Performance model and in-flight fault detection for Solid Rocket Motor

V.N. Smelyanskiy*, D.G. Luchinsky, V. Osipov,

D.A. Timucin, C. Kiris

NASA Ames Research Center

- Serdar Uckun (NASA Ames Research Center)
- Fred Culick (California Institute of Technology)

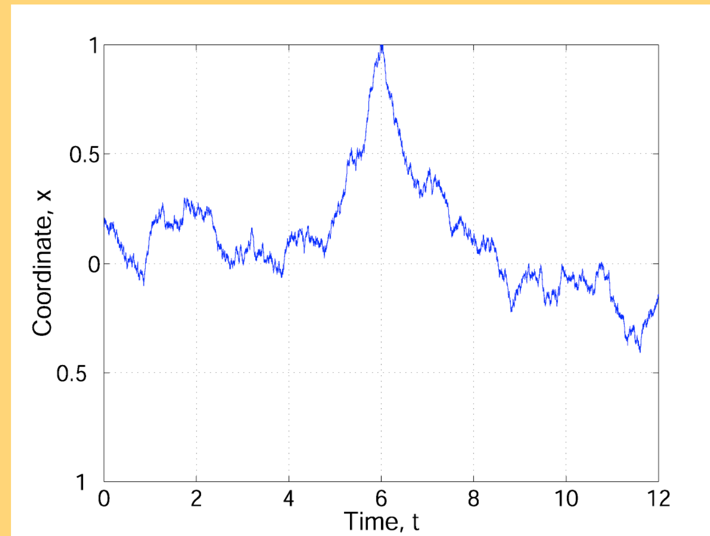
Contents

- **Our goal** is to predict the catastrophic events in SRM during the flight
- **High-fidelity model** is introduced to simulate known faults
- **Low-dimensional models** are derived to infer SRM parameters with redundancy and to calculate the probability of the catastrophe to occur at a given time
- **The information contents** of two sensors is evaluated using Bayesian model inference algorithm to ensure that we have multiple evidence of the fault. **Prognostics algorithm** is developed to predict the redline time.
- We present an **analysis of an example fault:** an overpressure due to various causes (i.e., bore chocking)
- Conclusions are drawn and the **future work** is discussed

Diagnostics and Prognostics: Identification of nonlinear stochastic models from data

Low-dimensional dynamical model

$$\dot{\vec{x}} = \vec{f}(\vec{x};\vec{c}) + \vec{\xi}(t)$$



Data record

$$I = \left\{ t_k, \vec{i}_k \right); \quad k = 0 : K \right\}$$

Measurement model

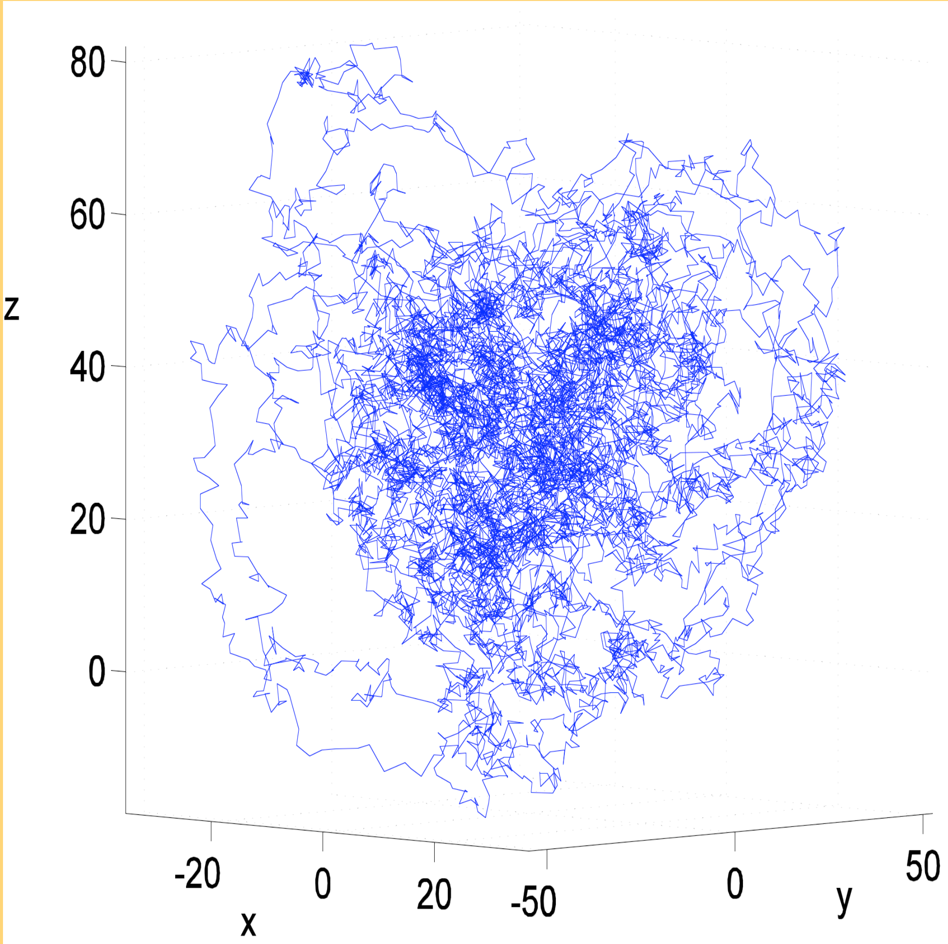
$$\vec{i}_k = \hat{\Gamma}\, \vec{x}(t_k) + \vec{\eta}_k \qquad \vec{\eta}_k \sim N(0, \sigma)$$

measurement error

Algorithm for reconstruction of system state, parameters of dynamical and measurement models from time series

Stochastic dynamical system output

$$\dot{x}_1 = \sigma(x_2 - x_3) + \xi_x(t),$$
$$\dot{x}_2 = rx_1 - x_2 - x_1x_3 + \xi_y(t),$$
$$\dot{x}_3 = x_1x_2 - bx_3 + \xi_z(t),$$
$$\langle \xi_i(t) \rangle = 0, \langle \xi_i(t)\xi_j(0) \rangle = D_{ij}\delta(t)$$

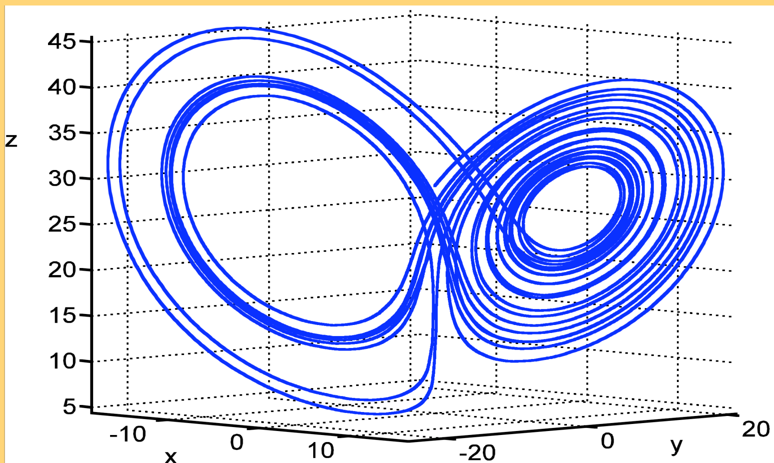


Noise intensity is greater than deterministic forces by ~ 1000 times

Lorenz system II

Inferential framework:

$$\dot{x}_i = \sum_j a_{ij} x_j + \sum_{k \neq m} b_{ikm} x_k x_m + \div_i(t),$$

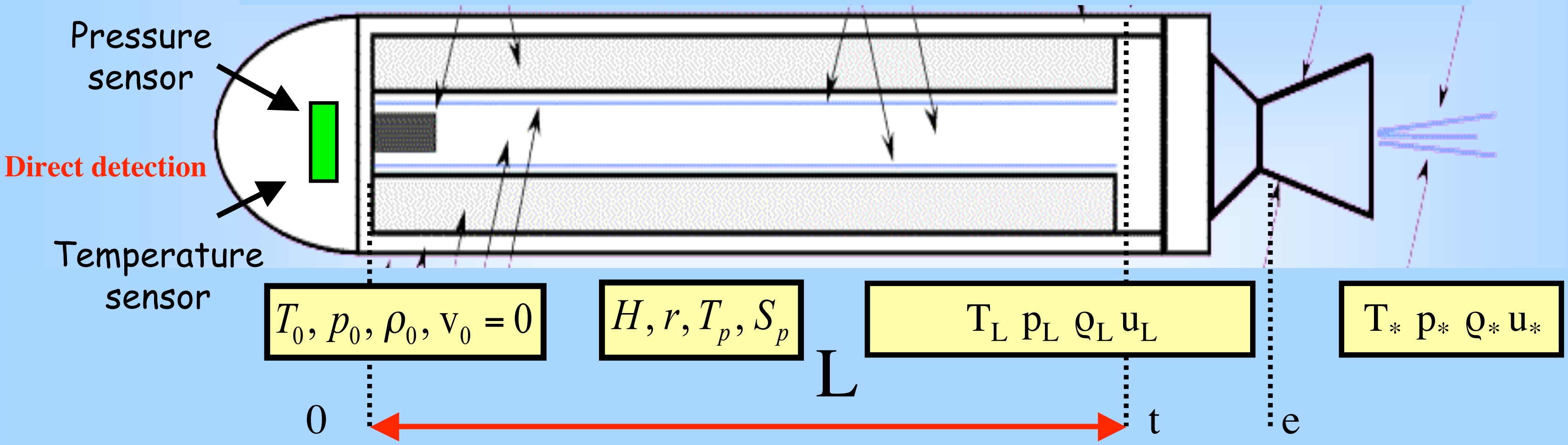


a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}	D_x
-10	10	0	0	0	0	1500
-10.55	10.77	0.065	0.013	0.002	-0.02	1522.1
a_{21}	a_{22}	a_{23}	a_{24}	a_{25}	a_{26}	D_y
28	1	0	0	-1	0	1600
27.53	-0.194	0.0011	0.0105	-0.985	-0.012	1621.5
a_{31}	a_{32}	a_{33}	a_{34}	a_{35}	a_{36}	D_z
0	0	-2.667	1	0	0	1700
-0.4294	0.569	-2.759	0.9951	0.0176	-0.016	1713.4

Inference of noise intensities and correlations

We use 50 blocks with 20000 points each to infer parameters shown in the table. Convergence can be improved by increasing the number of points.

Geometry of an Idealized Solid Rocket



Failure modes leading to overpressure

1. Bore choking: Bore choking occurs when the propellant deforms (bulge) radially inward and disrupts the exhaust gas flow, causing a choked flow condition inside the motor. Bore choking can be most likely realized near radial slots and segment joints between two sections with a smaller radius of the aft section. This critical effect is typically caused by localized areas of low pressure arising near such inhomogeneous. Bore choking has the potential of causing booster over-pressure and catastrophic failure.

2. Nozzle Failure: Nozzle failure will reduce the thrust being generated. Failures down stream of the throat will have no impact on the chamber pressure. A non uniform failure of the nozzle (such as loosing a chunk of the aft exit cone, or partially failing a joint) will result in a non-axial component of thrust. A failure would also result in the plume moving closer to the aft skirt causing increased heating the could adversely affect the TVC system.

3. Debonding: Potentially large parts of the propellant debond from the liner and got loose. They can bend and stick inside the bore. In the large rocket with the large aspect ratio of the bore volume the depleted propellant can significantly obscure the bore volume leading to chocking.

4. Propellant structural failure: Critical defects are cracks and voids in solid propellant and slots of booster joint segments. These defects can stimulate the increase of local burning rate that can result in abruption of lager enough piece of the propellant. This piece can stick to a narrow place of the burning propellant or choke minimum cross section of the nozzle. This can cause a sharp catastrophic jump of the booster trust and overpressure in the chamber head.

High-fidelity model of SRM (Parameters and variables)

Conservation of mass $$\partial_t \rho S = -\partial_x \rho u S + m(p) l(x) + \xi_1(t,x)$$

Momentum conservation $$\partial_t \rho u S = -\partial_x \rho u^2 S - S \partial_x p - f_0 \rho u^2 l(x) + \xi_2(t,x)$$

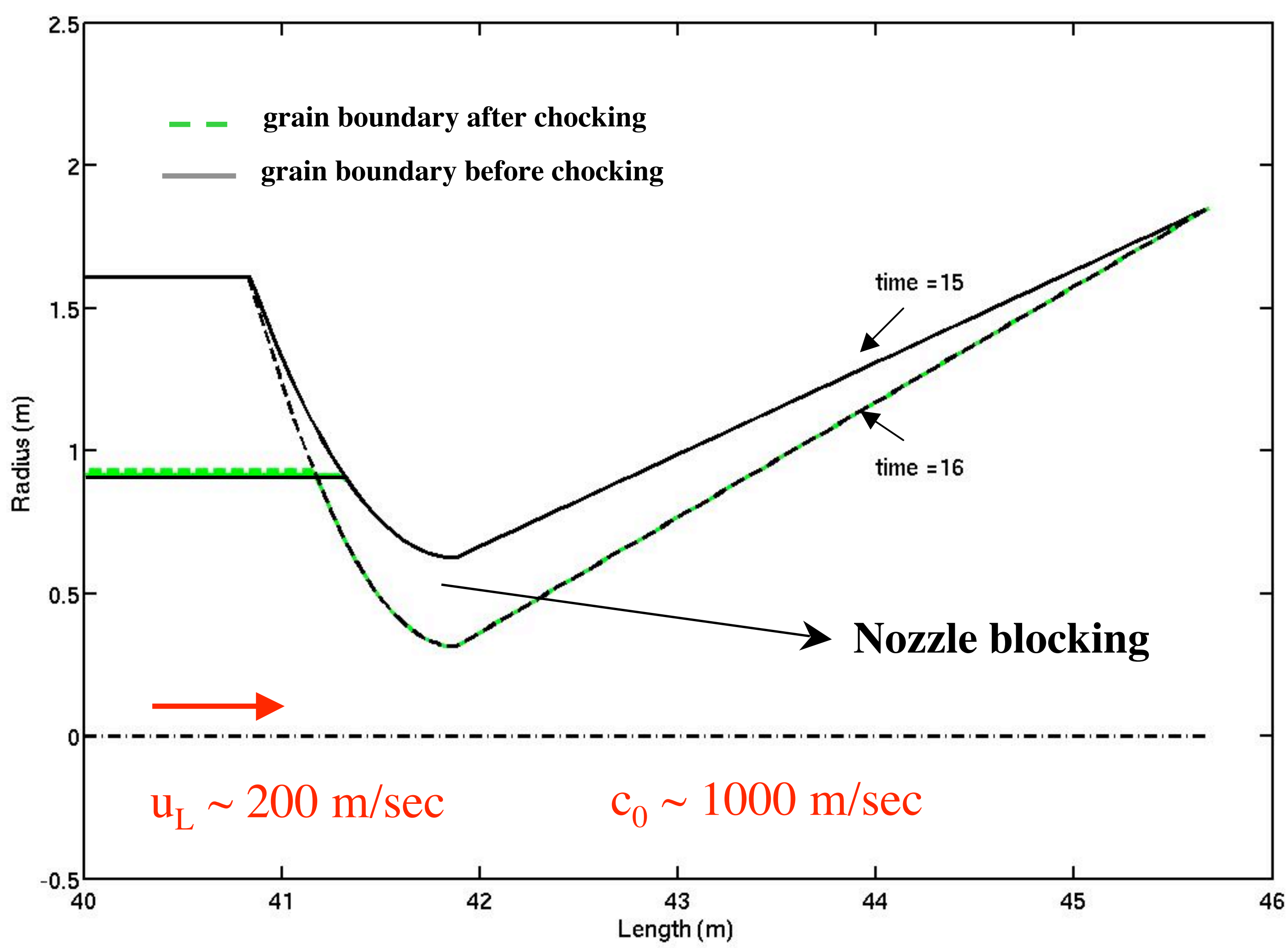
Energy conservation $$\partial_t \rho e S = -\partial_x \rho u S e - \partial_x p u S + h_0 m(p) l(x) + \xi_3(t,x)$$

Burning rate model $$r_b(p) = r_c (p / p_c)^n \equiv a_b \; p^n$$

1D time-accurate PDE model with dynamical noise sources: stochastic integration

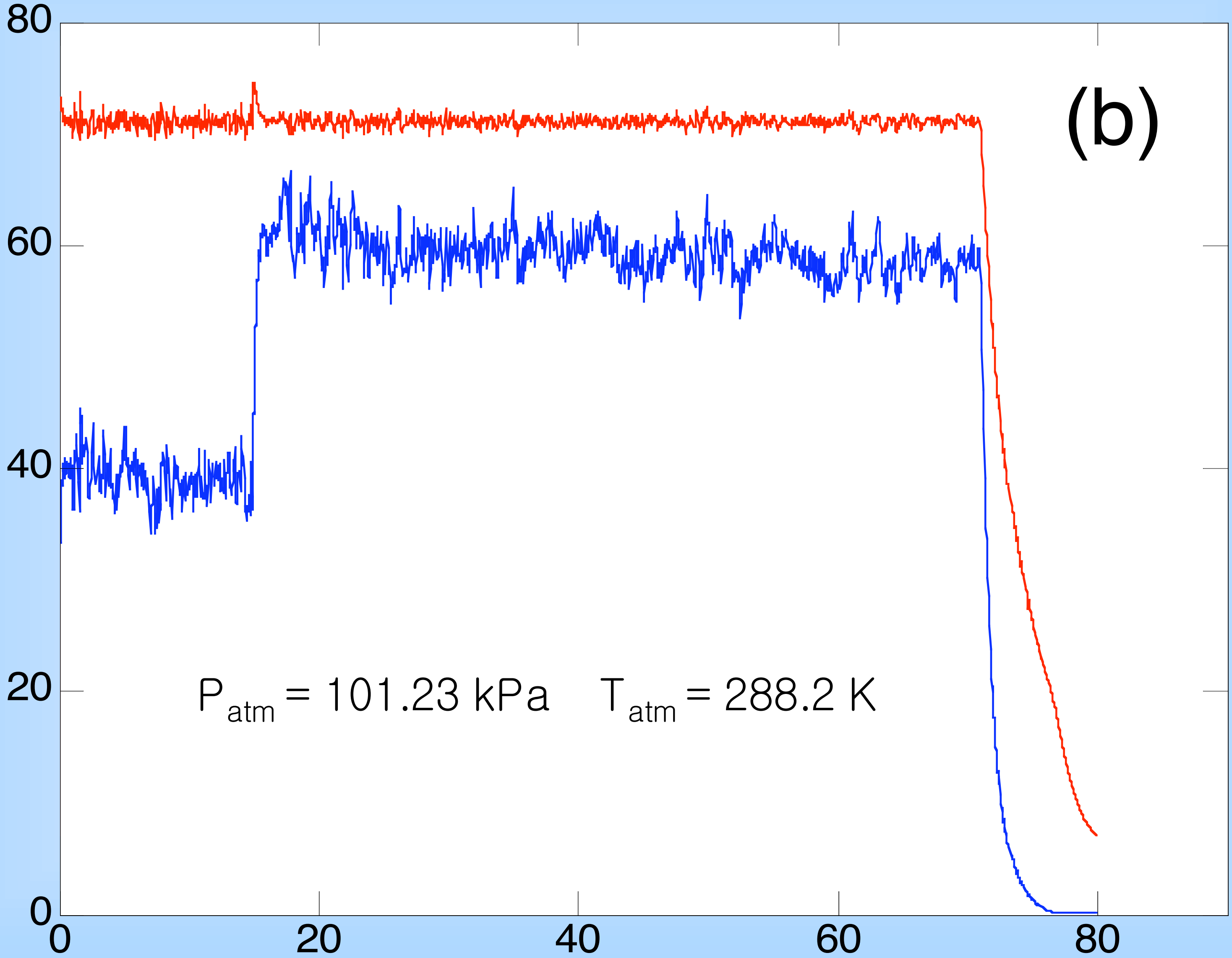
- ρ [kg·m^{-3}] is the gas density, ρ_p [kg·m^{-3}] is the propellant density
- u [m·s^{-1}] is the gas velocity
- p [N·m^{-2}] is the gas pressure
- S [m^2] is the cross-sectional area of the propellant; S_* is the minimum cross section of the nozzle
- L [m] is the length of the burning propellant
- $l(x)=2\pi R(x)$ [m] is the perimeter of the internal cross-section of the burning propellant
- F(x) [m^2] is the burning surface of the grain up to the point x along the axis of the SRB
- R(x) is the internal radius of the propellant
- $e = c_V T + u^2/2$ [J·kg^{-1}] is the specific energy of the gas
- $h = c_P T + u^2/2$ [J·kg^{-1}] is the specific enthalpy of the gas
- H [J·kg^{-1}] is the heat released by the burning propellant

Fault model: nozzle blocking



Chamber head temperature and pressure

Noise is enhanced during the chocking event



Low-dimensional dynamic model

Averaging over the combustion volume

1. The mass dynamics
2. The energy dynamics
3. The dynamics of the burning area

Noise sources

1. Gas flow density fluctuation in the throat
2. Gas flow density fluctuation in the burning surface

$$
\begin{cases}
\dfrac{d\rho}{dt} = -A(t)\sqrt{p\rho} + B(t)\left(\rho_p - \rho\right)p^n + \xi_1(t) \\
\dfrac{dp}{dt} = -\gamma A(t)p\sqrt{\dfrac{p}{\rho}} + B(t)\left(\gamma\rho_p - p\right)p^n + \xi_2(t) \\
\dfrac{dR}{dt} = r_b\left(p_0\right)p^n + \xi_3(t) \qquad p \to p(t)/p_0(0)
\end{cases}
$$

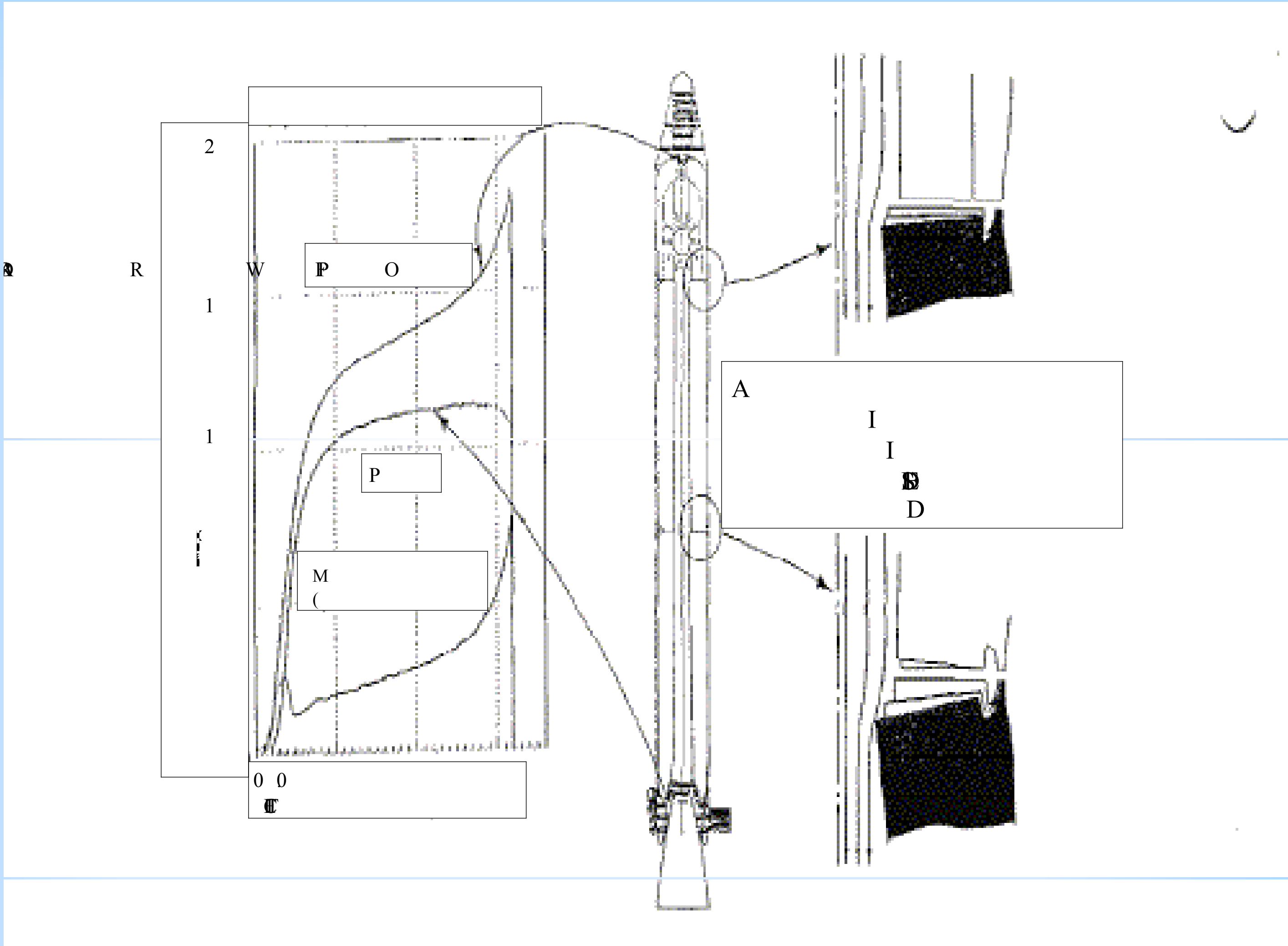
A(t) describes fast nozzle area change in chocking

$$A(t) = \frac{c_0\Gamma(\gamma)S_*(t)}{V(t)}$$

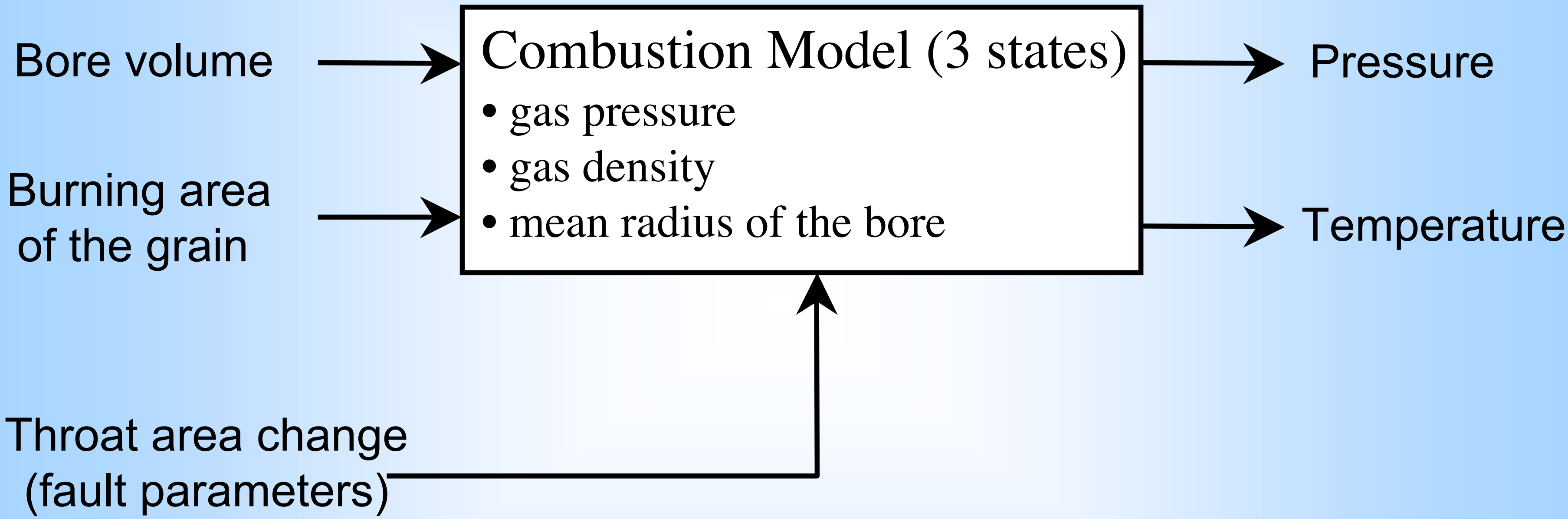
B(t) is a slow variable

$$B(t) = \frac{r_b\left(p_0\right)F(t)}{V(t)}$$

Fault model: Titan IV overpressure fault

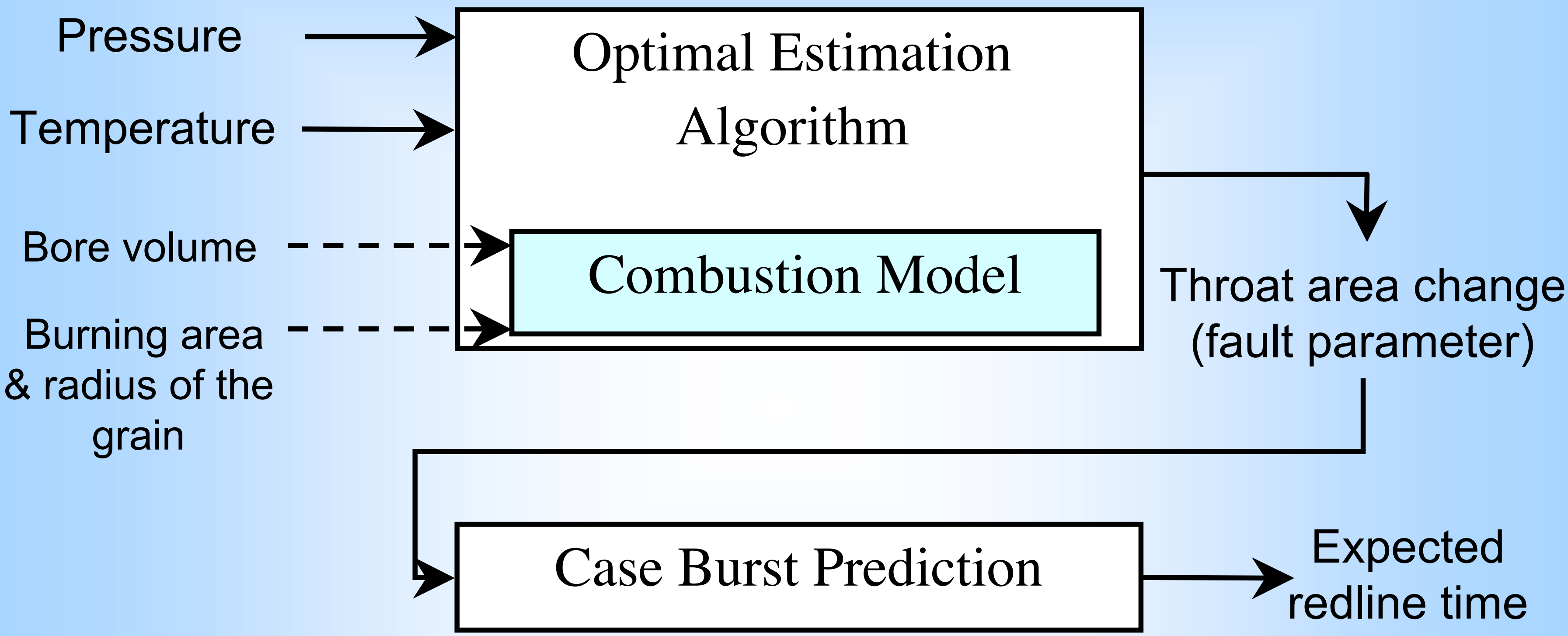


Low-dimensional dynamic model: System view



The model includes state driving noises, which are assumed to have unknown parameters. Noise characterization is important for designing the fault estimation algorithm.

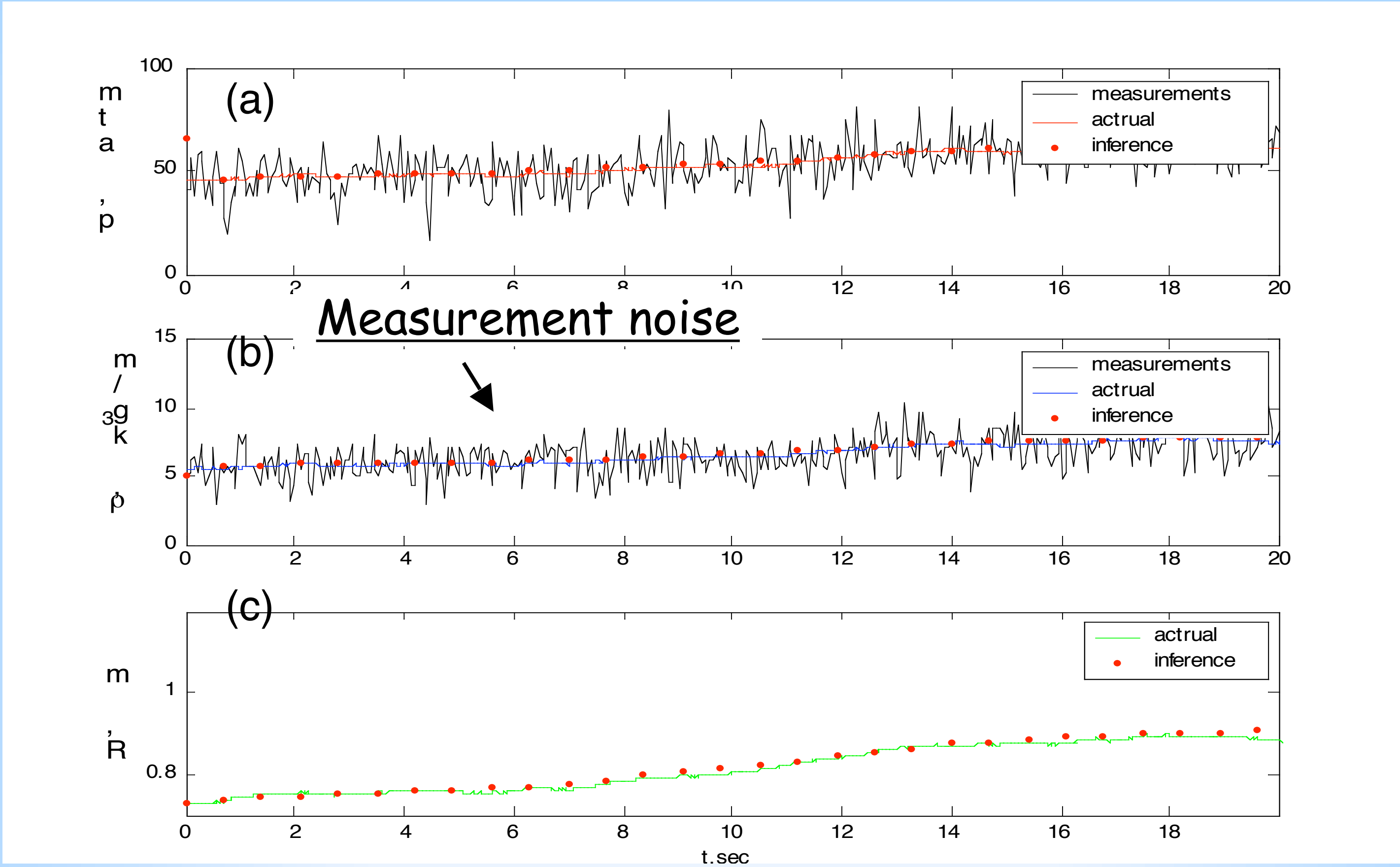
Fault estimation based on low-dimensional dynamic model



We used an algorithm for reconstruction of system state, parameters of dynamical and measurement models from time series, see V. N. Smelyanskiy, D. G. Luchinsky, D. A. Timucin, and A. Bandrivskyy, Physical Review E 72, 026202 (2005).

NOTE. That the algorithm of dynamical inference will be embedded into inferential learning framework that will continuously update the models and parameters beginning from the stage of development and firing tests.

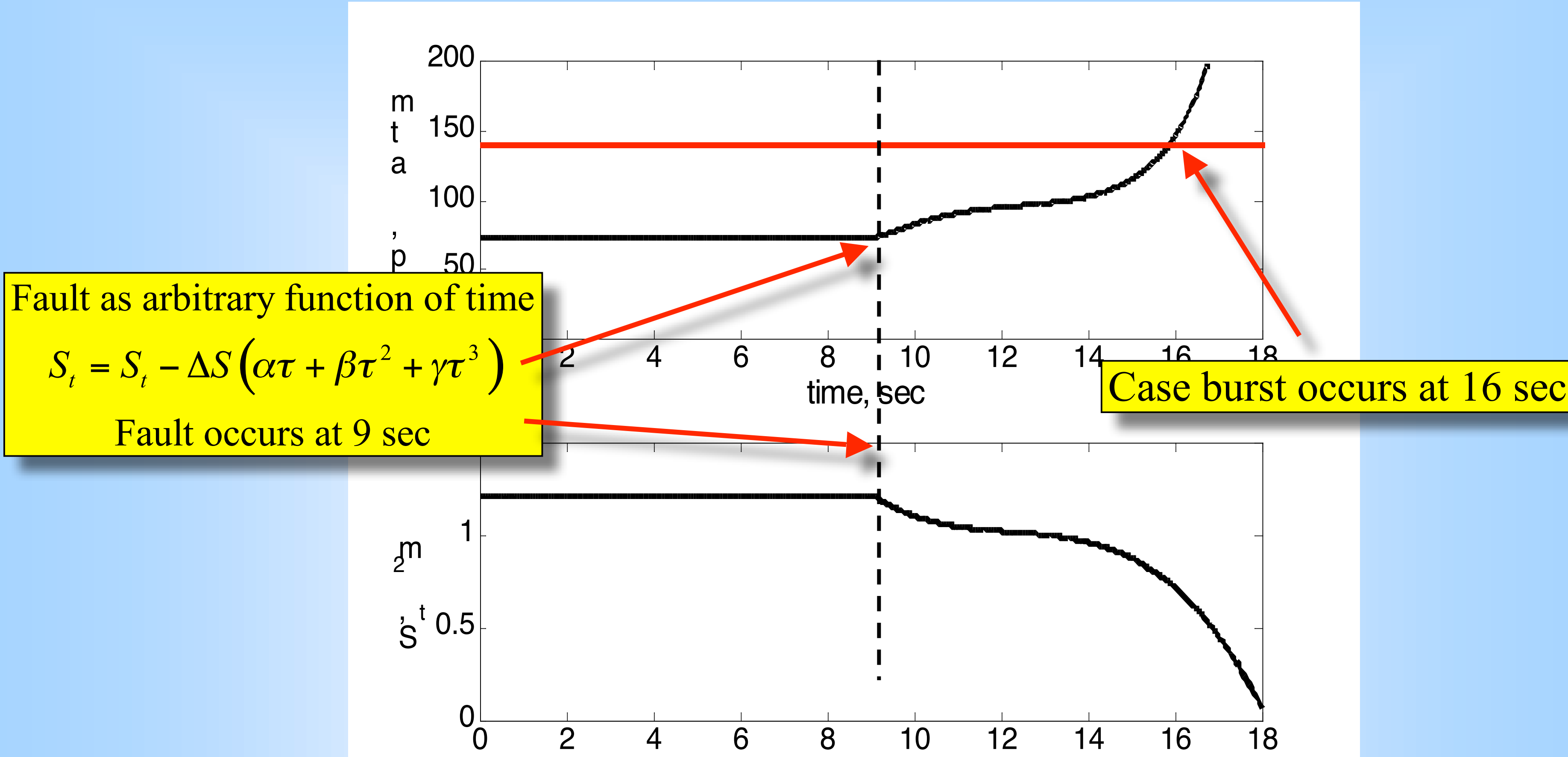
Inference of the hidden dynamical variables



In this figure we show preliminary results of the inference of the gas flow parameters in the case when the measurements of the pressure, p (a) and density, ρ (b) are corrupted by noise and the propellant radius, R (c) is not measured. The actual values of the pressure, density, and radius are shown by the red, blue and green solid lines correspondingly. The measured values of the pressure and density are shown by black juggling lines.

The inferred values of p, ρ, and R are shown by the red dotted lines.

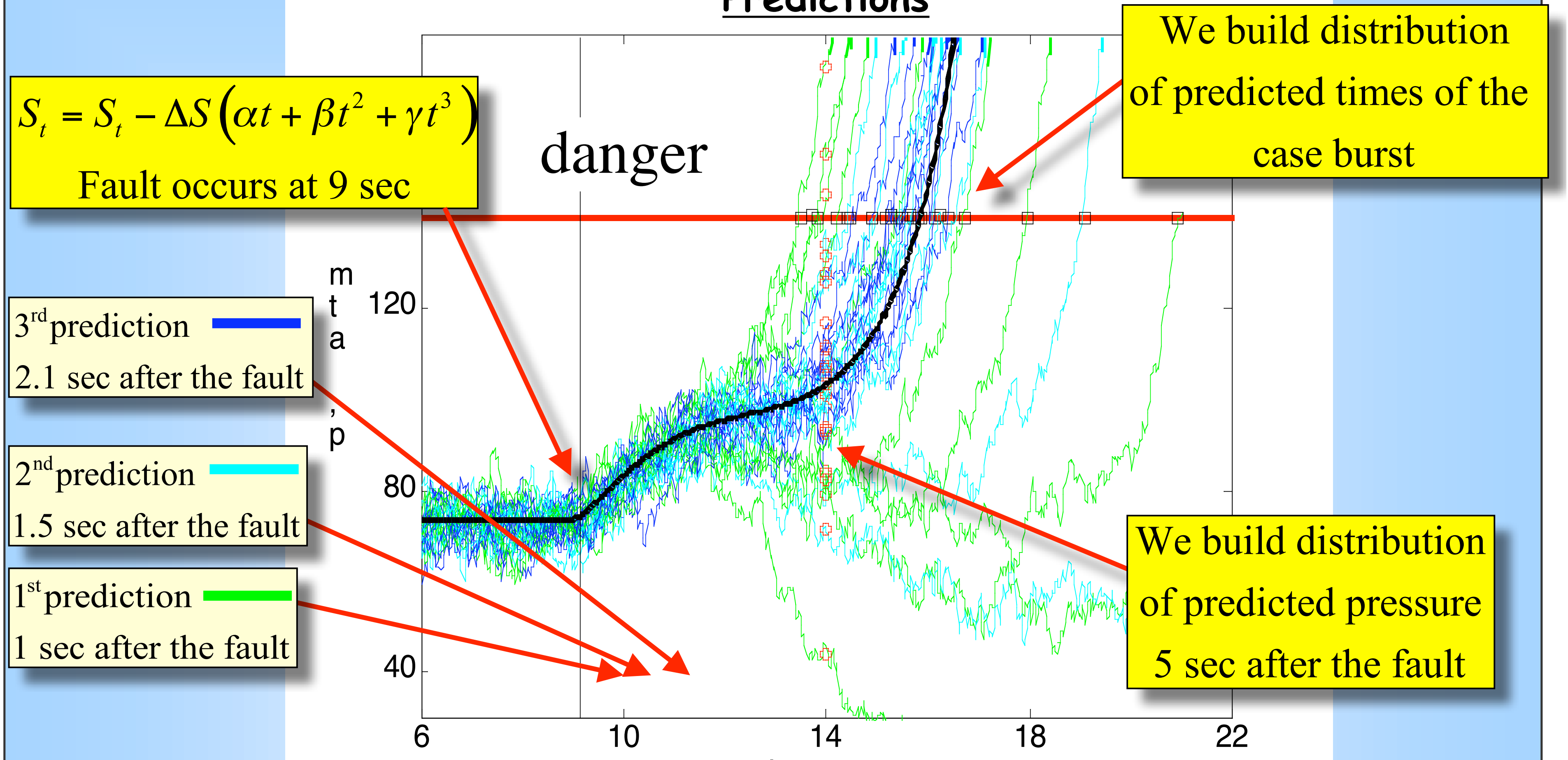
Prognostics for the neutral thrust curve



Consider fault which time evolution is arbitrary function of time with characteristic time scale of a few seconds.

As an example consider slowly varying area of the nozzle throat, which is described by the following function $f(\tau)= f(t-t_0)= a(t-t_0) +b(t-t_0)^2+c(t-t_0)^3$: a=-0.6, b=0.4, c=-0.1. Fault occurs at 9 sec. It leads to the case burst 7 sec later. The problem is how soon we can detect dangerous situation and predict the time of the case burst.

Predictions

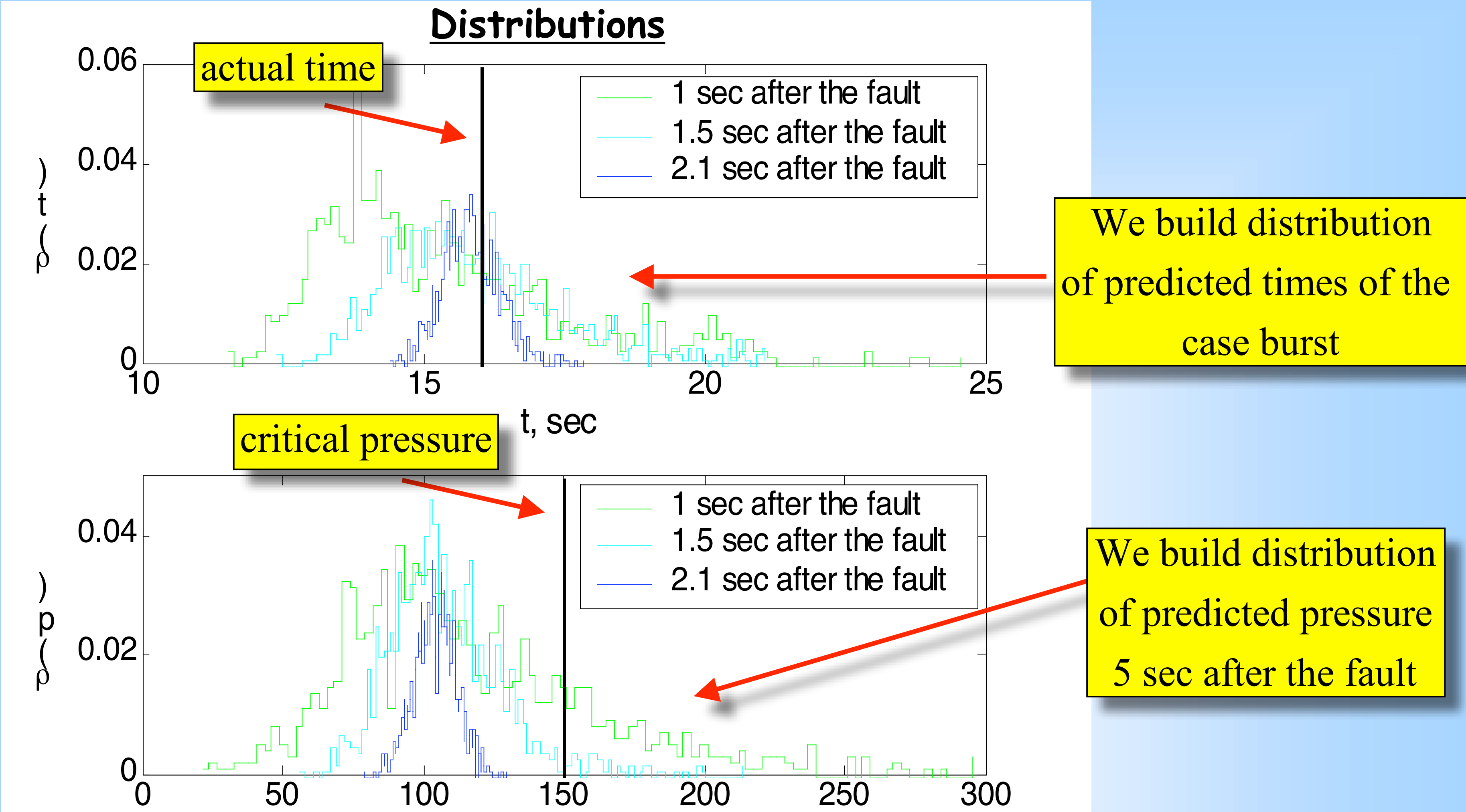


We infer parameters of the gas dynamics and use these parameters to predict time when pressure will cross the dangerous level. To estimate the accuracy of the prediction we also build the distribution of predicted pressure values which will occur 5 sec after the fault

Here presents the result of the prediction of the time of the case burst after

(i) 1 sec after the fault (greed lines); (ii) 1.5 sec after the fault (cyan lines); (iii) 2.1 sec after the fault (blue lines)

It is clear that prediction captures highly nonlinear behavior of the fault.



Here presents the resulting distributions of predicted time of the case burst (top figure) and the head pressure, which will occur 5 sec after the fault.

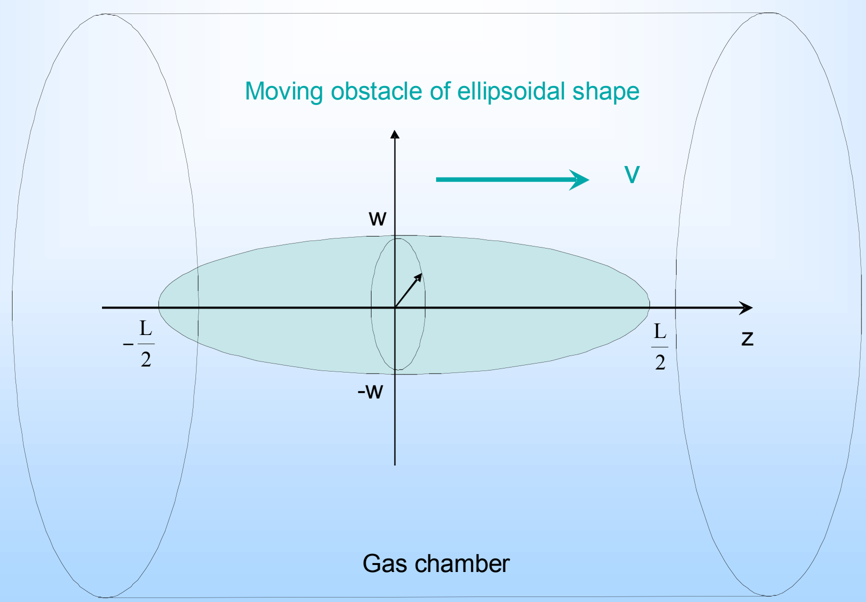
(i) 1 sec after the fault (red lines); (ii) 1.5 sec after the fault (black lines); (iii) 2.1 sec after the fault (blue lines)

It is clear that prediction becomes reasonable 1.5 sec after the fault. The accuracy of prediction can be father improved by adding measurements of the thrust.

Nonlinear dynamics of the bore clogging fault

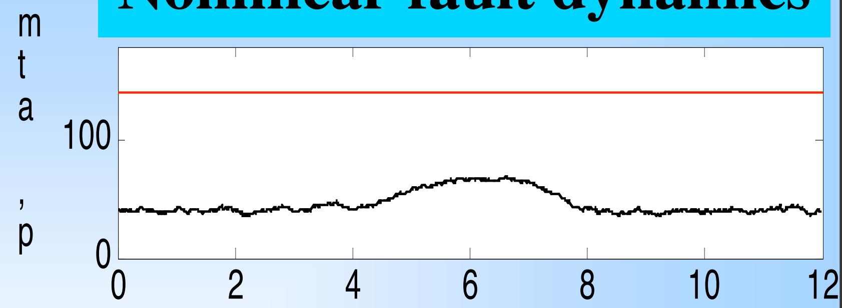
Consider an obstacle passing through the nozzle throat. E.g. it can represent a cloud of particles in the exhaust gases or a piece of the propellant. The effect of this nonlinear dynamical fault is an effective reduction of the nozzle area. Assume the fault have time evolution corresponding to an obstacle of elliptical shape passing through the nozzle throat. Then in the first approximation we can write

$$f(t) = \frac{S_0(0) - S(t)}{S_0(0)} = \begin{cases} 1 - \varepsilon_0 \left(1 - \left(\dfrac{t}{t_0/2} \right)^2 \right), & \text{for} \quad -t_0/2 \le t \le t_0/2 \\ 0, \quad \text{for} \quad |t| > t_0/2 \end{cases}$$



Low-dimensional dynamic model

Nonlinear fault dynamics



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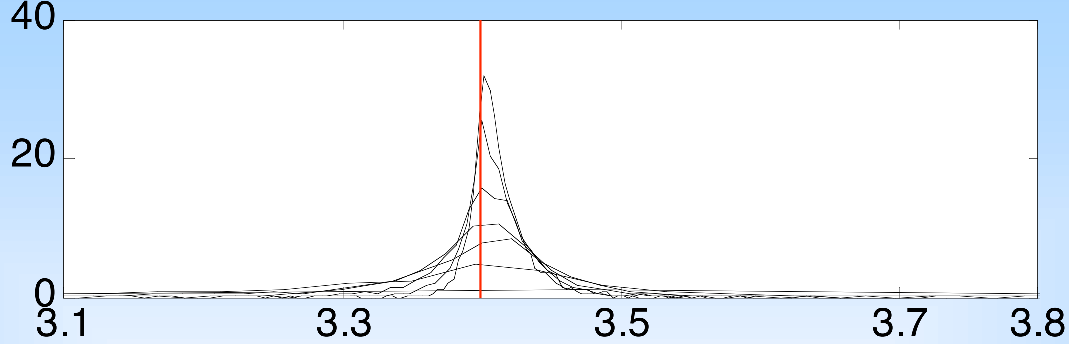
$$\begin{cases} \dfrac{d\rho}{dt} = -A(t)\sqrt{p\rho} + B(t)(\rho_p - \rho)p^n + \xi_1(t) \\ \dfrac{dp}{dt} = -\gamma A(t) p \sqrt{\dfrac{p}{\rho}} + B(t)(\gamma\rho_p - p)p^n + \xi_2(t) \\ \dfrac{dR}{dt} = r_b(p_0)p^n + \xi_3(t) \qquad p \to p(t)/p_0(0) \end{cases}$$

Inference of the model parameters



Convergence of the model parameters: 7 lines from the top to the bottom correspond to the results of the inference 1.5, 1.3, 1.1, 0.9, 0.7, 0.5, 0.3 sec after the fault. Red lines show actual values of the parameters.

Model space for nonlinear dynamical faults of the nozzle clogging

1. Base functions

$$\left[p^n;\ \frac{p^n}{R};\ \frac{p^{n+1}}{R};\ \frac{p^n\rho}{R};\ \frac{\rho\sqrt{p/\rho}}{R};\ \frac{p\sqrt{p/\rho}}{R};\ \frac{p\sqrt{p/\rho}}{R}t;\ \frac{p\sqrt{p/\rho}}{R}t^2;\ \frac{\rho\sqrt{p/\rho}}{R}t;\ \frac{\rho\sqrt{p/\rho}}{R}t^2 \right]$$

These base functions span part of the model space corresponding to the dynamics of model clogging fault

2. Model parameters

$$\begin{bmatrix} 0 & 2\gamma\rho_1 & -2 & 0 & 0 & -a_1\gamma p_0 & -a_1\gamma p_1 & -a_1\gamma p_2 & 0 & 0 \\ 0 & 2\gamma\rho_1 & 0 & -2 & -a_1 p_0 & 0 & 0 & 0 & -a_1 p_1 & -a_1 p_2 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

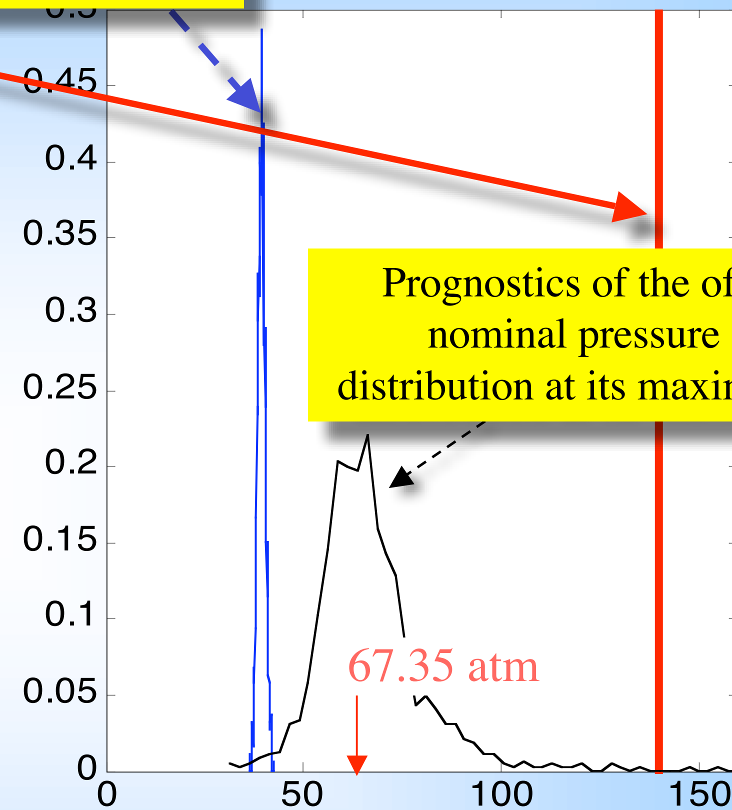
These model parameters correspond to the dynamics of model clogging fault

$$a_1 = \frac{c_0 \Gamma S_{th}^0}{\pi L r_{b0} R^*}$$

Continuous prognostic of the nonlinear dynamical fault

Critical level of case burst

Nominal distribution



Prognostics of the off-nominal pressure distribution at its maximum

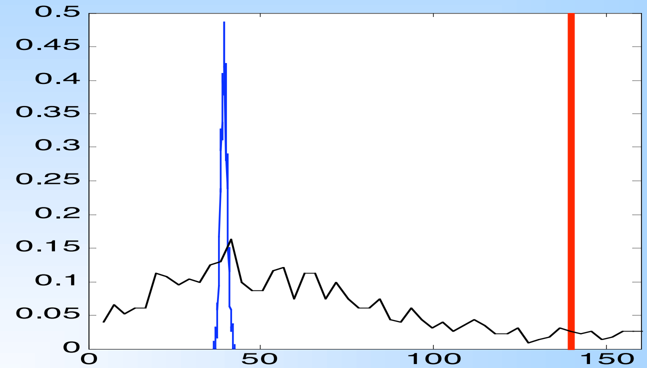
Standard threshold for mission cancellation due to the overpressure

Nonlinear dynamical inference predicts, however, that the fault is not critical and that pressure will not exceed 90 atm maxim and then will relax to the normal value

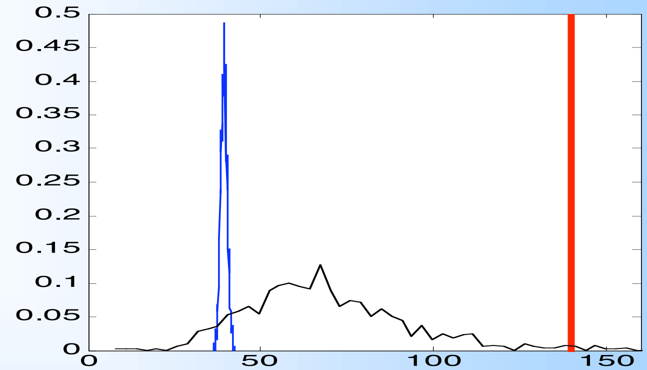
Continuous prognostic of the fault evolution (we predict the value of the maximum pressure for the fault)

Prediction time line

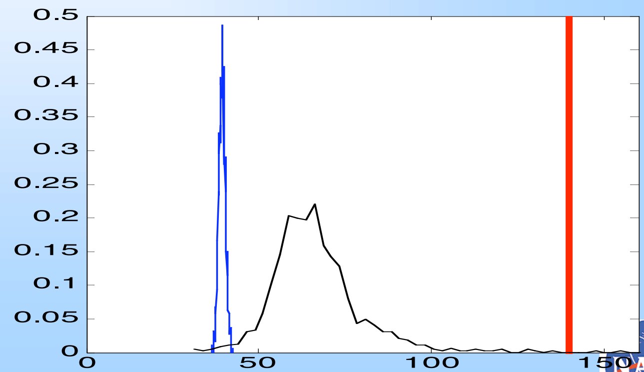
0.5 sec



0.9 sec



1.3 sec



Conclusions and Further Work

- **Low-dimensional model** gives accurate representation of the transient overpressure phenomena in a number of important fault modes
- Accordingly a **Bayesian framework** is introduced for prognostics and diagnostics of these failure modes
- **Case breach**
- **Bore chocking**
- **Gas leak**

DIRECT DETECTION of faults

Changing of an efficient cross section of the nozzle $S_*(t)$ can be found from measurement of the averaged head pressure $p_0(t)$, thrust $F_{th}(t)$ or pressure jump at a rapid chocking: $dS_*/d(t)<0$ at the nozzle blocking and $dS^*/d(t)>0$ at the burning case

(1) pressure variation

$$(2a)\quad \frac{\langle F_{th}(t)\rangle / S_{ex} + p_a}{\langle p_0(t)\rangle} = \left(1+\frac{\gamma-1}{2}M^2\right)^{-\frac{\gamma+1}{2(\gamma-1)}}\left[1+\gamma M^2\left(1+\frac{\gamma-1}{2}M^2\right)^{-\frac{1}{2}}\right]$$

$$(2b)\quad \frac{S_*(t)}{S_{*0}} = \frac{M}{\Gamma}\left(1+\frac{\gamma-1}{2}M^2\right)^{-\frac{\gamma+1}{2(\gamma-1)}}$$ – variations of thust and pressure

$$(3)\quad \frac{S_{*1}}{S_{*0}} = 1-\frac{\langle p_1(t)\rangle-\langle p_0(t)\rangle}{\langle p_0(t)\rangle}\frac{c_0}{\gamma u_L}$$ – pressure jump due to the shock wave

Time-resolved picture of fast nozzle blocking