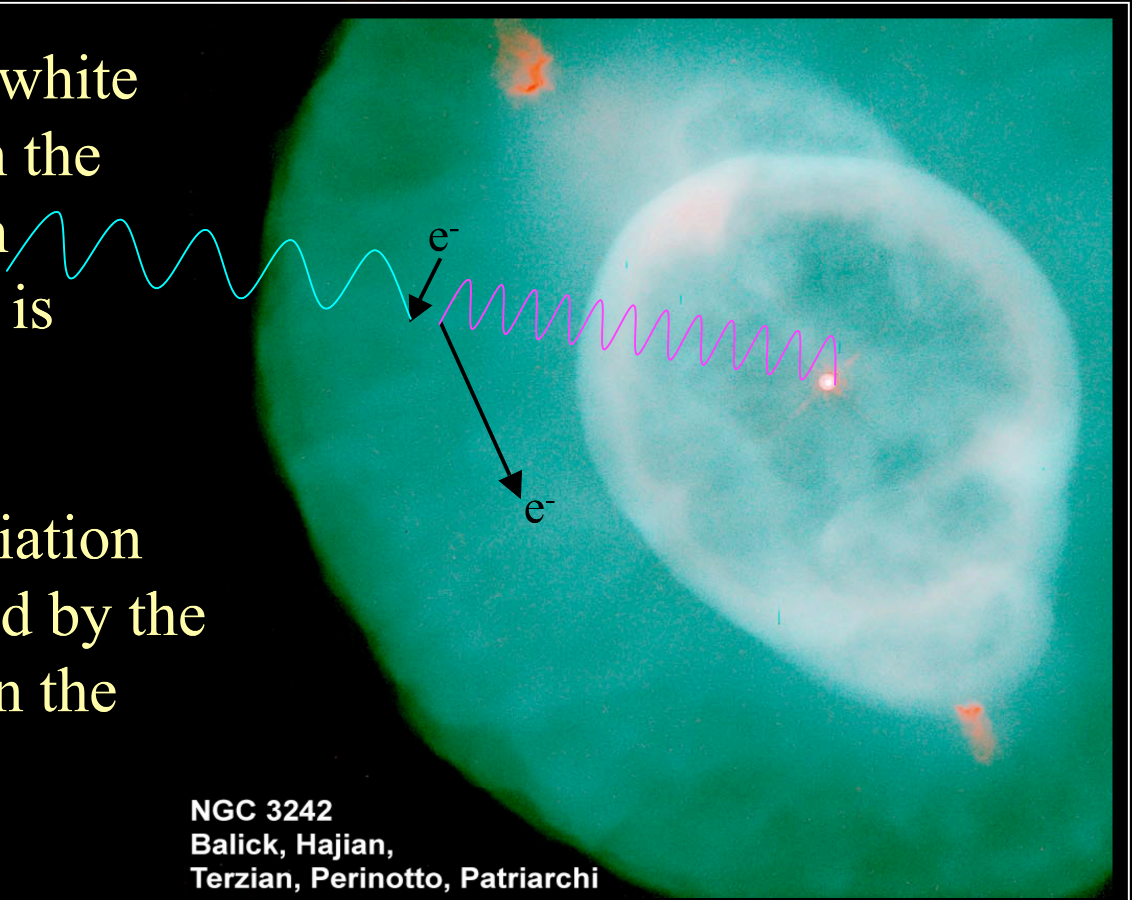


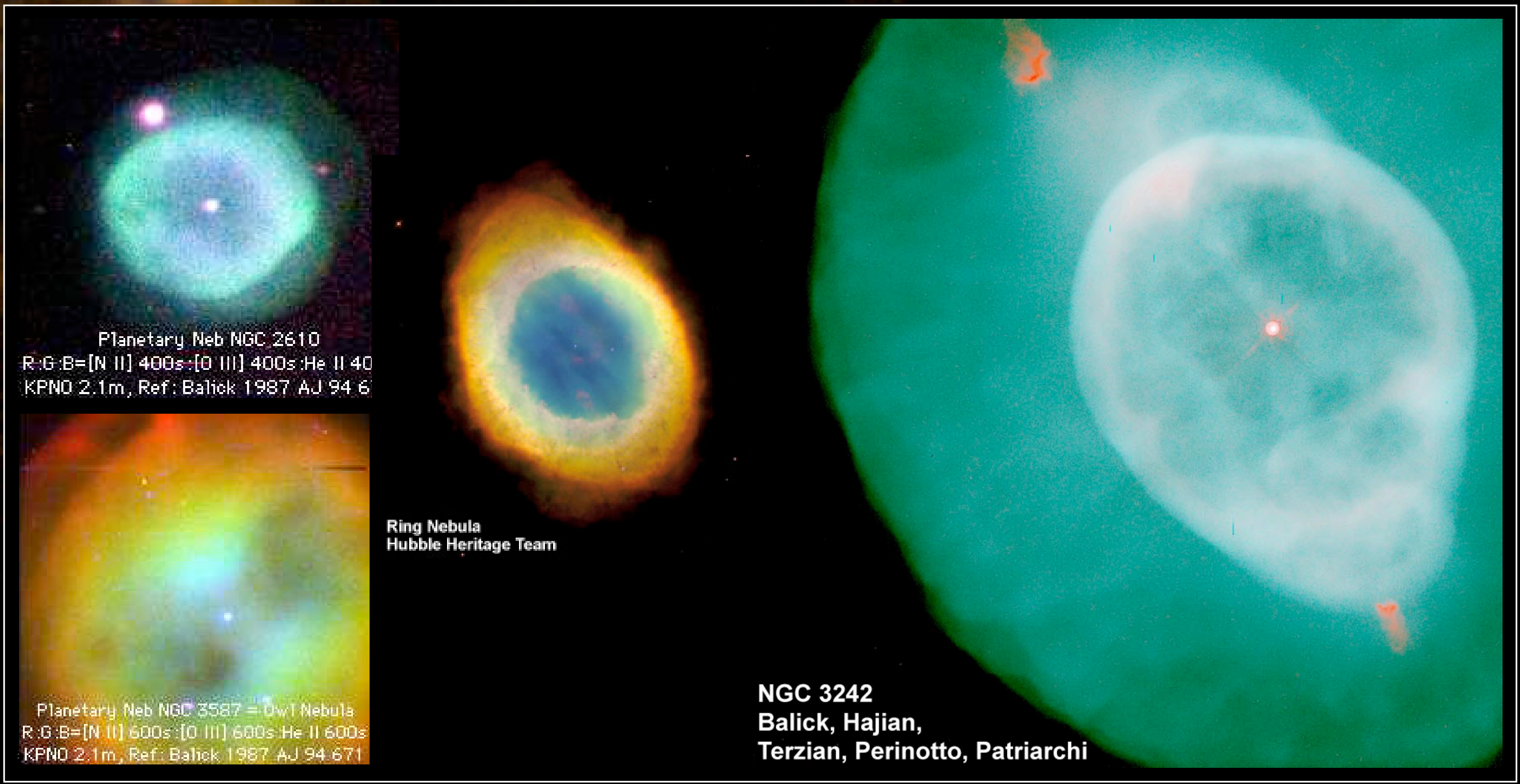
How Is a PN Illuminated?

Radiation from the hot white dwarf ionizes the gas in the nebula. Recombination occurs and visible light is emitted.

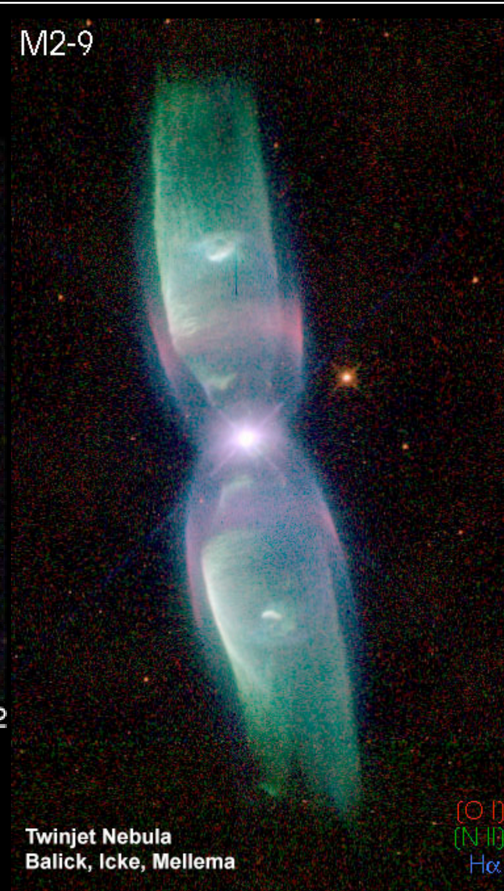
Hence, the ionizing radiation from the star is absorbed by the nebula and re-emitted in the visible range.

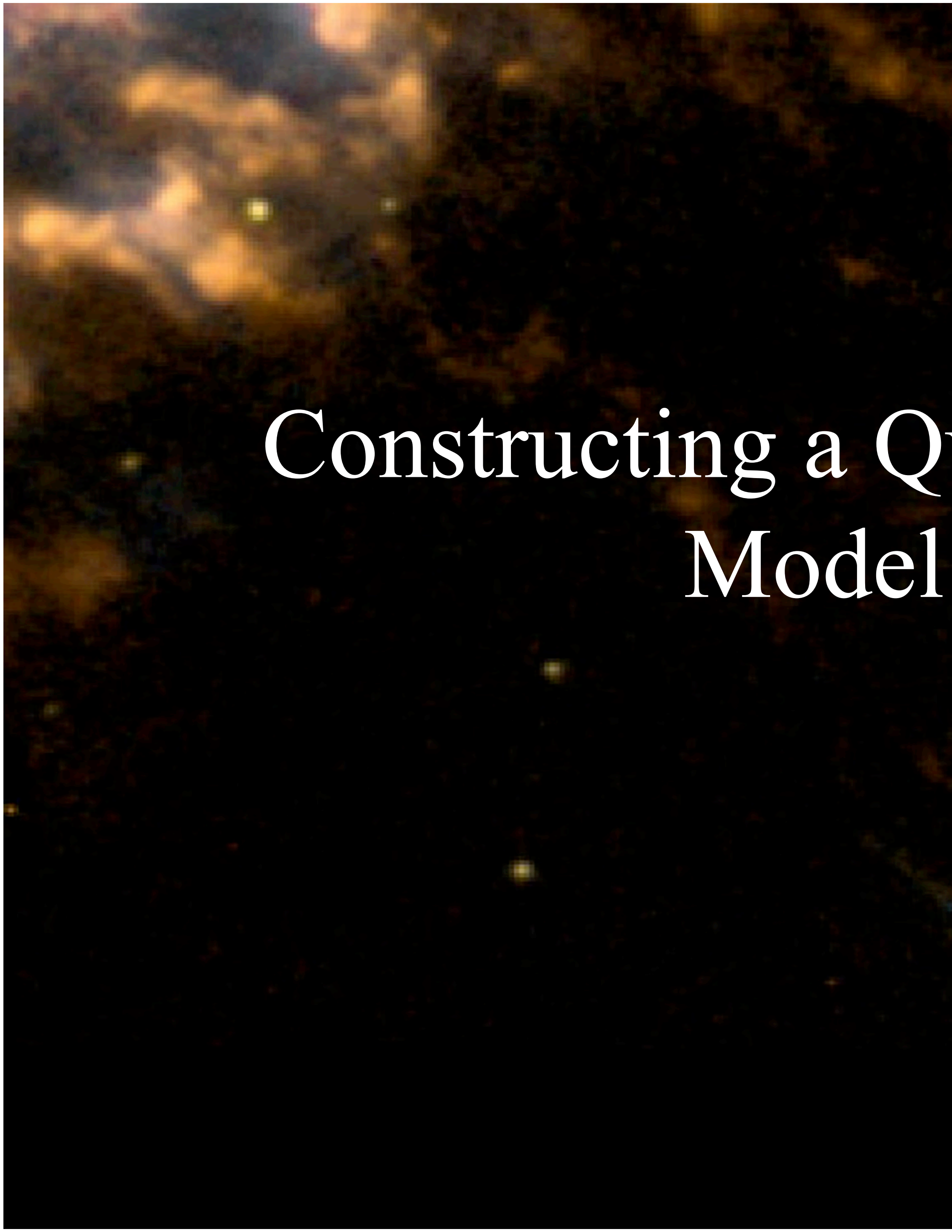


Various Morphologies of PNe



Various Morphologies of PNe





Constructing a Quantitative Model

What Do We Want to Know?

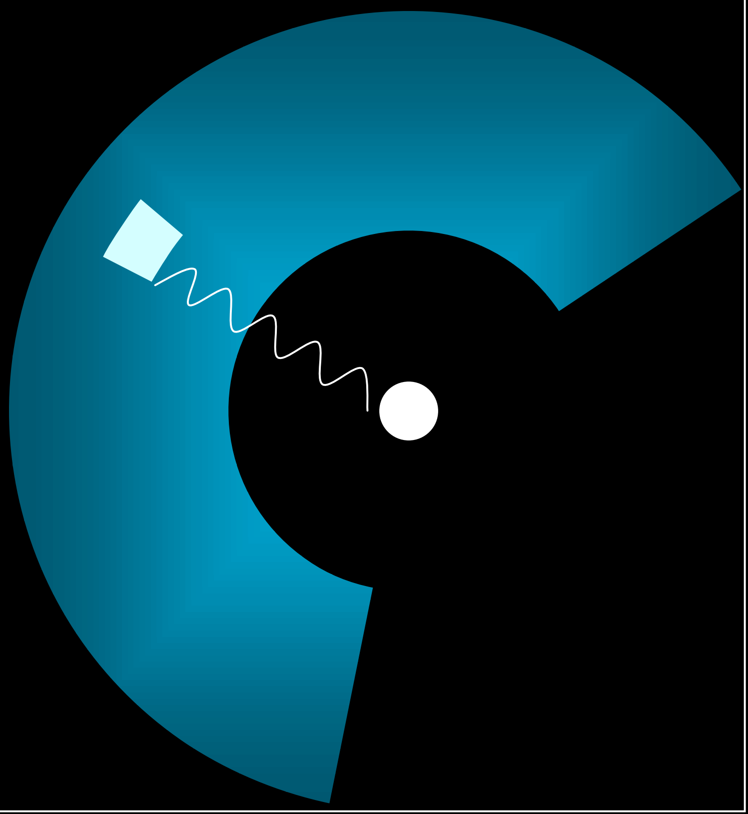
- General Morphology of the Nebula
- Orientation of the Nebula
- Composition and Density Profile of the Nebula
- Luminosity of the Central Star
- Distance of the Nebula to Earth

The Physics of Illumination

High energy photons ionize the gas in the nebula.

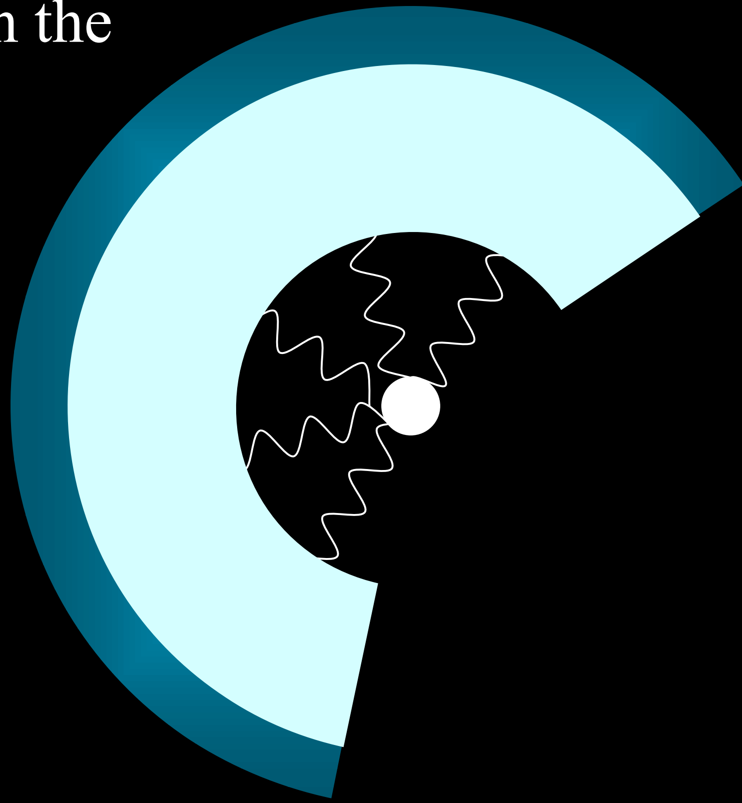
Every volume element of gas absorbs a fraction of the incoming photons.

The luminosity of a volume element is proportional to the square of its density.



#1 – PN is Ionization Bounded.

If, throughout the nebula, the gas density is great enough, all the ionizing photons are absorbed within the nebula itself.



Thus only part of the actual nebula will be illuminated

Such a nebula is said to be

IONIZATION BOUNDED

#2 – Evacuated Bubble is Ellipsoidal

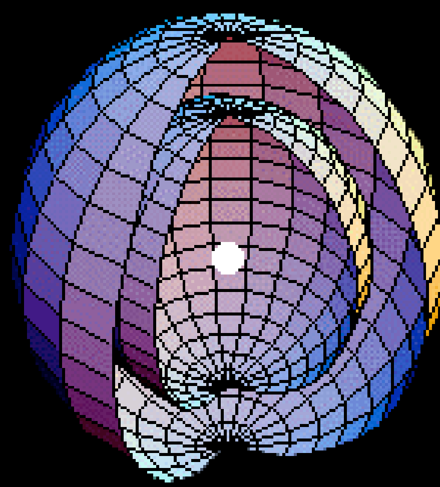
IBPES Model

Ionization-Bounded Prolate Ellipsoidal Shell Model

Masson (1989, 1990), Aaquist & Kwok (1996), Zhang & Kwok (1998)

The inner bubble is assumed to be a prolate spheroid of radius $R_{inner}(\theta,\phi)$.

The outer radius $R_{outer}(\theta,\phi)$ of the visible gaseous shell is determined by density profile of the nebula and by applying the ionization-bounded condition.



IBPES Model: PN Illumination

The number of photons emitted by a volume element of gas are

$$\#\, photons = \alpha_B \quad n^2(r,\theta,\phi) \quad r^2 \sin\theta\, dr\, d\theta\, d\phi$$

Recombination parameter

Density squared

Volume element

For an ionization bounded nebula, the total ionizing luminosity emitted by the star is equal to the total number of photons emitted by the nebula

$$\frac{dL}{d\Omega} = \int_{R_{inner}}^{R_{outer}} \alpha_B\, n^2(r,\theta,\phi) r^2\, dr$$

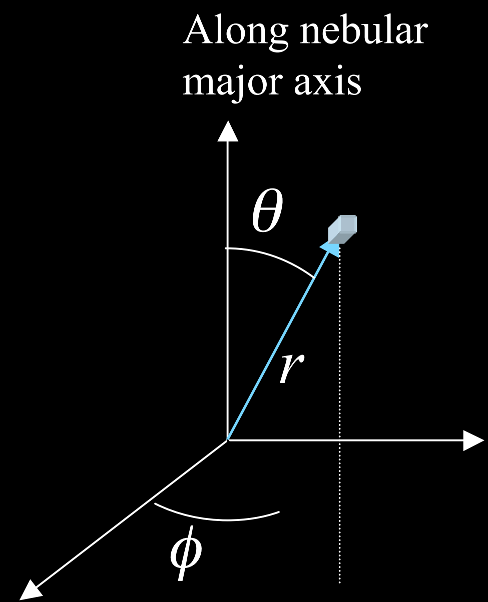
IBPES Model: Density Profiles

The nebular density is generally not uniform.

We adopt the density profiles used by Aaquist & Kwok 1996.
Given a density profile of the separable form

$$n(r,\theta,\phi) = n_o\,\eta(r)\eta(\theta)\eta(\phi)$$

Equatorial density at inner radius – n_o



Density Profiles

Radial density varies as a power law: $\eta(r) = \left(\frac{r}{R_{\text{inner}}} \right)^{-\gamma}$

Cylindrical symmetry $\eta(\phi) = \eta_\phi$

Latitudinal density variation is parameterized

$$\eta(\theta) = \begin{cases} \left(\frac{2\theta}{\pi} \right)^{\alpha} (1 - \beta) + \beta & \text{if} \quad 0 \le \theta \le \frac{\pi}{2} \\ \left(\frac{2\pi - 2\theta}{\pi} \right)^{\alpha} (1 - \beta) + \beta & \text{if} \quad \frac{\pi}{2} \le \theta \le \pi \end{cases}$$

Radial density exponent – γ
Latitudinal density gradient – α
Pole-to-equator density ratio – β

Azimuthal density – η_ϕ

Outer Boundary

By applying the ionization-bounded condition …

$$\frac{dL}{d\Omega} = \int_{R_{inner}}^{R_{outer}} \alpha_B \, n_o^2 \, \eta_\phi^2 \, \eta_\theta^2(\theta) \left(\frac{r}{R_{inner}} \right)^{-2\gamma} r^2 dr$$

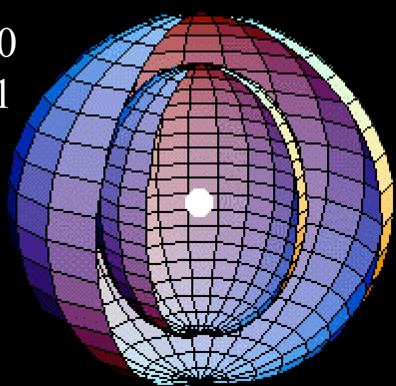
…the outer boundary of the nebula can be solved for as

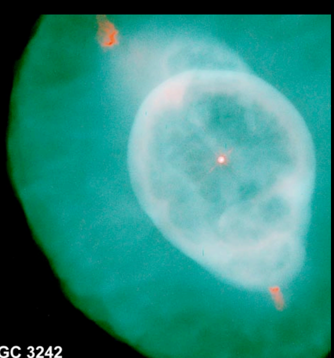
$$R_{\text{outer}}(\theta) = R_{\text{inner}}(\theta) \left[\left(\frac{(-2\gamma + 3) R_{\text{inner}}^{-3}(\theta)}{\eta^2(\theta)} \right) \left(\frac{1}{\alpha_B \, n_o^2 \, \eta_\phi^2} \frac{dL}{d\Omega} \right) + 1 \right]^{\frac{1}{-2\gamma+3}}$$

Reproducing Morphology

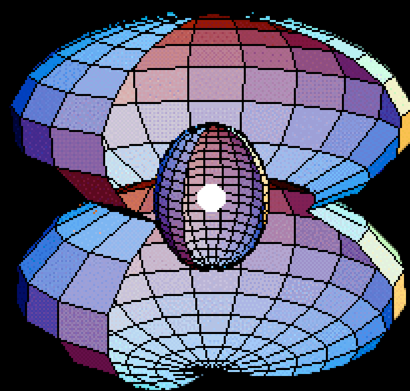
By varying the latitudinal density, one can reproduce the basic morphologies

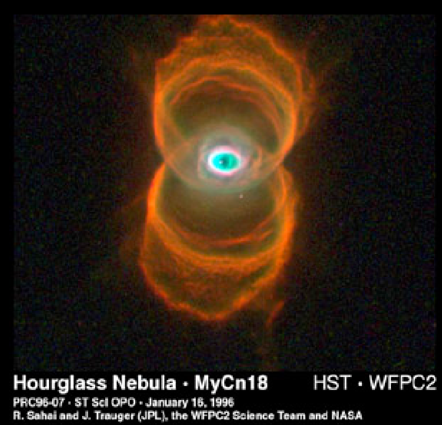
$\alpha=0$
$\beta=1$



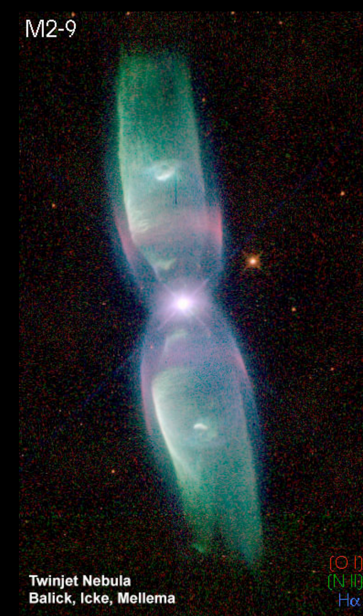


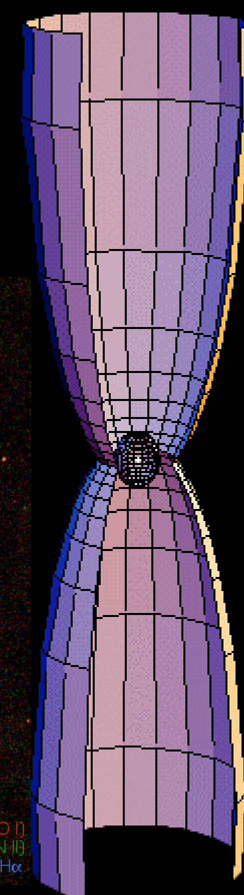
$\alpha=10$
$\beta=0.5$





$\alpha=1.5$
$\beta=0.1$





Reviewing the Model

Model Parameters

Position and Orientation

y_o, z_o - *angular position of nebula center in sky*

ι, ω - *inclination and orientation of polar axis*

d - *distance to earth*

Morphological

R_{inner} - *equatorial radius of interior*

ε - *eccentricity of interior bubble*

γ - *radial density exponent*

α - *gradient of latitudinal density*

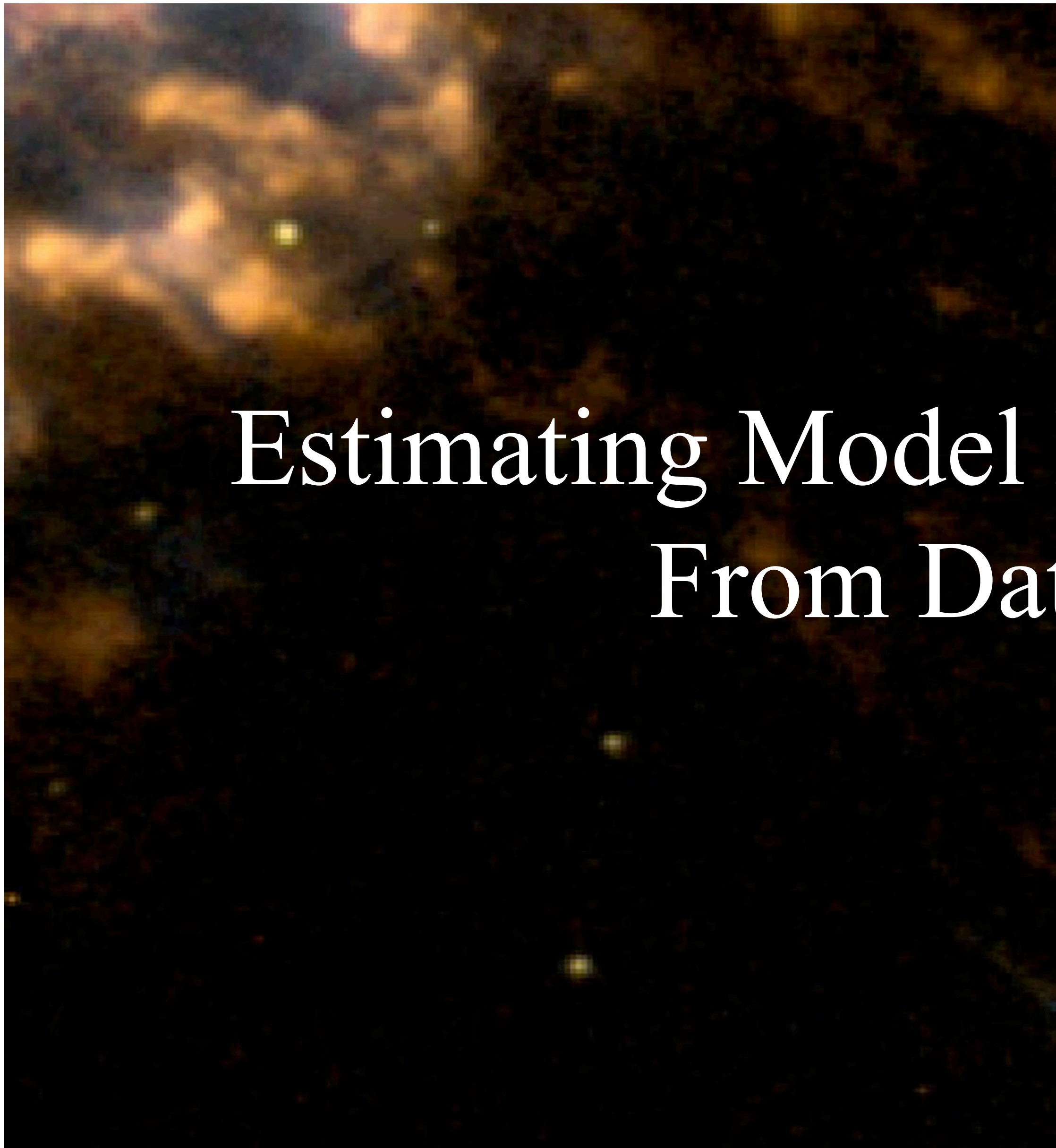
β - *pole-to-equator density ratio*

ξ - *density/luminosity parameter*

$$\xi \equiv \frac{1}{\alpha_B \, n_o^2 \, \eta_\phi^2} \frac{dL}{d\Omega}$$

Dynamical

v_o - *expansion velocity*



Estimating Model Parameters From Data

Bayes Theorem

All of the prior information

Likelihood of the data given the model

$$P(model \mid data, I) = P(model \mid I)\,\frac{P(data \mid model, I)}{P(data \mid I)}$$

Posterior Probability

Prior Probability

Evidence

Application to Our Problem

For this problem, we can write Bayes' Theorem as a proportionality

$$P(model \mid image, I) \;\propto\; P(model \mid I)\,P(image \mid model, I)$$

Our first attempt has involved using a Gaussian likelihood and uniform priors – Maximum Likelihood

$$P(model \mid image, I) \;\propto\; Exp\left(-\frac{1}{2\sigma^2}\sum_{i=\{pixels\}}(\text{image}_i - \text{model}_i)^2\right)$$

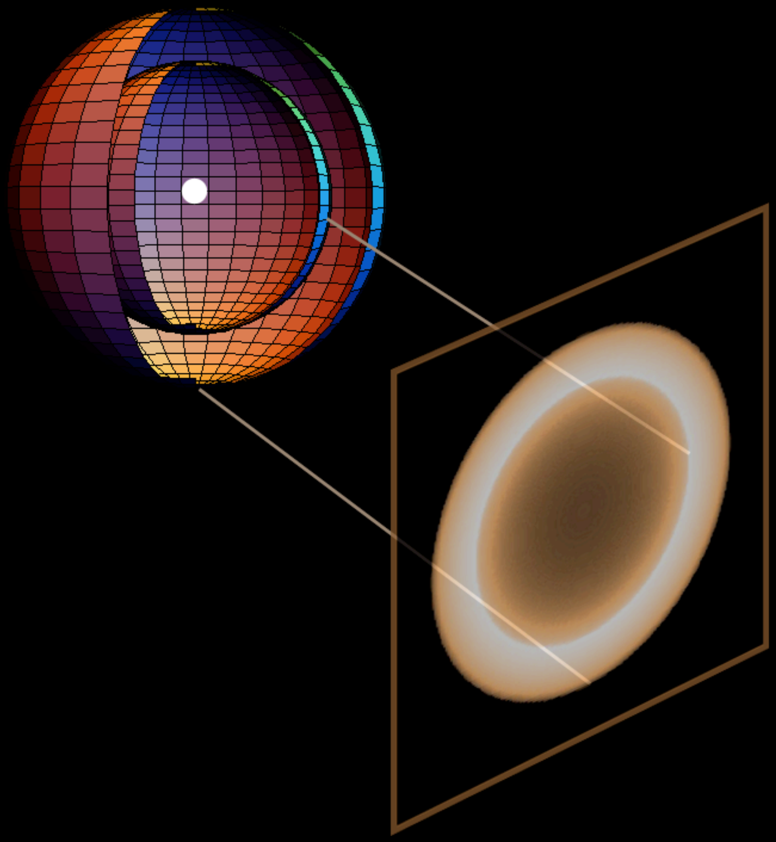
Maximizing P is the same as minimizing the least-square error

Synthetic Images From Models

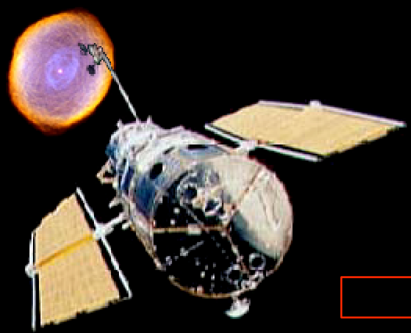
A three-dimensional model of the nebula is made using a set of 400 × 400 × 400 voxels.

If a voxel lies between the inner and outer radii of the nebula, it is assigned an intensity based on the square of the density.

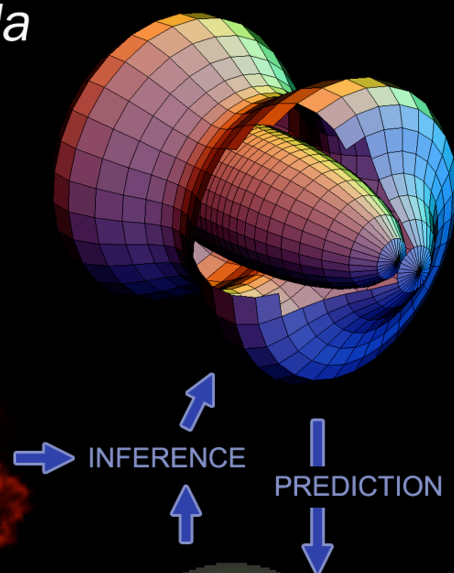
Summing the intensities along the line of sight creates an image from the model parameters.

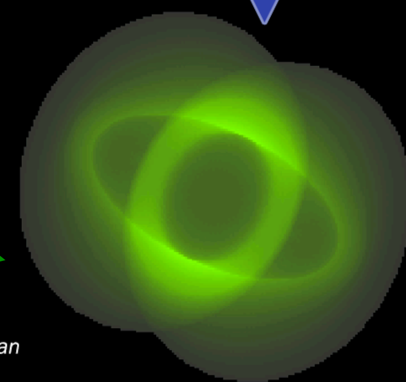


Calculating the Likelihood



The Cat's Eye Nebula
NGC 6543





The likelihood that the data could have originated from an object that is well-described by the model can be calculated from this pixel-wise comparison.

$$P(\text{model} \mid \text{image}, I) \propto \exp\left(-\frac{1}{2\sigma^2} \sum_{i=\{pixels\}} (\text{image}_i - \text{model}_i)^2 \right)$$

HST Image Credits: P. Harrington, K.J. Borkowski, STScI, NASA and recolored by B. Balick
Model Image Credits: K.A. Huyser, K.H. Knuth, A.R. Hajian

Search is SLOW!

5 min per 400x400x400 image on 1.2GHz processor

Gradient descent with 12 parameters takes

2 hours per learning step!!

Search is SLOW!

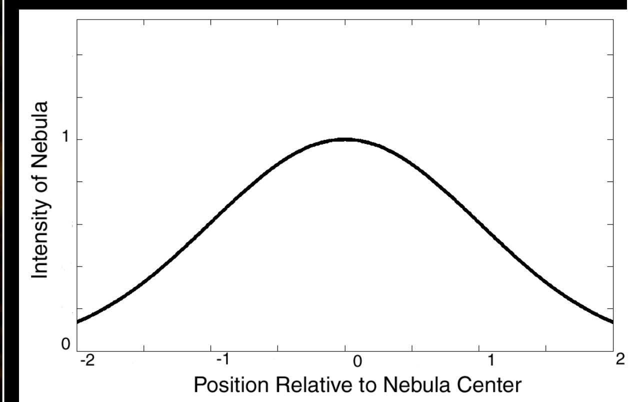
5 min per 400x400x400 image on 1.2GHz processor

Gradient descent with 12 parameters takes
2 hours per learning step!!

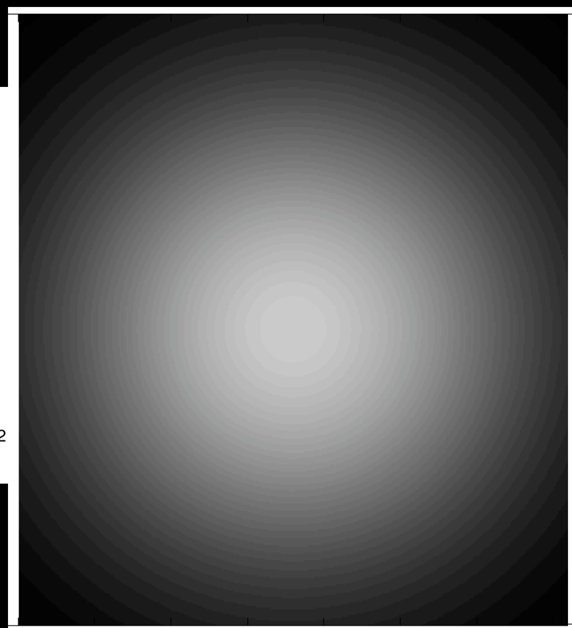
>> Use a Hierarchy of Models <<

Gaussian Model - GAUSS

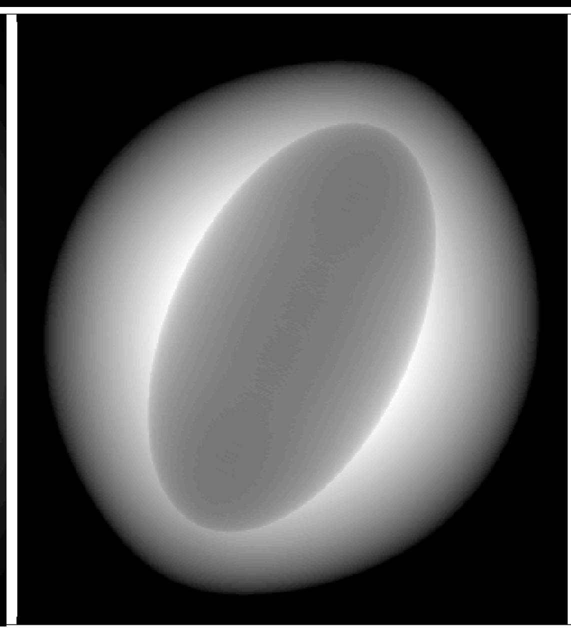
Discovers the center and general extent of the nebula.



Profile of Gauss



Circular Gaussian Fit

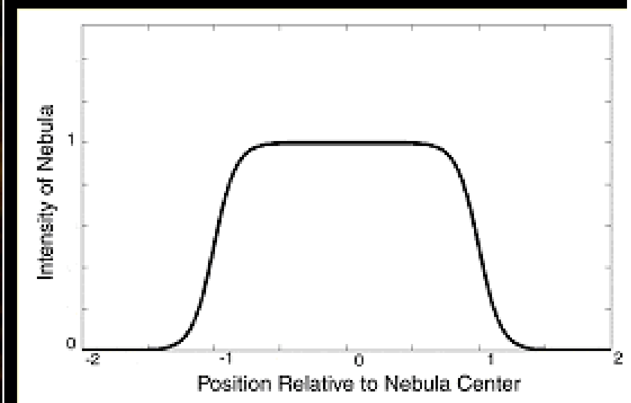


IBPES synthetic nebula

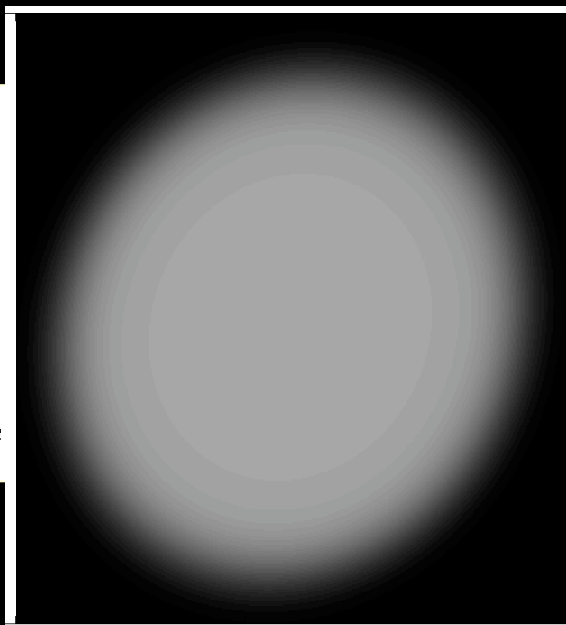
0.25 seconds per learning step

Sigmoid Hat Model - SIGHAT

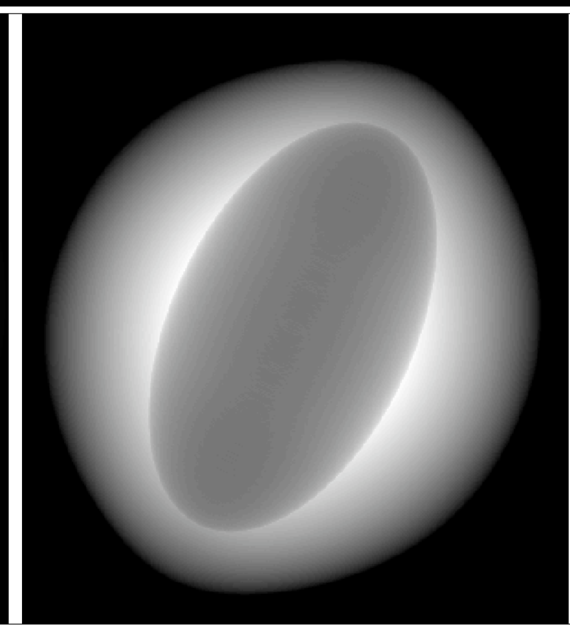
SigHat captures eccentricity, orientation and extent.



Profile of SigHat



Sigmoid Hat Fit

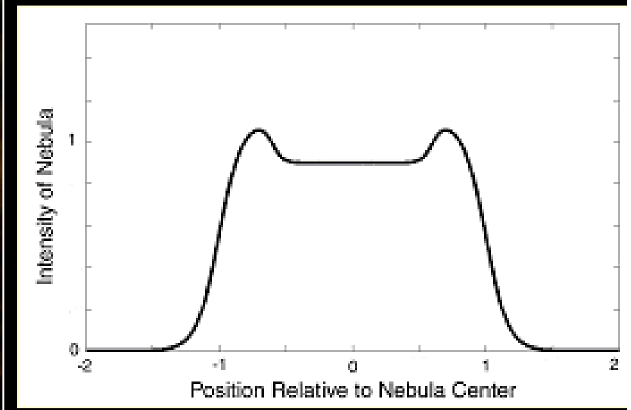


IBPES synthetic nebula

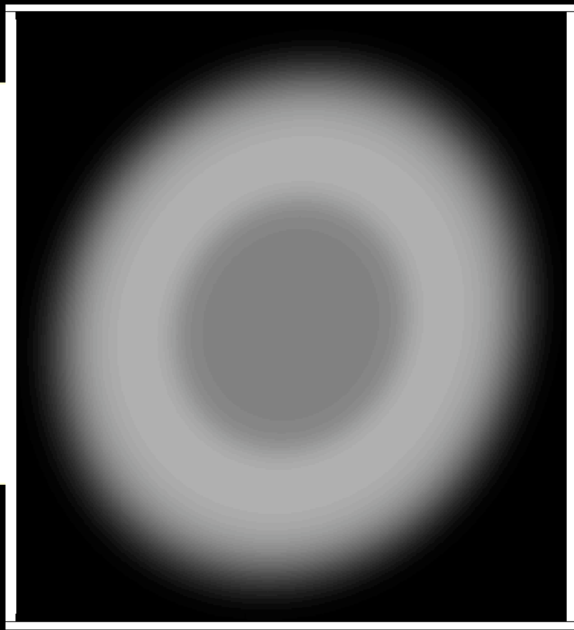
0.6 seconds per learning step

Dual SigHat Model - DUALSIG

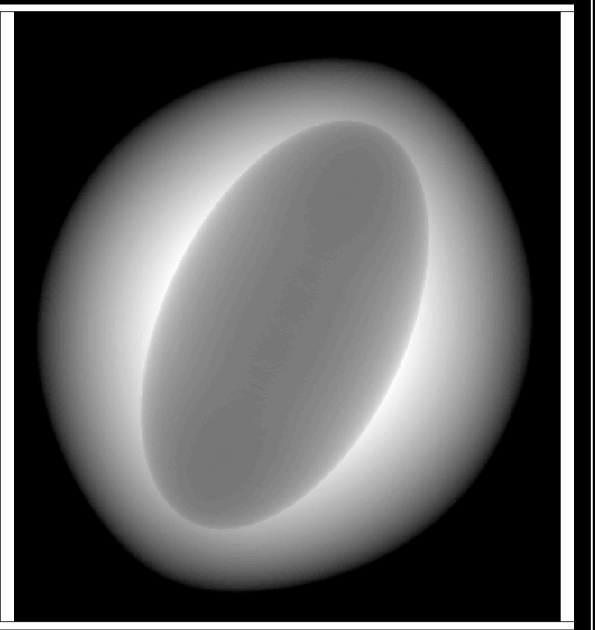
Dual SigHat estimates the shell thickness.



Profile of Dual SigHat



Dual SigHat Fit



IBPES synthetic nebula

0.8 seconds per learning step

Scaled Ellipsoidal Shell - FastSES

FastSES repeats Dual SigHat in three dimensions.

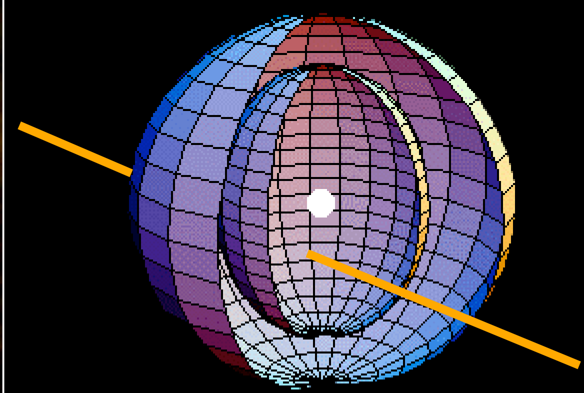
$$F(y,z) = I_0\, n^2 \left\{ d_{xo} - d_{xi} \right\}$$

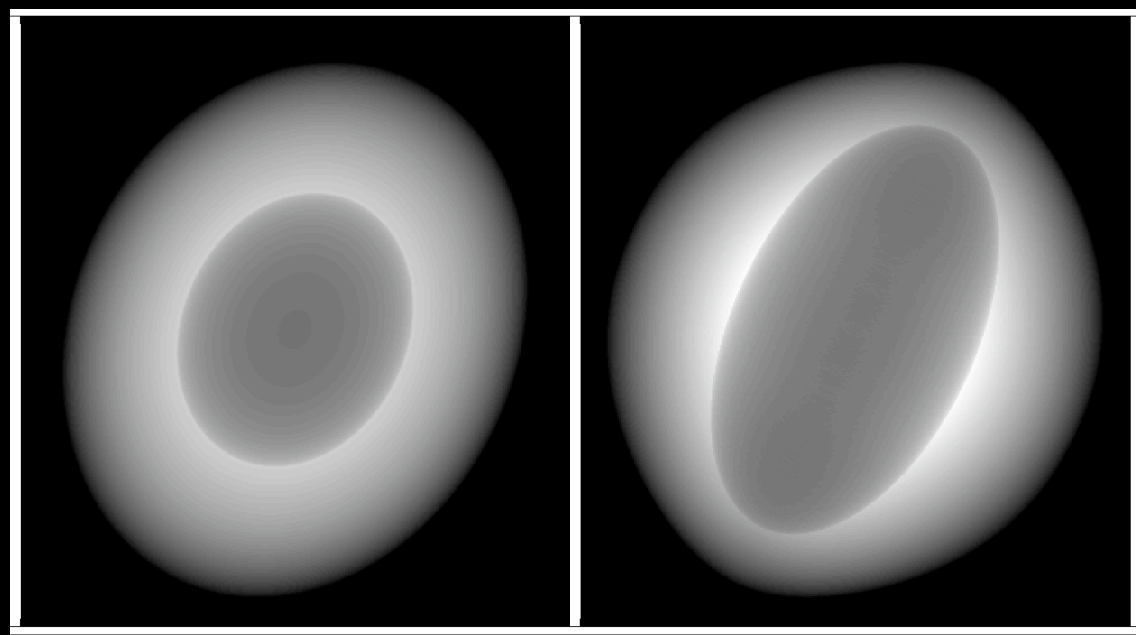
Analytically computes the distance d_x through an outer and an inner ellipsoid along the line-of-sight. Ellipsoids are three-dimensional, concentric, co-axial, and equi-eccentric.

The gradient vector is computed numerically.

Scaled Ellipsoidal Shell - FastSES

FastSES estimates the equatorial density.



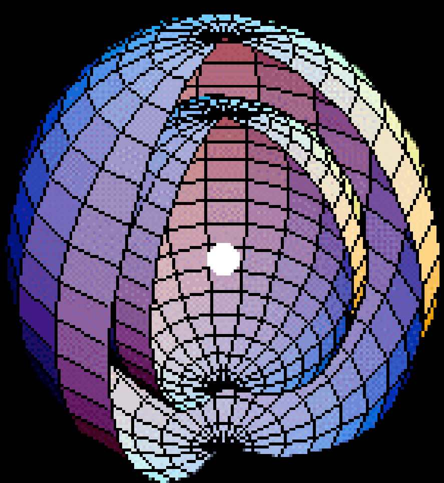


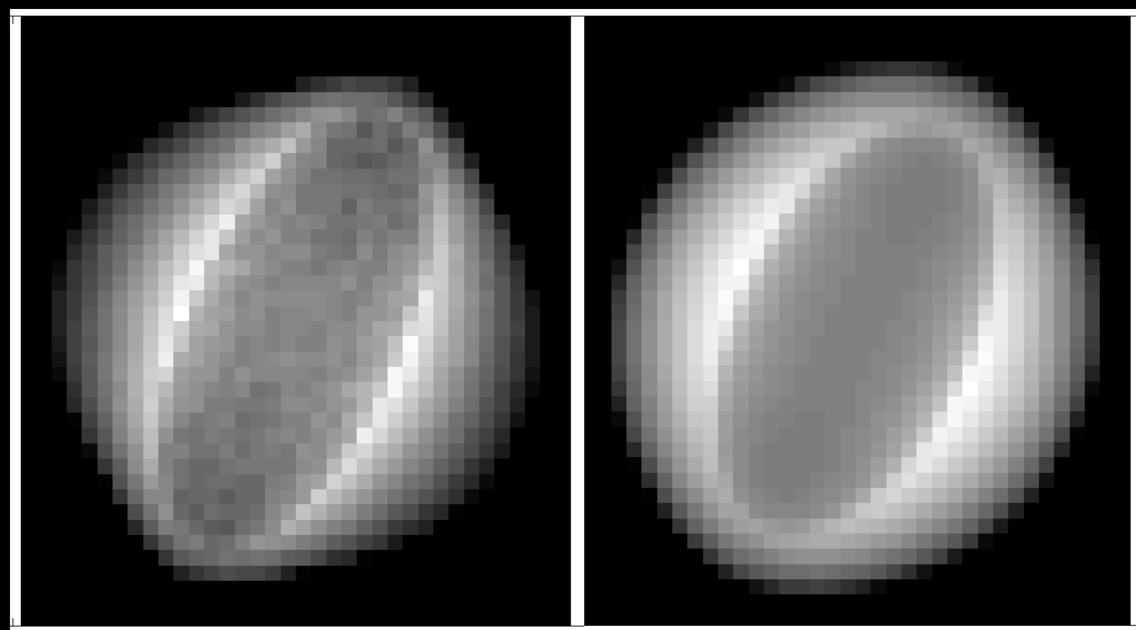
FastSES Fit IBPES synthetic nebula

12 seconds per learning step

IBPES Model

IBPES calculations are so slow, we DECIMATE the images first.





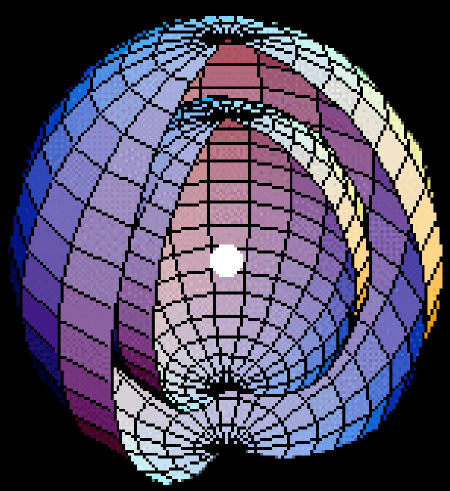
IBPES/10 Fit

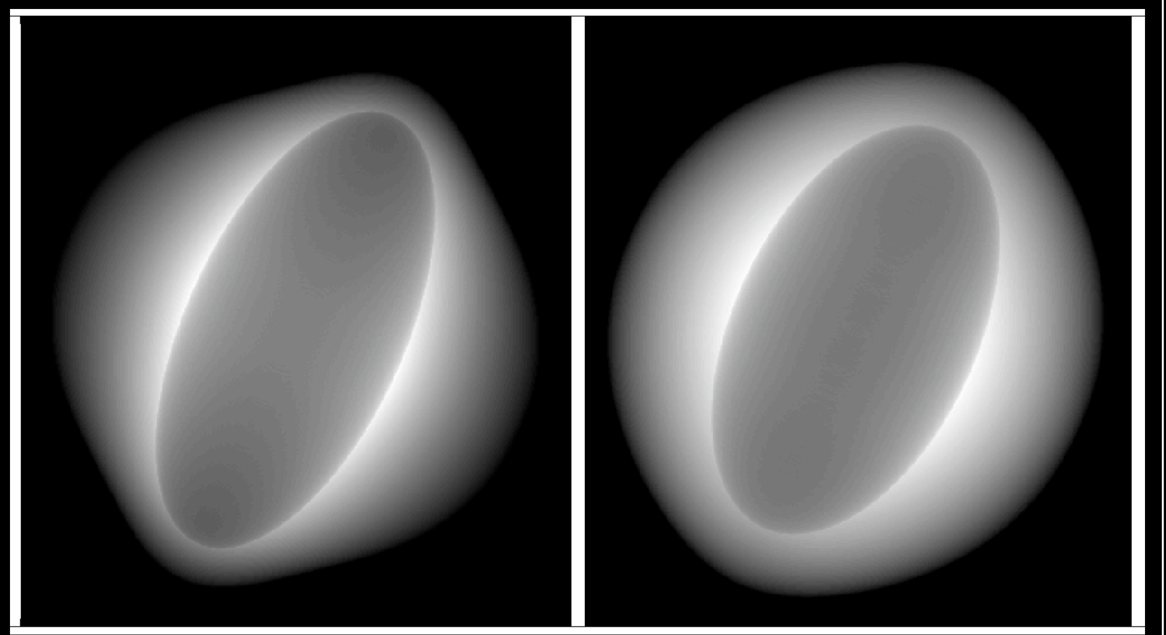
IBPES/10 synthetic nebula

7 seconds per learning step

IBPES Model

IBPES fits all remaining parameters and adjusts the others.





IBPES Fit IBPES synthetic nebula

7000 seconds per learning step if NOT decimated

Exploring the Parameter Space

Exploring the Parameter Space

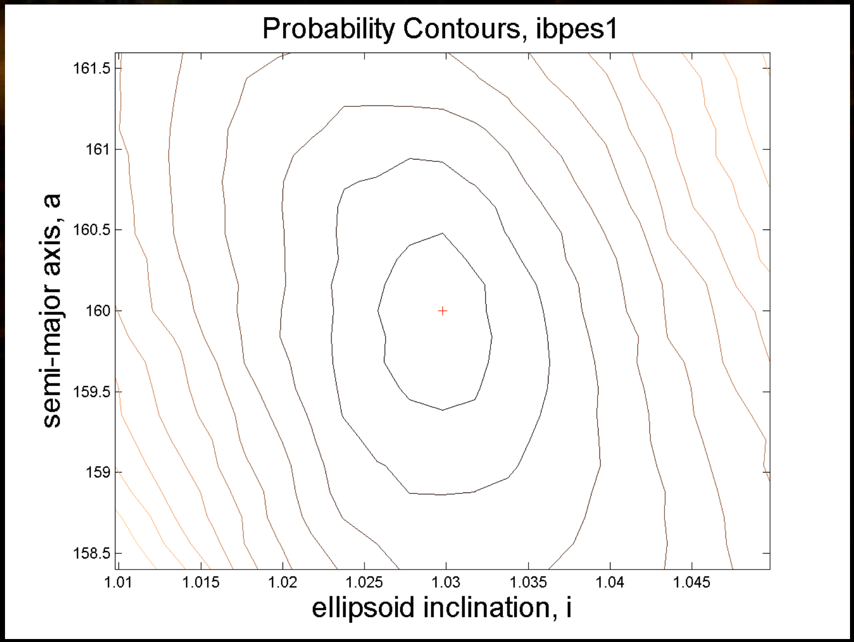
What do the probability contours look like in a perfect world?

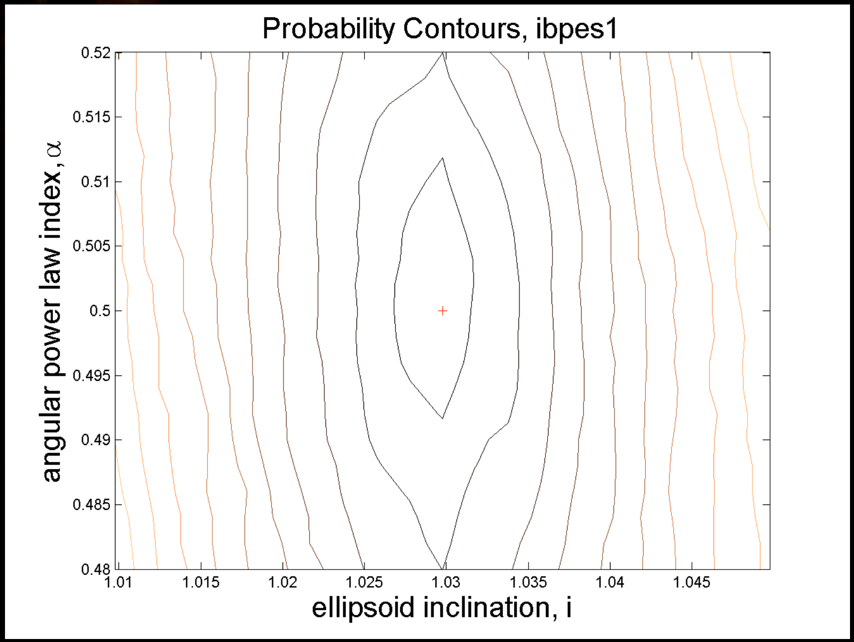
- Use Synthetic Nebula IBPES
- Explore changes to the log probability around the known IBPES solution.

Exploring the Parameter Space

Parameters ι and a are separable despite being related through the apparent major axis, a_p:

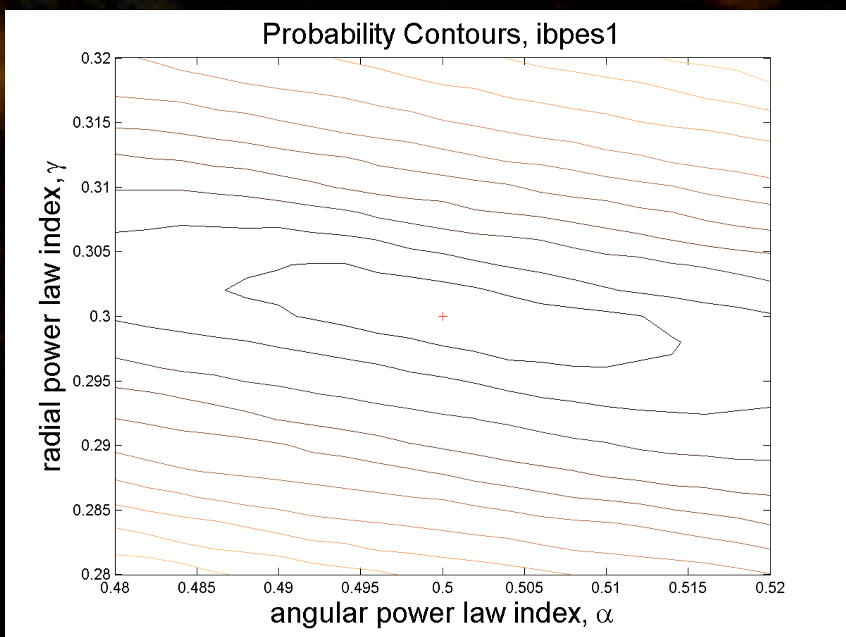
$$a_p = a \sin(\iota)$$

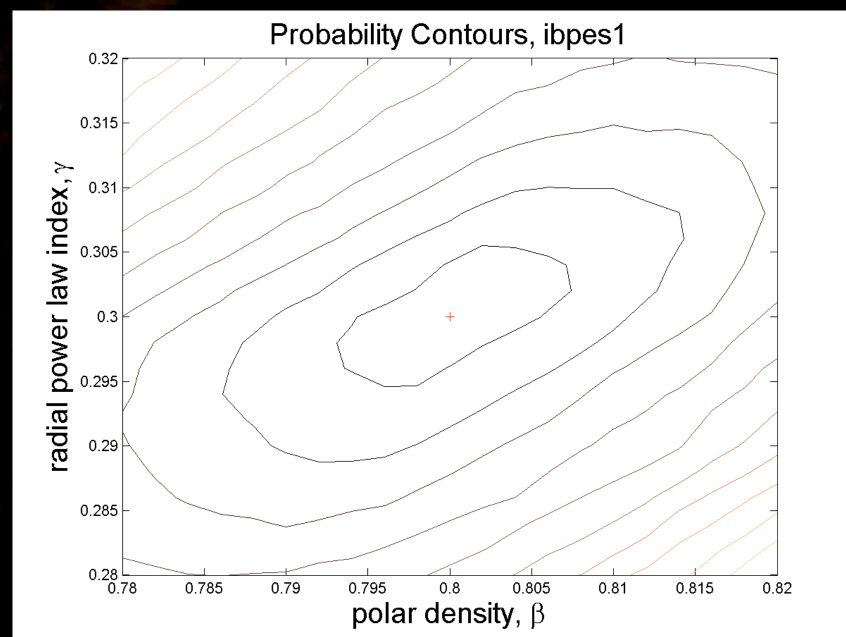




Exploring the Parameter Space

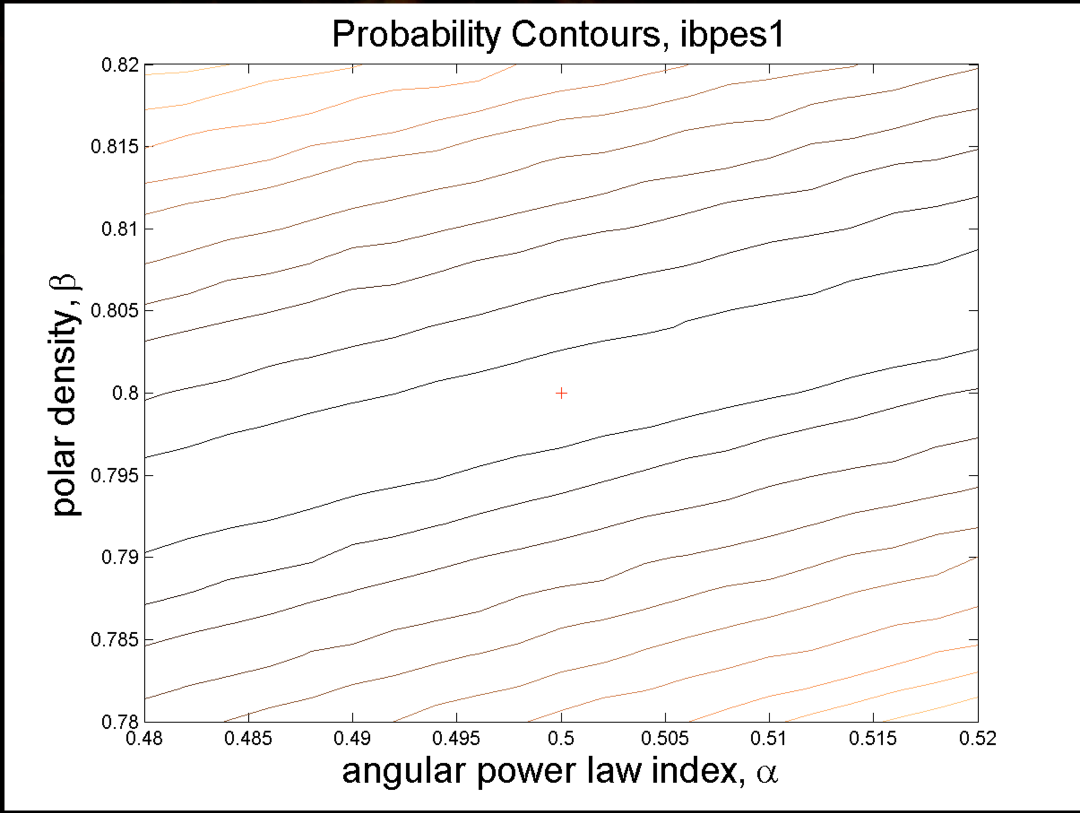
Model parameter γ is related to both α and β, although a maximum is clearly evident.



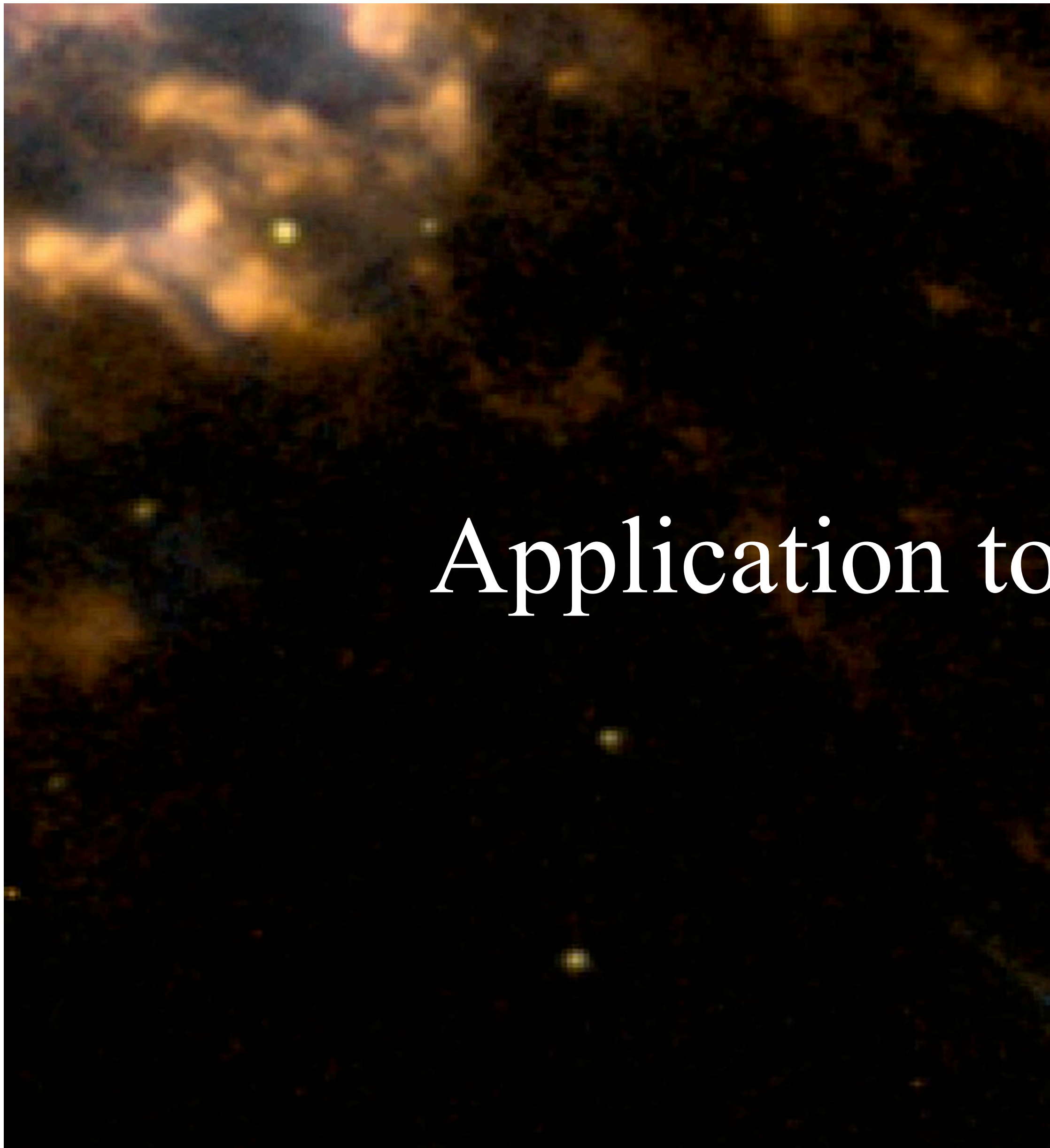


Exploring the Parameter Space

Parameters α and β turn out to be strongly correlated. Learning might improve if η(θ) were constructed from orthogonal functions such as spherical harmonics.



Application to IC418



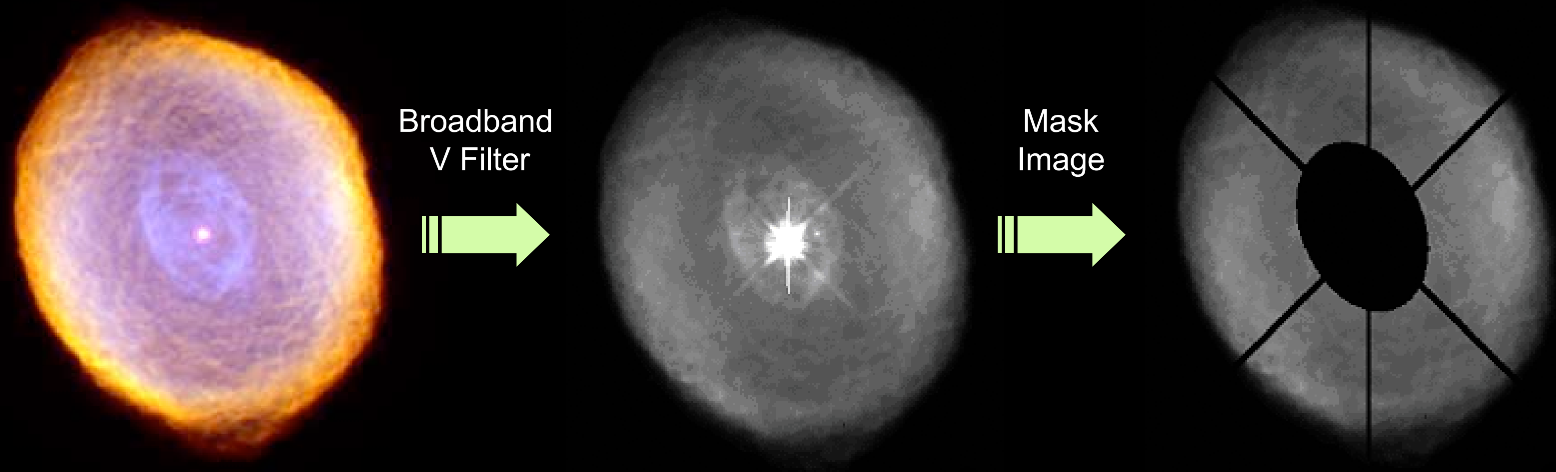
Application to IC418

Sahai, Trauger, Hajian, Terzian, Balick, Bond, Panagia
Hubble Heritage Team

Application to IC418: Masking

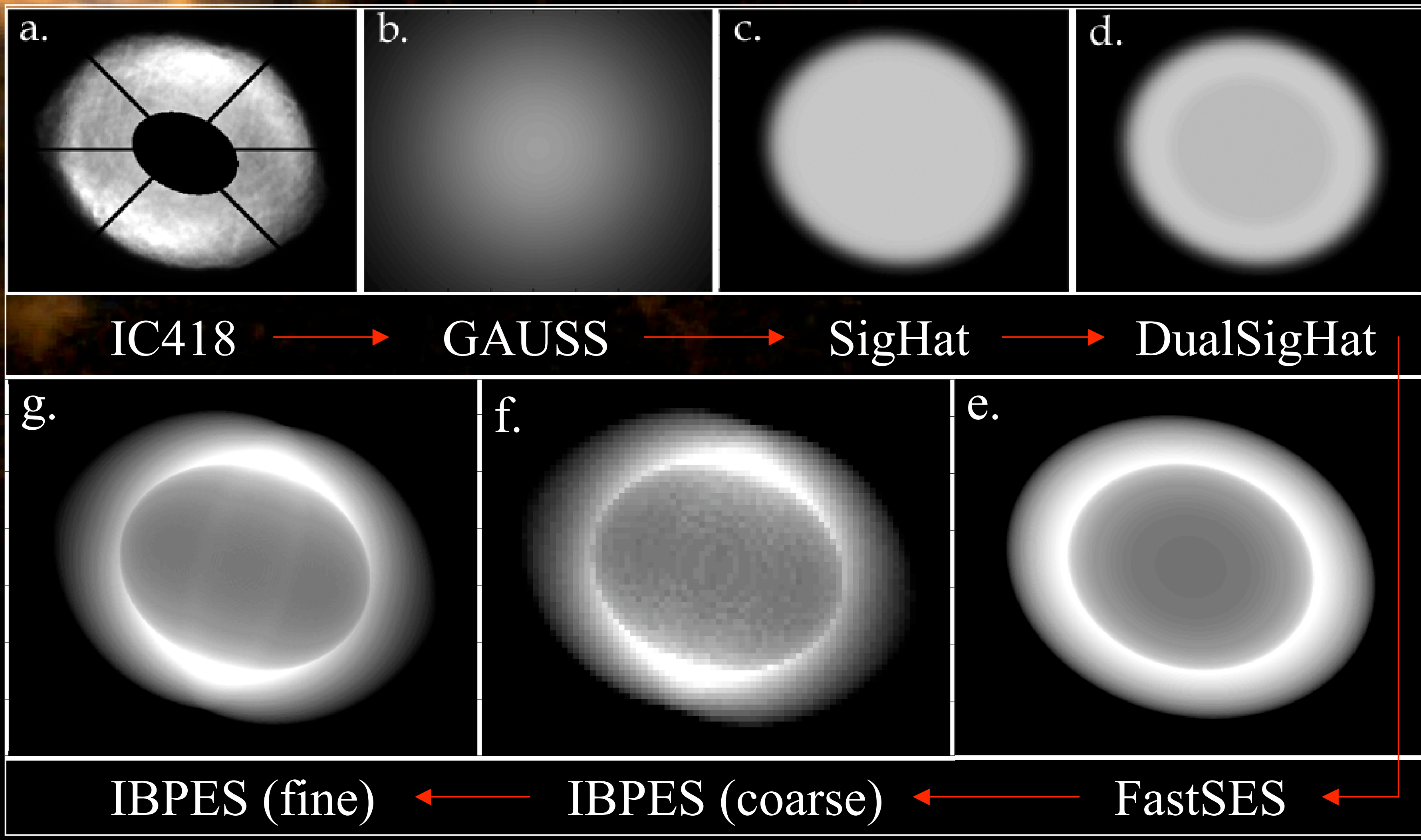
An image of a PN is collected and the central star and its diffraction spikes are masked out.

IC418



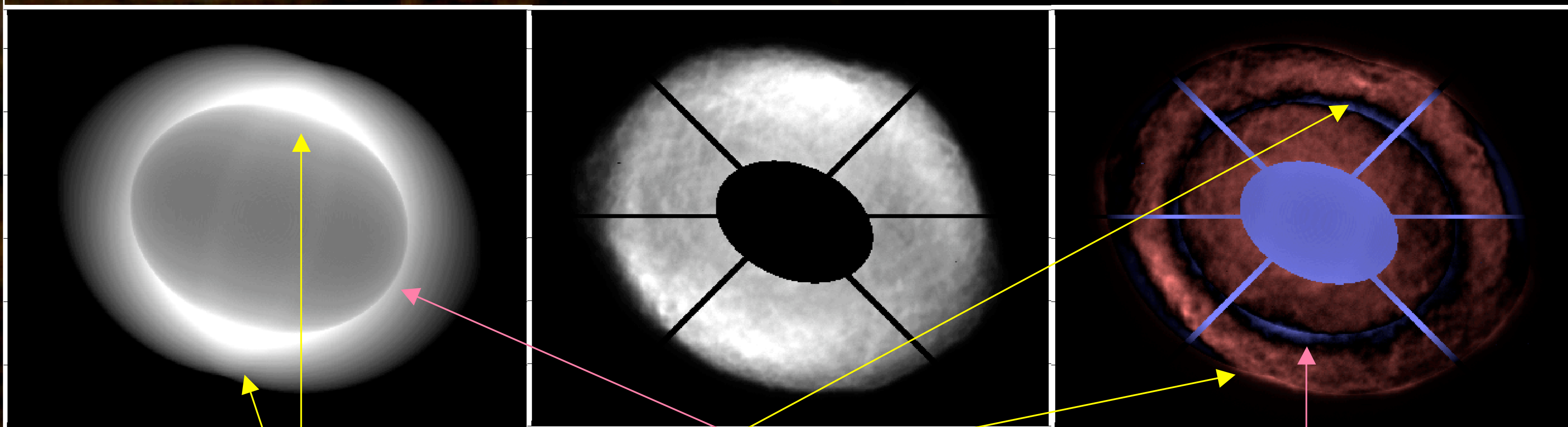
Sahai, Trauger, Hajian, Terzian, Balick, Bond, Panagia
Hubble Heritage Team

Application to IC418: Hierarchy



Application to IC418: Solution

model M | data D | difference M-D



Limb brightening fairly good

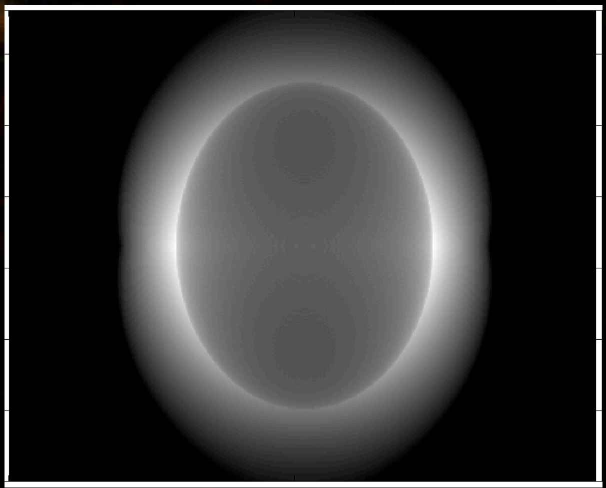
Red < 0 | Blue > 0

Overall extent fairly accurate

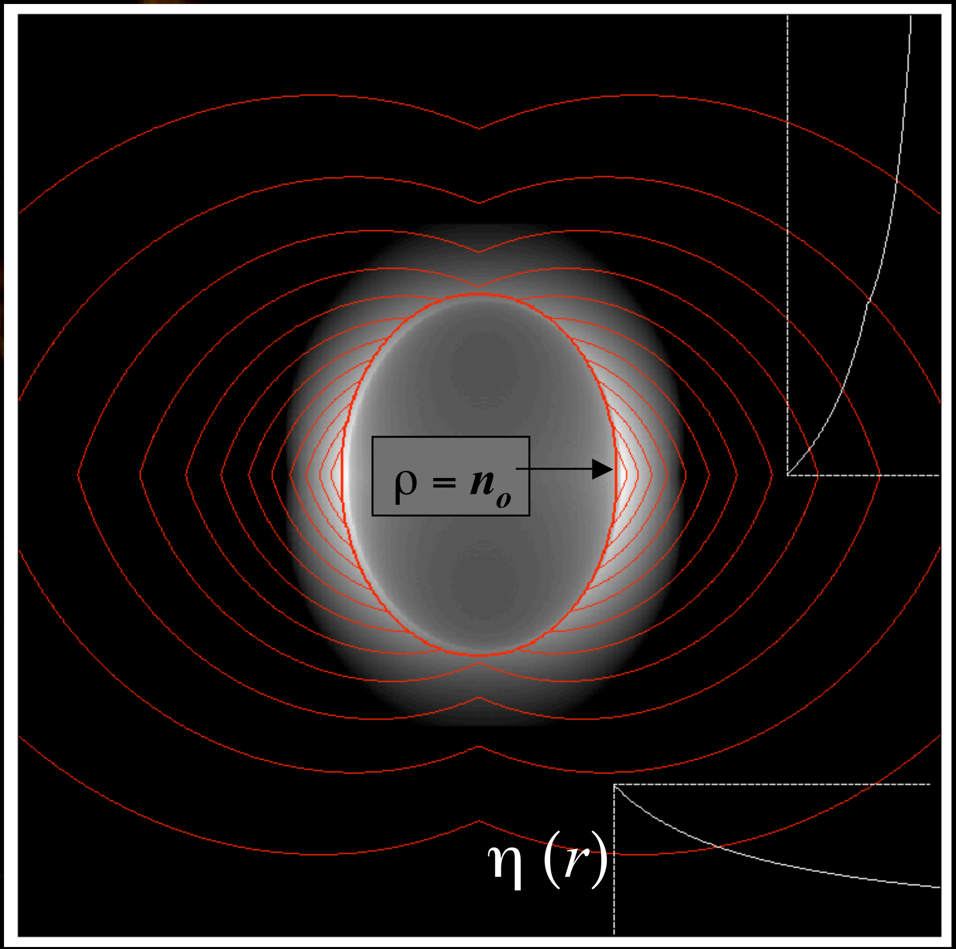
Inner bubble dimensions off

Application to IC418: Densities

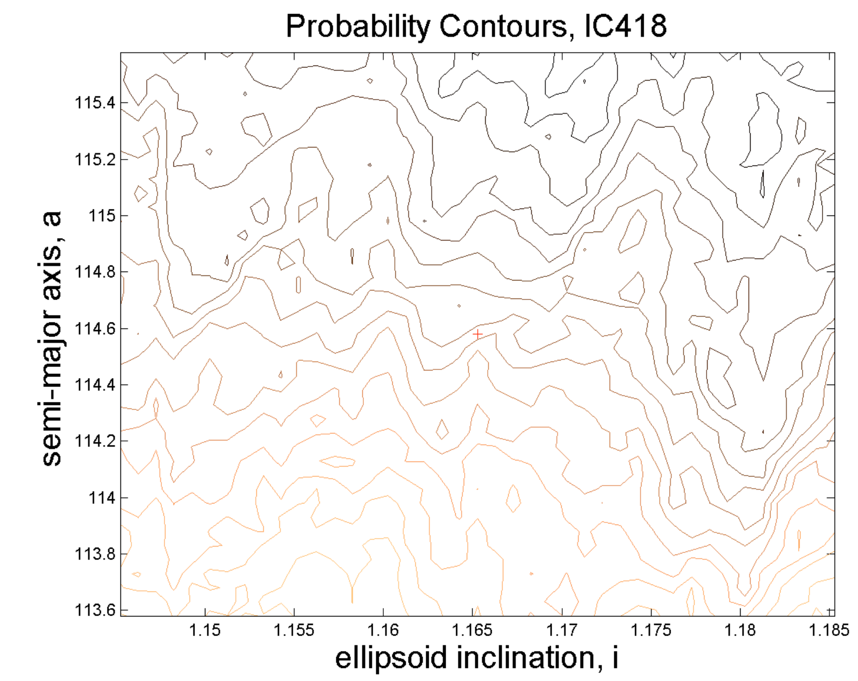
Use the model to examine the density profile of the nebula.

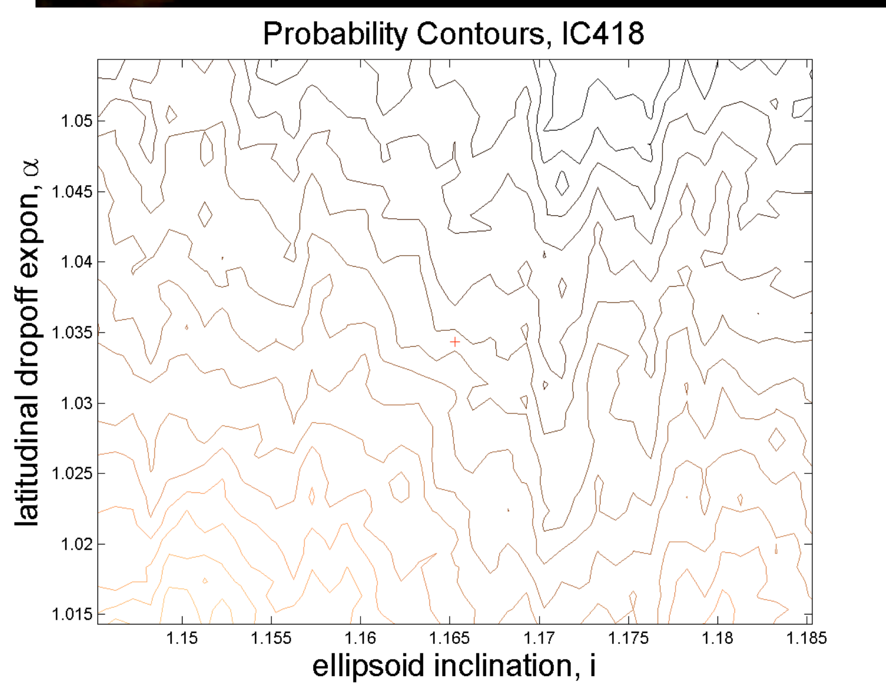


Unrotated IC418 model.



Probability Contours for IC418





Next Steps

- Improve the search method (simplex, conjugate gradient). Try MCMC on NASA Supercomputers.
- Initial conditions are important in IBPES. May help to search in parallel from a dozen canonical morphologies.
- Add information from other sources:
    1) Estimate angular expansion by modeling a pair of images taken years apart.
    2) Use long-slit spectra to estimate radial velocity of expansion (Doppler shift). Obtains distance to earth and absolute estimates of volume, mass and density.

Conclusions

A Hierarchical Set of Models allows one to efficiently estimate several of the parameters using simple models while gradually increasing accuracy and complexity. The implementation effort comes in the form of finding a fast way to characterize the set of probable solutions.

The IBPES Model is capable of modeling a wide variety of PN morphologies. It is not easy to learn automatically, unfortunately. In particular, the strong correlation between α and β argues for a reformulation of the latitudinal density term $\eta(\theta)$.

Acknowledgements

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>> Use a Hierarchy of Models <<

Inclination Alters Rim Brightness